On the Navier-Stokes flow with infinite energy and its applications

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It has been widely known that the existence of a global-in-time smooth solution for the three-dimensional Navier-Stokes initial value problem with non small initial data is an important open problem. In fact, this is one of seven millennium problems posed by the Clay Institute. To attack this problem various non blow-up criteria have been proposed. My talk today is concerned with non blow-up criteria by developing L^∞ theory of the Navier-Stokes equations.

Usually, the solvability of the Navier-Stokes initial value problem is discussed for initial data decaying at space infinity for example of finite kinetic energy. However, it is also important to consider nondecaying initial data so we survey L^∞ theory. We give an application to non blow-up criteria. We show that continuous vorticity alignment implies non blow-up without assuming that the kinetic energy is finite. This provides a different view point for a famous result of Constantin-Fefferman in 1993, where integral estimates play a key role.