

Manifolds with k -positive Ricci curvature

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Abstract: We begin this talk with a review (for the non-experts) of the curvature and Ricci curvature of a Riemannian manifold. The Ricci curvature at a point is a symmetric bilinear form. If, at each point, all its eigenvalues are positive we say that the Ricci curvature is positive. The trace of the Ricci curvature is the scalar curvature. Manifolds with positive Ricci curvature and manifolds with positive scalar curvature have been extensively studied over the last 40 years.

We say a Riemannian manifold has k -positive Ricci curvature if, at each point, the sum of the k smallest eigenvalues of the Ricci curvature is positive. Note that n -positive Ricci curvature is equivalent to positive scalar curvature and one-positive Ricci curvature is equivalent to positive Ricci curvature. We first describe some basic connect sum and surgery results for k -positive Ricci curvature that are direct generalizations of the well known results for positive scalar curvature (n -positive Ricci curvature). Using these results we construct examples that motivate questions and conjectures in the cases of 2-positive and $(n-1)$ -positive Ricci curvature. In particular, we conjecture:

“If M is a closed n -manifold that admits a metric with 2-positive Ricci curvature then the fundamental group of M is virtually free.”

We discuss approaches to proving this conjecture and some of the results that are now known that will be needed in its proof.