

Global Schrödinger maps in dimensions $d \geq 2$: small data in the critical Sobolev spaces

Ioan Bejenaru

Abstract. We consider the Schrödinger map initial-value problem

$$\begin{cases} \partial_t \phi = \phi \times \Delta \phi & \text{on } \mathbb{R}^d \times \mathbb{R}; \\ \phi(0) = \phi_0, \end{cases}$$

where $\phi : \mathbb{R}^d \times \mathbb{R} \rightarrow \mathbb{S}^2 \hookrightarrow \mathbb{R}^3$ is a smooth function. In all dimensions $d \geq 2$, we prove that the Schrödinger map initial-value problem admits a unique global smooth solution $\phi \in C(\mathbb{R} : H_Q^\infty)$, $Q \in \mathbb{S}^2$, provided that the data $\phi_0 \in H_Q^\infty$ is smooth and satisfies the smallness condition $\|\phi_0 - Q\|_{\dot{H}^{d/2}} \ll 1$.

The result is joint work with A. Ionescu, C. Kenig and D. Tataru.