A Lie Groups Approach to the Analysis of Kolmogorov-Fokker-Planck Equations

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Part I. Classical Kolmogorov and Degenerate Ornstein-Uhlenbeck Equations.

Abstract. *We first survey a series of papers dealing with classes of linear second order PDE's of Kolmogorov type, arising in diffusion processes and in Finance. The common feature of these works is the use of a Lie groups approach to the regularity theory and to a-priori estimates. We then show how this approach also works in studying a class of degenerate Ornstein-Uhlenbeck operators arising. e.g. as generators of Markovsemigroups associated to stochastic differential equations.*

Part II. Lie Groups Constructed from Hormander Operators. Fundamental Solutions and Applications to Kolmogorov-Fokker-Planck Equations

Abstract. *Let L be a Hormander-type operator, sum of squares of vector fields+drift. We show sufficient conditions on the vector fields and on the drift term for the existence of a Lie group structure G such that L is left invariant on G. We also investigate the existence of a global fundamental solution for L, providing results that ensure a suitable left invariance property. We will show several examples of operators to which our results apply: some are new, some appear in recent literature usually quoted as Kolmogorov-Fokker-Planck operators. Our examples arise in several theoretical and applied settings, such as diffusion theory, computer and human vision, phase noise Fokker-Planck equations, curvature Brownian motion.*