An Introduction to Correlation in the Financial Markets

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The Basic Idea of Correlation in Finance

- Mathematically, if X and Y are two random variables on the same space, $\mathrm{corr}(X,Y) \equiv E\left[(X-\bar{X})(Y-\bar{Y})\right] / \sigma(X)\,\sigma(Y)$
- In financial applications, we typically consider the case where X, Y represent the <u>changes</u> in two different asset prices between specified times T_0 and T_1
- $\mathrm{corr}(X,Y)$ takes values between -1 and 1, where higher values represent a higher tendency for price changes in the same direction
- We will examine some important financial products where correlation plays a vital role

Significance of the LIBOR Curve

- The LIBOR curve reflects the interest rates paid by the most creditworthy banks for term borrowing of USD
- LIBOR rates are higher than US Treasury rates because the US government is viewed as the most creditworthy borrower
- For USD fixed-income arbitrage analysis the LIBOR rate curve is very important because it reflects the cost of money for the market makers and their associated trading desks

200<GO>to view this page in Launchpad Index **BBAM**

PAGE MAY NOT UPDATE AT 11AM LONDON TIME DUE TO DELAYS IN REUTERS CALCULATIONS

10:44 28AUG09 THOMSON REUTERS BBA LIBOR RATES Page 1 of 4 LIBOR01

BRITISH BANKERS ASSOCIATION INTEREST SETTLEMENT RATES Alternative to <3750>

[28/08/09] RATES AT 11:00 LONDON TIME 28/08/2009 Disclaimer <LIBORDISC>

BBA Guide <BBAMENU>

	USD	GBP	CAD	EUR	JPY	EUR 365
O/N	0.22938	0.50750	0.27000	0.26875 SN	0.12125	0.27248
1WK	0.24875	0.52000	0.29000	0.30500	0.14000	0.30924
2WK	0.25438	0.52563	0.29833	0.33500	0.15625	0.33965
1MO	0.25875	0.53375	0.30167	0.42750	0.20000	0.43344
2MO	0.28000	0.56625	0.41667	0.63000	0.30063	0.63875
3MO	0.34750	0.69000	0.55667	0.79563	0.39063	0.80668
4MO	0.52000	0.76875	0.71333	0.89750	0.48125	0.90997
5MO	0.66750	0.83500	0.87000	0.98750	0.54250	1.00122
6MO	0.75500	0.90688	1.05500	1.07688	0.59875	1.09184
7MO	0.86750	0.96250	1.12833	1.11500	0.65750	1.13049
8MO	0.97188	1.01375	1.19000	1.16500	0.70250	1.18118
9MO	1.06500	1.07000	1.25667	1.20313	0.73750	1.21984
10MO	1.15125	1.12375	1.35333	1.23813	0.76125	1.25533
11MO	1.23750	1.17625	1.45333	1.26875	0.78438	1.28637
12MO	1.33000	1.22625	1.55167	1.30000	0.81500	1.31806

<HELP> for explanation. Index **NWAX**

282 - Save a copy to your NW list

Ticker	Bid	Ask	Mid	Chng
US Semi 30/360				
2) 1 YR	.6320	.6520	.6420	-.0170
3) 2 YR	1.3750	1.3800	1.3775	-.0120
4) 3 YR	2.0190	2.0240	2.0215	-.0035
5) 4 YR	2.4790	2.4830	2.4810	-.0150
6) 5 YR	2.8250	2.8310	2.8270	-.0170
7) 6 YR	3.0920	3.0960	3.0920	-.0175
8) 7 YR	3.2980	3.3030	3.3000	-.0120
9) 8 YR	3.4550	3.4580	3.4565	-.0080
10) 9 YR	3.5760	3.5810	3.5780	-.0080
11) 10 YR	3.6740	3.6800	3.6775	-.0075
12) 15 YR	3.9610	3.9680	3.9645	-.0090
13) 20 YR	4.0530	4.0580	4.0555	-.0105
14) 25 YR	4.0920	4.0960	4.0935	-.0165
15) 30 YR	4.1150	4.1215	4.1178	-.0132

Change on day

IYC4 I52<GO>

Change on Month

IYC6 I52<GO>

Pricing based on XDF<GO> settings

Ticker	Bid	Ask	Mid	Chng
US SPREADS				
22) 1 YR	20.59	22.59	21.54	-.11
23) 2 YR	35.88	36.38	36.13	+1.25
24) 3 YR	48.13	48.75	48.44	+.69
25) 4 YR	48.63	48.88	48.75	+.19
26) 5 YR	37.25	37.75	37.50	+.50
27) 6 YR	32.00	32.38	32.13	-.56
28) 7 YR	20.63	21.00	20.76	-1.63
29) 8 YR	24.50	25.00	24.69	-1.00
30) 9 YR	24.75	25.25	24.94	-.75
31) 10 YR	22.75	23.25	23.00	-.25
32) 15 YR	32.38	33.13	32.76	-.13
33) 20 YR	22.63	23.38	23.00	+.25
34) 25 YR	7.63	8.25	7.94	+.13
35) 30 YR	-9.13	-7.88	-8.44	+.31

Change on day

IYC4 I48<GO>

Change on Month

For US Govt Yield Curve, type {IYC1 I25 <GO>}

For US swap Curve, type {IYC1 I52 <GO>}

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000
H148-842-1 28-Aug-09 13:52:52

Vanilla USD Interest-Rate Swaps

- Two counterparties, one pays fixed and the other pays float
- Floating payments tied to three-month LIBOR (actual/360)
  - payments made at three-month intervals
  - rate used is set three months + 2 days prior to payment date (rates are set on reset dates)
  - short stub (if any) computed using linear interpolation on money market rates
- Fixed payment made semiannually (30/360)
- Notional principal stays fixed throughout, no exchange of principal at maturity
- A <u>par swap</u> has no initial value (value zero at inception)

In the standard course of business, financial institutions often find it advantageous to trade assets they naturally accumulate for other types of assets to reduce risk

<u>Example</u>: Mortgage GSEs

How Mortgage GSEs Work

Fannie and Freddie securitize mortgages bought from issuers. They

(1) Hold mortgages in their own portfolio

(2) Issue debt securities

(3) Resell whole mortgages

(4) Create MBS which they sell

Key problems with mortgages and many MBS derivatives:

$$\left.\begin{array}{l}(1)\ \text{Rates} \downarrow \Rightarrow \text{Refinancing} \uparrow \Rightarrow \text{duration} \downarrow \\ (2)\ \text{Rates} \uparrow \Rightarrow \text{Refinancing} \downarrow \Rightarrow \text{duration} \uparrow\end{array}\right\}\ \text{Negative Convexity}$$

Hedging Mortgage Portfolios

Negative convexity has two implications for mortgage portfolios:

(1) It is expensive if you do not adjust your hedge, so

(2) You need to adjust your hedge regularly if you want to stay alive

Traditionally, GSE's get some protection by issuing callable debt

More recently, they are hedging portfolios more and more with interest-rate swaps and swaptions

Hedging Against Rate Increase with a Pay-Fixed Swap

(1) Fannie Mae auctions 3-month benchmark bills weekly

(2) By entering into a payer swap, Fannie converts short-term floating-rate debt into long-term fixed rate debt

(3) Note that the 3-month floating LIBOR payments received on the swap will pay the interest on the benchmark bills, protecting them against increases in rates at the short end

(4) Net effect: like issuing a long-term benchmark note

(5) Benefit: <u>easy to unwind the swap if rate outlook changes</u>

From 2008 Annual Report

Freddie Mac

Quarterly mark-to-market items

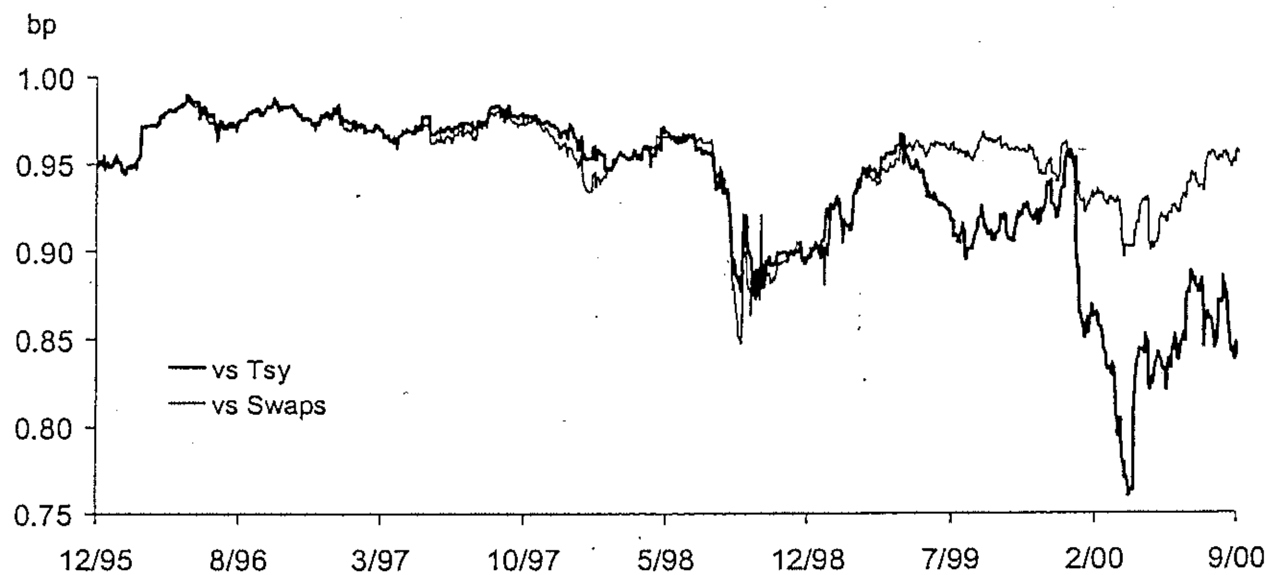
	($ Millions)	4Q 2007	3Q 2008	4Q 2008	4Q 2008 vs 3Q 2008
1	Losses on derivatives excluding accrual of periodic settlements and swaps denominated in foreign-currency	($2,253)	($1,448)	($11,806)	($10,358)
2	Mark-to-market on guarantee asset	(843)	(1,291)	(4,721)	(3,430)
3	Gains (losses) on trading securities	203	(932)	3,195	4,127
4	**Subtotal interest-rate related items (Lines 1, 2, 3)**	**(2,893)**	**(3,671)**	**(13,332)**	**(9,661)**
5	Losses on certain credit guarantees	(1,269)	(2)	-	2
6	Losses on loans purchased	(736)	(252)	(1,211)	(959)
7	**Subtotal credit-related items (Lines 5, 6)**	**(2,005)**	**(254)**	**(1,211)**	**(957)**
8	AFS security impairments	(13)	(9,106)	(7,465)	1,641
9	Other	(46)	103	76	(27)
10	**Total mark-to-market items**	**($4,957)**	**($12,928)**	**($21,932)**	**($9,004)**

- **Line 1: Declines in swap interest rates resulted in fair value losses in the net pay-fixed swaps portfolio, partially offset by gains on swaptions.**
- **Line 2: Declines in interest rates resulted in mark-to-market losses on the company's guarantee asset.**
- **Line 3: Net gains on trading securities reflect mark-to-market gains from declining interest rates and mark-to-market losses due to spread widening.**
- **Line 6: An increase in volume and declines in loan prices related to modified loans purchased from PC pools drove increased losses during 4Q 2008.**
- **Line 8: The company recognized additional impairment losses on AFS securities in 4Q 2008 due to sustained deterioration in the performance of the underlying collateral.**

Historically

Swaps are Better than Treasuries for Hedging Mortgage Rates

Figure 7. **Mortgage Rates Are Less Correlated with Treasuries**
Rolling 30-day Correlation of Change in Mortgage Rates with Change in 10-year Off-the-Run Treasury/Swap Yields



What Went Wrong with Fannie & Freddie?

- Collapse of housing & mortgage-backed securities markets in Fall 2008 caused unbearable losses
- Fannie & Freddie were put into conservatorship by Federal government so that US housing market could survive
- Under conservatorship many policies and practices have changed
- Swap hedges are now being marked to market
  $\Rightarrow$ \$ 11^{+} Billion losses in swap book in 4Q 2008 when interest rates dropped

The spread between 30 Yr Interest-rate swaps and the yield on the 30 Yr US Treasury Bond went negative in Nov 08

The result is an arbitrage opportunity

when the price spread between related pairs of assets move out of line, speculators trade the spread to make money

<u>Example</u>: Swap spread trade 3/30/09
(30 Yr US Treasury vs. Interest Rate Swap)

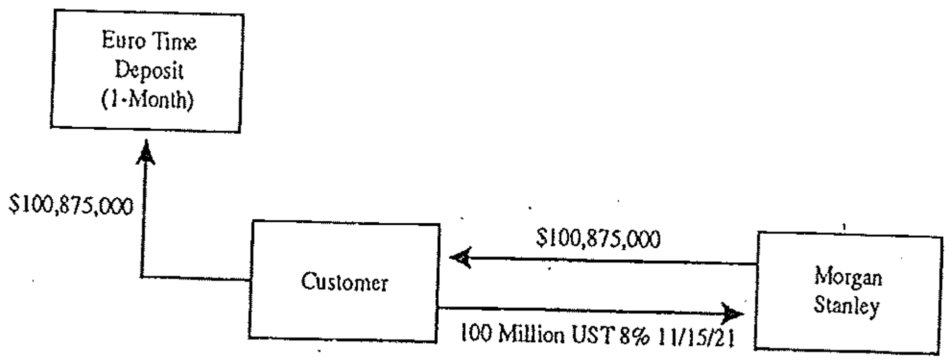
Exhibit 7

Yield Pickup Arbitrage: An Example

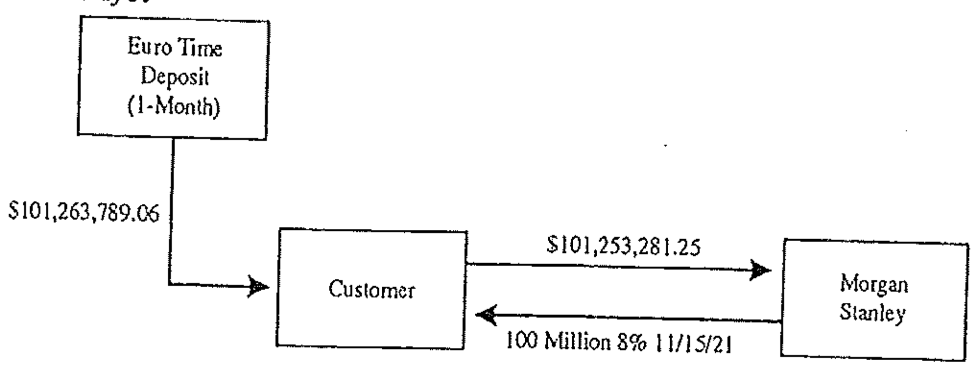
Assumptions

- An unleveraged investor owns (fully paid for) $100 million par value of the 8% 11/15/21.
- Dealer bids 4 1/2% for these securities for one month.
- Investor reinvests cash in Euro Time Deposits (or any other higher yielding instrument) at a rate of 4 5/8% for 1-month.
- Customer earns a spread of 12.5 basis points for the period.
- Market price of bond = 101
- Accrued interest = 0.125
- Haircut = 0.25

Day 1



Day 30



Calculations

- All-in Price of Bond = 101 + 0.125 - 0.25 = 100 7/8
- Principal = $100 million x 100 7/8 = $100,875,000
- Interest Expense @ 4 1/2% = $\$100{,}875{,}000 \times 0.045 \times \frac{30}{360} = \$378{,}281.25$
- Interest Income from Euro Time Deposits @ 4 5/8% = $\$100{,}875{,}000 \times 0.04625 \times \frac{30}{360} = \$388{,}789.06$
- Profit on Trade = $10,507.81

Note: This trade is assumed to transpire between coupon payments.
Source: Morgan Stanley

Complete Ticket for Swap Spread Trade 3/30/09

We Buy \$100mm T 3.5 02/15/39 at 98-31^{+}, yield 3.56%

Term Repo \$100mm T 3.5 02/15/39 to 11/15/09 at 0.35%

Pay fixed on swap, \$93.52mm notional, 3.21% fixed rate

swap starts 4/01/09, ending 02/15/39

short stub rate 1.08529% through 5/15/09

Hedge: Buy 93 EDK9 at 98.845

Buy 93 EDQ9 at 98.870

note correction, Aug <u>not</u> Sep

Swap Spread Mark to Market 8/20/09

UST 3.5 02/15/39 Position

Date		Quantity	Clean Price	Clean Cost	Accrued Interest	Total Value
3/30/2009	$	100,000,000	98.984375	$98,984,375	$425,414	$99,409,789
8/20/2009	$	100,000,000	87.265625	$87,265,625	$57,065	$87,322,690

UST MTM P&L	-$12,087,099

Eurodollar Futures Hedges

3/30/2009	93	98.845
5/18/2009	93	99.215
3/30/2009	93	98.87
8/17/2009	93	99.5687

EDK9 P&L	$86,025
EDQ9 P&L	$162,448

Interest-Rate Swap Position

3/30/2009	-$23,084
8/20/2009	$14,691,879

IR Swap P&L	$14,714,963

Cash Flows and Accrued Repo

Date			
5/15/2009		IRS Receive Float	$80,703
8/17/2009	Short-Term	IRS Receive Float	$215,653
8/17/2009		Accrued Repo Cost	-$130,475
8/17/2009		Remaining Repo MTM	-$24,852
8/17/2009	Long-Term	IRS Pay fixed	-$1,134,086
8/17/2009		Bond Coupon	$1,750,000

Total P&L 8/20/2009

Net P&L, no interest $3,633,279

Swap MTM P&L Reconciliation

	DV01	Par Swap Rate	Change in Rate	Average DV01	Rough Swap P&L
3/30/2009	-$180,222	3.21			
8/20/2009	-$143,560	4.12	91	-$161,891	$14,732,081

Bond MTM P&L Reconciliation

	Modified Duration	Yield	Change in Yield	Average DV01	Rough Bond P&L
3/30/2009	$182,960	3.56			
8/20/2009	$174,030	4.263	70.3	$178,495	-$12,548,199

	P&L from Spread Change	$2,627,864
	Linear P&L from Spread	$2,183,883
	P&L from Cash Flows	$756,942
3-month ED outperformed term repo, so	P&L from repo hedge	$248,473

When dealers find themselves stuck with large levels of correlated portfolio risk that they cannot easily hedge, they sometimes offload some of this risk in the form of a derivative product.

<u>Examples</u>: Correlation Swap
Variance Swap Dispersion

Definition of Realized Volatility

Let $S_0, S_1, \ldots, S_N$ be closing prices of a security on $N+1$ consecutive trading days

The j^{th} return $r_j \equiv \log\left(S_j / S_{j-1} \right)$ for $j = 1, \ldots, N$

$$\left.\begin{array}{ll} \text{The mean drift} & \mu = \frac{1}{N} \sum_{i=1}^{N} r_i \\ \text{The variance} & V = \frac{1}{N} \sum_{i=1}^{N} (r_i - \mu)^2 \end{array}\right\} \text{ refer to daily changes}$$

Industry standard uses annualized estimate of variance:

$$\boxed{\text{Realized Volatility } \sigma = \sqrt{252\, V}}$$

Definition of Realized Covariance and Correlation

Given a set of M different securities, let $r_{ij} = j^{th}$ return of i^{th} security

$$\underline{X} = \begin{bmatrix} r_{11} & \cdots & r_{1N} \\ \vdots & & \vdots \\ r_{M1} & \cdots & r_{MN} \end{bmatrix}$$ matrix of daily returns, $\mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_M \end{bmatrix}$

Put $$\underline{Y} = \begin{bmatrix} r_{11}-\mu_1 & \cdots & r_{1N}-\mu_1 \\ \vdots & & \vdots \\ r_{M1}-\mu_M & \cdots & r_{MN}-\mu_M \end{bmatrix},$$ then the <u>Realized</u> <u>Covariance</u> is

$$\Sigma = \frac{252}{N} \underline{Y}\,\underline{Y}^T,$$ note that $\Sigma_{ii} = \sigma_i^2$ so put $D = \begin{pmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_M \end{pmatrix}$

Then <u>Correlation</u> $$\rho = D^{-1} \Sigma D^{-1} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1M} \\ \vdots & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ \rho_{1M} & \cdots & \cdots & 1 \end{pmatrix}$$

Observations Concerning Covariance, Correlation

(1) When securities are not linearly dependent, $\Sigma > 0$ $\begin{pmatrix}\text{positive} \\ \text{definite}\end{pmatrix}$

(2) So $\rho = D^{-1} \Sigma D^{-1}$ is also positive definite, implying constraints on $\{\rho_{ij}\}$

(3) In particular, $$(1, \ldots, 1) \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1m} \\ \rho_{12} & 1 & & \\ \vdots & & \ddots & \\ \rho_{1m} & & & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = m + 2 \sum_{i<j} \rho_{ij} > 0$$

So $$\rho_{AVG} \equiv \frac{2}{m(m-1)} \sum_{i<j} \rho_{ij} > -\frac{1}{m-1}$$

(4) When m is large (say 30 or more) the <u>average</u> correlation between pairs of securities cannot be much smaller than 0.

The Correlation Swap Derivative Security

Pick a basket of M securities, pick a time window of (N+1) consecutive trading days starting tomorrow (Monday)

Parties A and B agree upon a contract: after N+1 days

A pays B the amount $\max(0, \rho_{AVG} - \rho_{STRIKE}) * \text{Notional}$

B pays A the amount $\max(0, \rho_{STRIKE} - \rho_{AVG}) * \text{Notional}$

So only one party pays (the other receives) an amount proportional to the Notional amount <u>and</u> the difference between ρ_{AVG} and ρ_{STRIKE}

Basic Variance Swaps

Again, let $r_i = i^{th}$ return of a security, $i = 1, \dots, N$

Abuse notation: __Realized__ __Variance__ $V = \frac{252}{N} \sum_{i=1}^{N} r_i^2$

Parties A and B agree upon a contract: after $N+1$ days

A pays B the amount $\max(0, V - V_{STRIKE}) * \text{Notional}$

B pays A the amount $\max(0, V_{STRIKE} - V) * \text{Notional}$

So only one party pays (the other receives) an amount proportional to the Notional amount __and__ the difference $V - V_{STRIKE}$

Variance Dispersion Trade for an Index

As a way to trade correlation, put on

(1) Long positions of variance notionals $\{N_i\}$ of single names

(2) Short variance notional of N of index variance swap

$$\text{Net Vega} = \begin{pmatrix} \frac{\partial \text{Value}}{\partial \bar{\sigma}_1} \\ \vdots \\ \frac{\partial \text{Value}}{\partial \bar{\sigma}_m} \end{pmatrix} = 2 \begin{bmatrix} N_1 - N\lambda_1^2 & -N\lambda_1\lambda_2\bar{\rho}_{12} & \cdots & -N\lambda_1\lambda_m\bar{\rho}_{1m} \\ -N\lambda_1\lambda_2\bar{\rho}_{12} & N_2 - N\lambda_2^2 & & \\ \vdots & \vdots & & \\ -N\lambda_1\lambda_m\bar{\rho}_{1m} & \cdots & & N_m - N\lambda_m^2 \end{bmatrix} \begin{bmatrix} \bar{\sigma}_1 \\ \vdots \\ \bar{\sigma}_m \end{bmatrix}$$

$$\text{Vega neutral} \iff N_i = N\lambda_i \left(\sum_{j=1}^{m} \lambda_j \bar{\rho}_{ij} \frac{\bar{\sigma}_j}{\bar{\sigma}_i} \right) \longleftarrow$$

makes position insensitive to changes in $\bar{\sigma}_j$

Note: If $\bar{\rho}_{ij}$ increase the Value decreases, so we are short correlation

Example SPX Varswap Dispersion

Other Examples of Correlation Trades

- Commodity price spreads
- CDO's and CMO's
- 10-Yr Treasury futures basis trade
- Best-of puts, worst-of calls
- Yield-curve arbitrage - flatteners, steepeners & butterflies
- Dispersion options

Summary

- Correlation trades play an important role in financial markets
- Correlation models are vital for assessing risk in portfolios, yet the models are not very reliable
- Despite theoretical difficulties, there remain correlation trades in practice that seem to offer favorable reward vs. risk
- Correlation modeling is a fertile area for new mathematical research

An Introduction to Correlation in the Financial Markets

Blaise Morton, Whitebox Advisors

Abstract:

We cannot reliably predict how the price of a single asset will fluctuate over time; but sometimes there are predictable patterns in the relative movements of related asset prices. The tendency of relative asset prices to move in predictable ways is called correlation. While we do not have a general theory of correlation, there are some basic examples that are important and easy to understand. In this presentation I will present some of these basic examples as motivation for some of the general modeling and analysis problems associated with correlation.

The simplest example of a correlation position is a spread trade between two assets. Basis trading, swap trading and relative-value trading are strategies for reducing risk (hedging) when the spread is fairly priced, or for making money (speculating) when the price spread between pairs of assets move out of line.

In the standard course of business, financial institutions often find it advantageous to trade some of the assets they naturally accumulate for other types of assets in order to diversify their risk. Correlation models are necessary to quantify the benefits of these hedging and risk management activities.

When dealers find themselves stuck with large levels of correlated portfolio risk that they cannot easily hedge (e.g. catastrophic risk), they sometimes repackage and offload some of this risk in the form of a derivative product. The counterparty (typically a hedge fund) will want an attractive expected payoff for taking the other side of the trade. One example of such a product is a correlation swap. On account of financial engineering inventions like these, correlation trading has become a growing market with its own set of specialized, evolving products.