Placement Test 2a (Topology)

You may cite standard theorems below, without proof. These include Tychonoff's theorem, the Baire Category Theorem and the Urysohn metrization theorem.

1. Define "topological space".

2. Let X and Y be topological spaces. Define what it means for a function $f : X \to Y$ to be "continuous".

3. Let X and Y be topological spaces. Define what it means for a function $f : X \to Y$ to be a "homeomorphism".

4. Give an example of a bijective continuous map that is not a homeomorphism.

5. Let X be a topological space and let S be a subset of X. Define the "relative topology" on S, inherited from X.

6. Define a "base" of a topology. Define a "subbase" of a topology.

7. Let X be a topological space. Define what it means for X to be "compact".

8. Let X be a topological space and let $\mathcal{V}$ be a subbase for the topology of X. Assume that every $\mathcal{V}$-open cover of X has a finite subcover. (That is, assume, for any $\mathcal{U} \subseteq \mathcal{V}$, that: $\cup\mathcal{U} = X$ implies that there is a finite $\mathcal{F} \subseteq \mathcal{U}$ such that $\cup\mathcal{F} = X$.) Show that X is compact.