Placement Test 2b (Topology)

You may cite standard theorems below, without proof. These include Tychonoff's theorem, the Baire Category Theorem and the Urysohn metrization theorem.

9. Let X be a topological space. Define what it means for X to be "Hausdorff".

10. Let X be a topological space. Define what it means for X to be "connected". Define what it means for X to be "discrete".

Note: A subset of a topological space is connected iff it is connected in the relative topology.

11. Give an example of a Hausdorff topological space which is not discrete, but such that no connected subset has more than one point. (You do not need to prove that your answer is correct.)

12. Let X and Y be topological spaces. Define the topological space $X \times Y$.

13. Let X be a Hausdorff topological space. Let $C_1, C_2, \ldots$ be a sequence of closed subsets of X. Assume, for all integers $i \geq 1$, that the interior of C_i is empty. Is it possible that $X = \bigcup_{i=1}^{\infty} C_i$? Explain your answer. (*Warning:* We are making no assumptions about X except that X is Hausdorff.)

14. Let X be a topological space.

(a) Define what it means for X to be "separable".

(b) Define what it means for X to be "metrizable".

(c) Define what it means for X to be "second countable".

(d) Assume that all one-point subsets of X are closed. Define what it means for X to be "normal".

15. Show that every separable, metrizable topological space is second countable.