Number theory and harmonic analysis on non-Euclidean spaces

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In 1859 Riemann observed that conjectural facts about the asymptotic distribution of prime numbers are exactly equivalent to questions about the location of zeros of the zeta function. He speculated that all the interesting zeros lie on the line where the real part of the complex variable is 1/2. This is the notorious Riemann Hypothesis. Since that time, many more such connections have been observed, but with very little progress in understanding either the number-theoretic or the analytical side. Regarding Riemann's original example, it is very difficult to directly address questions about primes with any precision, and, on the other hand, it is almost as difficult to prove non-trivial things about zeta functions. As usual in mathematics, we want to find zeta functions arising in some -other-context, that provides-other-meaning to them, giving new information. Harmonic analysis on certain special non-Euclidean spaces gives a bit of new insight. This is part of the theory of modular or automorphic forms. I will gives some historical and mathematical background, and sketch some recent progress.