

MULTIPLE SCALING METHODS
IN CHEMICAL REACTION NETWORKS

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In this talk, we construct a general method of multiscale approximations in chemical reaction networks. A continuous time Markov jump process is used to describe the state of the chemical reactions.

In general chemical reactions, the chemical species numbers and the chemical reaction rate constants will have various orders of magnitude. Two different scaling exponents are used to normalize the numbers of molecules of the chemical species and to scale the chemical reaction rate constants. Applying a time change, we have different time scales for the limiting processes in the reduced subsystems. The law of large numbers for Poisson processes is applied to approximate non-integer-valued processes. In each time scale, the slow processes act as constant and the fast processes are averaged out. Then the limit of the intermediate processes is obtained in terms of the averaged fast processes and the initial values of the slow processes.

The general method of multiscale approximations is applied to a model of heat shock response. We analyze the system and obtain limiting processes in each simplified subsystem, which approximates the normalized processes in the system with different time scales. Error estimates of the difference between the normalized processes and the limiting processes are given. Simulation results are given to compare the evolution of the processes in the system and the evolution of the approximated processes using the limiting processes in each simplified subsystem. Applying the martingale central limit theorem and using the averaging, we obtain a central limit theorem for deviation of the normalized processes from their limiting processes in the heat

shock response model.