FM 5001 Fall 2007, Midterm #1
Handout date: Wednesday 17 October 2007

1. Definitions: Complete the following sentences.

a. (5 pts.) Let S be a subset of $\mathbb{R}^n$. A point $p \in \mathbb{R}^n$ is a **boundary point** of S if ...

b. (5 pts.) A subset S of $\mathbb{R}^n$ is a **subspace** if ...

c. (5 pts.) The **span** of a subset A of $\mathbb{R}^n$ is ...

2. True or False. (No partial credit.)

a. (5 pts.) Any spanning set is linearly independent.

b. (5 pts.) Let $\mathcal{B}$ be a basis of a subspace of $\mathbb{R}^n$. If a linear combination of the elements of $\mathcal{B}$ is equal to zero, then all the coefficients are equal to zero.

c. (5 pts.) If the kernel of a linear transformation $L : \mathbb{R}^n \to \mathbb{R}^k$ is $\{0\}$, then $L : \mathbb{R}^n \to \mathbb{R}^k$ is a bijection.

d. (5 pts.) Every conjugate of a nilpotent matrix is nilpotent

e. (5 pts.) Every compact set is closed.

3. Computations. (Answers typically must be exactly correct. No partial credit, except in unusual situations.)

a. (5 pts.) Eliminate the linear term in $y = -(x^2/2) + 4x - 2$.

b. (5 pts.) Compute $\displaystyle\int_{-\infty}^{\infty} x^2 e^{-x^2/2}\,dx$.

c. (5 pts.) Compute $\displaystyle\frac{d}{dt}[(\tan t)^t]$.

d. (5 pts.) Find $P + Q$, where

$$P := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad Q := \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}.$$

e. (5 pts.) Find $P \oplus Q$, where

$$P := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad Q := \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}.$$

f. (5 pts.) Find AB, where

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B := \begin{bmatrix} 5 \\ 6 \end{bmatrix}.$$

g. (5 pts.) Compute e^X, given the following data:

$$X := \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}.$$

h. (5 pts.) Compute **the** $(1,1)$**-entry** <u>**ONLY**</u> of $e^{CXC^{-1}}$, given the following data:

$$C := \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \quad X := \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}, \quad C^{-1} := \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

(No need to simplify; your answer may involve powers of e.)

i. (5 pts.) Compute $(I+N)^{100}$, given the following data:

$$I := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad N := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

4. Miscellaneous.

a. (5 pts.) Give an example of a 3×3 Jordan block.

b. (5 pts.) Let $S \subseteq \mathbb{R}^4$ be the span of $\{(1,3,4,2),(2,1,2,-1)\}$. Construct an isomorphism $F : \mathbb{R}^2 \to S$. (To get credit, you must write out $F(x,y)$ explicitly.)

c. (5 pts.) Let $Z := \{1,2,3,4,5,6,7\}$. Among the 2^7 subsets of Z, how many of them have exactly four elements?