FM 5001 Fall 2007, Midterm #2
Handout date: Wednesday 14 November 2007

1. Definitions: Complete the following sentences.

a. (5 pts.) The **determinant** of a matrix $M \in \mathbb{R}^{n \times n}$ is the number d such that, for any oriented n-paralellpiped P, we have: ...

b. (5 pts.) The **kernel** of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^k$ is $\ker(T) = \cdots$

c. (5 pts.) A **homogeneous quadratic polynomial** in x, y, z is a linear combination of the following list of monomials ...

(Your answer should be a finite list of monomials formed from the three variables x, y, z.)

d. (5 pts.) A **basis** of a subspace S of $\mathbb{R}^n$ is a set B of vectors in S such that ...

e. (5 pts.) A **dimension** of a subspace S of $\mathbb{R}^n$ is ...

2. True or False. (No partial credit.)

a. (5 pts.) If a square matrix has a left inverse, then it has a right inverse.

b. (5 pts.) Every square matrix has positive determinant.

c. (5 pts.) Every square matrix with nonzero determinant is invertible.

d. (5 pts.) Every invertible square matrix has nonzero determinant.

e. (5 pts.) Let $A \in \mathbb{R}^{5 \times 3}$. Then there cannot be a matrix $B \in \mathbb{R}^{3 \times 5}$ Such that AB is the 5×5 identity.

f. (5 pts.) Let $A \in \mathbb{R}^{5 \times 3}$. Assume that $L_A : \mathbb{R}^3 \to \mathbb{R}^5$ is injective. Then there *must* exist a matrix $B \in \mathbb{R}^{3 \times 5}$ such that BA is the 3×3 identity.

g. (5 pts.) An elementary row or column operation can't change whether a matrix has a left inverse.

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3. Computations. (Answers typically must be exactly correct. No partial credit, except in unusual situations.)

a. (5 pts.) How many monomials of degree ≤ 5 can be formed using the eight variables a, b, c, d, e, f, g, h? (You need not list them.)

b. (5 pts.) Find the dimensions of the kernel and image of $L_A : \mathbb{R}^4 \to \mathbb{R}^3$, where

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 7 & 8 & 10 & 12 \end{bmatrix}.$$

c. (5 pts.) Find the determinant of B, where

$$B = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 7 & 5 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 1 & -3 \end{bmatrix}.$$

d. (5 pts.) Let B be the matrix in 3.c. Find $\det(BB^t)$.

e. (5 pts.) Let

$$C := \begin{bmatrix} 4 & 8 & 12 & -2 & 0 & 8 \\ 1 & -1 & -4 & 1 & 5 & 5 \\ 12 & 18 & 22 & -3 & 10 & 30 \\ 7 & 11 & 14 & -2 & 5 & 17 \end{bmatrix}.$$

Using elementary row and column operations, put C in fully canonical form.

f. (5 pts.) Let C be the matrix in 3.e. Let $v_1, \ldots, v_4 \in \mathbb{R}^6$ be the vectors whose entries are the entries of the first through fourth rows (respectively) of C. That is,

$$\begin{aligned} v_1 &= (4, 8, 12, -2, 0, 8), \\ v_2 &= (1, -1, -4, 1, 5, 5), \\ v_3 &= (12, 18, 22, -3, 10, 30), \\ v_4 &= (7, 11, 14, -2, 5, 17). \end{aligned}$$

Find a subset of $\{v_1, \ldots, v_4\}$ which is a basis of the subspace of $\mathbb{R}^6$ spanned by $v_1, \ldots, v_4$.

g. (5 pts.) Let C be the matrix in 3.e. Compute the dimensions of the kernel and image of $L_C : \mathbb{R}^6 \to \mathbb{R}^4$.

h. (5 pts.) Find the inverse of

$$D = \begin{bmatrix} 3 & 2 & 1 & 2 \\ 7 & 5 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & -3 \end{bmatrix}.$$