FM 5001 Fall 2008, Final Exam
Handout date: Wednesday 10 December 2008

PRINT NAME:

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.
Show work; a correct answer, by itself, may be insufficient for credit.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN NAME:

1. Definitions: Complete the following sentences.

a. (5 pts.) Two matrices S and S' are said to be **t-equivalent** if ...

b. (5 pts.) A subset S of a Euclidean space is a **subspace** if ...

c. (5 pts.) Let $n \geq 1$ be an integer. Let $Q : \mathbb{R}^n \to \mathbb{R}$ be a positive semidefinite quadratic form, and let $B : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be its polarization. Then the Cauchy-Schwarz inequality states that ...

d. (5 pts.) Let V and W be subspaces of Euclidean spaces. Let $T : V \to W$ be a linear transformation. The **kernel** of T is ...

e. (5 pts.) Let $n \geq 1$ be a positive integer. Let $M \in \mathbb{R}^{n \times n}$. An **eigenvalue** of M is a scalar a such that ....

f. (5 pts.) Let $n \geq 1$ be a positive integer. Let $M \in \mathbb{R}^{n \times n}$. Let a be an eigenvalue of M. The a-eigenspace of M is ....

2. True or False. (No partial credit.)

a. (5 pts.) Let $n \geq 1$ be an integer. Let $N \in \mathbb{R}^{n \times n}$ be a nilpotent matrix. Then $L_N : \mathbb{R}^n \to \mathbb{R}^n$ cannot be bijective.

b. (5 pts.) Each eigenvalue of a matrix is a real number, provided all the entries of the matrix are real.

c. (5 pts.) Let $n \geq 1$ be an integer. A linear map $L : \mathbb{R}^n \to \mathbb{R}^n$ is onto iff it is bijective.

d. (5 pts.) Every symmetric matrix is diagonalizable.

e. (5 pts.) Let $n \geq 1$ be an integer, and let $v, w \in \mathbb{R}^n$. Then $|v \cdot w| = [|v|][|w|]$. (In words: The absolute value of the dot product is the product of the lengths.)

f. (5 pts.) Every square matrix is conjugate to a diagonal matrix.

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1.

2.

3a.

3bc.

3d.

3e.

3f.

3g.

3hi.

3. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Answers typically must be exactly correct. No partial credit, except in unusual situations.)

a. (5 pts.) How many monomials are there of degree ≤ 5 in seven variables? (Write your answer as a quotient of products of integers; do not leave it as a binomial coefficient.)

b. (5 pts.) Let $X := \begin{bmatrix} 6 & 7 & 14 & 27 & 0 \\ 3 & 1 & 2 & 6 & 0 \\ 5 & 5 & 10 & 20 & 0 \\ 10 & 5 & 10 & 25 & 0 \end{bmatrix}$. Using elementary row and column operations, put X in fully canonical form. (Show all of your work, and indicate which elementary row and column operations you are using.)

c. (5 pts.) Let X be as in 3b. Find the dimensions of the kernel and image of $L_X : \mathbb{R}^5 \to \mathbb{R}^4$.

d. (5 pts.) Let $M := \begin{bmatrix} 1/2 & -\sqrt{3/2} & \sqrt{3/2} \\ \sqrt{3}/2 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$. Find a rotation matrix K such that the rows of KM are orthogonal to one another. Be sure to show your work. (Hint: $MM^t = \begin{bmatrix} 13/4 & -3\sqrt{3}/4 \\ -3\sqrt{3}/4 & 7/4 \end{bmatrix}$, which has trace 5 and determinant 4.)

e. (5 pts.) Find the inverse of $A := \begin{bmatrix} 2 & 3 & 0 & 0 \\ 5 & 7 & 0 & 0 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & -5 & -2 \end{bmatrix}$.

f. (5 pts.) Let $A := \begin{bmatrix} 2 & 3 & 0 & 0 \\ 5 & 7 & 0 & 0 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & -5 & -2 \end{bmatrix}$. Let $B := \begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Let $C := ABA^{-1}$. Compute $A^{-1}C^{10}A$.

g. (5 pts.) Let $A := \begin{bmatrix} 2 & 3 & 0 & 0 \\ 5 & 7 & 0 & 0 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & -5 & -2 \end{bmatrix}$. Let $B := \begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Let $C := ABA^{-1}$.
Compute the eigenvalues of C. For each eigenvalue λ of C find a basis of the λ-eigenspace of C. (Hint: For any eigenvalue λ, the kernel of $C - \lambda I$ is the image, under A, of the kernel of $B - \lambda I$; that is, $\ker(C - \lambda I) = A(\ker(B - \lambda I))$.)

h. (5 pts.) Give an example of a matrix with real entries which is complex diagonalizable, but which is not real diagonalizable.

i. (5 pts.) Suppose X is a PCRV such that $\Pr[X = 1] = 0.7$ and such that $\Pr[X = 5] = 0.3$. Compute $\mathrm{E}[X]$.