FM 5001 Fall 2008, Midterm #1
Handout date: Wednesday 15 October 2008

1. Definitions: Complete the following sentences.

a. (5 pts.) A set in $\mathbb{R}^n$ is **closed** if ...

b. (5 pts.) A function: $f : \mathbb{R}^n \to \mathbb{R}$ is **compactly supported** if ...

c. (5 pts.) A set of vectors in $\mathbb{R}^n$ is **linearly independent** if ...

d. (5 pts.) Let V and W be subspaces of Euclidean spaces. A map $L : V \to W$ is **linear** if ....

e. (5 pts.) A matrix is a **scalar matrix** if ...

2. True or False. (No partial credit.)

a. (5 pts.) The only clopen subsets of $\mathbb{R}$ are $\emptyset$ and $\mathbb{R}$.

b. (5 pts.) For any two $n \times n$ matrices A and B, we have: $AB = BA$.

c. (5 pts.) For any three $n \times n$ matrices A, B and C, we have: $A(BC) = (AB)C$.

d. (5 pts.) For any two vectors $v, w \in \mathbb{R}^n$, we have: $v \cdot w = w \cdot v$.

e. (5 pts.) For any linear map $L : \mathbb{R}^n \to \mathbb{R}^k$, there is a unique matrix $M \in \mathbb{R}^{k \times n}$ such that $L = L_M$.

f. (5 pts.) A linear transformation is onto iff its image is equal to $\{0\}$.

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

1.

2.

3abc.

3de.

3fg.

3hi.

3. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Answers typically must be exactly correct, and can be left in any form easily calculated on a calculator. No partial credit, except in unusual situations.)

a. (5 pts.) Find $P \otimes Q$, where

$$P := \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}, \quad Q := \begin{bmatrix} 3 & 2 \\ -3 & -2 \end{bmatrix}.$$

b. (5 pts.) Give an example of a 4×4 Jordan block.

c. (5 pts.) Give an example of a nilpotent 3×3 matrix.

d. (5 pts.) Eliminate the linear term in $y = -(x^2/2) + 6x - 8$. (Indicate your change of variables.)

e. (5 pts.) Find AB, where

$$A := \begin{bmatrix} 1 & 0 \\ 1 & 4 \end{bmatrix}, \quad B := \begin{bmatrix} 5 \\ 6 \end{bmatrix}.$$

f. (5 pts.) Compute $(CXC^{-1})^{100}$, given the following data:

$$C := \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \quad X := \begin{bmatrix} \sqrt{-1} & 0 \\ 0 & -\sqrt{-1} \end{bmatrix}, \quad C^{-1} := \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

g. (5 pts.) Compute $\displaystyle\int_{-\infty}^{\infty} x^9 e^{-x^2/2}\, dx$.

h. (5 pts.) Compute the binomial coefficient $\binom{10}{3}$.

i. (5 pts.) Compute e^{2I+N}, given the following data:

$$I := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad N := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$