FM 5001 Fall 2008, Midterm #2
Handout date: Wednesday 12 November 2008

1. Definitions: Complete the following sentences.

a. (5 pts.) The **determinant** of a matrix $M \in \mathbb{R}^{n \times n}$ is the number d such that, for any oriented n-paralellpiped P, we have: ...

b. (5 pts.) The **image** of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^k$ is $\text{im}(T) = \cdots$

c. (5 pts.) A **homogeneous cubic polynomial** in x, y is a linear combination of the following list of monomials ... ("Cubic" means "degree 3".)

d. (5 pts.) The **dimension** of a subspace S of $\mathbb{R}^n$ is ...

e. (5 pts.) An $n \times n$ **Jordan block** is ...

f. (5 pts.) An **ordered basis** of a subspace S of $\mathbb{R}^n$ is ...

2. True or False. (No partial credit.)

a. (5 pts.) The determinant of the sum of two square matrices (of the same size) is the sum of their determinants.

b. (5 pts.) The determinant of the product of two square matrices (of the same size) is the product of their determinants.

c. (5 pts.) Any two spanning sets of a subspace of Euclidean space have the same number of elements.

d. (5 pts.) Every square matrix is a conjugate of a standard nilpotent matrix.

e. (5 pts.) The product of any two nilpotent matrices (of the same size) is nilpotent.

f. (5 pts.) The product of two invertible matrices (of the same size) is invertible.

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1.

2.

3abc.

3de.

3fg.

3h.

3. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Answers typically must be exactly correct, and can be left in any form easily calculated on a calculator. No partial credit, except in unusual situations.)

a. (5 pts.) Find the determinant of A, where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ -3 & 7 & 1 & 0 \\ 6 & 6 & 7 & -3 \end{bmatrix}.$$

b. (5 pts.) How many monomials of degree ≤ 22 can be formed using the six variables a, b, c, d, e, f? (You need not list them. You may leave your answer as a binomial coefficient; you need not compute it.)

c. (5 pts.) Let $M \in \mathbb{R}^{3\times 5}$, let $v \in \mathbb{R}^5$ and assume that $L_M(v) = (2, 4, -1)$. Let $A := M^t M$. Compute $(L_A(v)) \cdot v$, *i.e.*, compute the dot product of $L_A(v)$ with v.

d. (5 pts.) Put the following matrix in row canonical form.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 6 & 8 & 10 & 12 & 15 \\ 3 & 6 & 9 & 11 & 15 & 18 & 21 \\ 4 & 8 & 12 & 1000 & 20 & 24 & 2000 \end{bmatrix}$$

e. (5 pts.) Put the following matrix in fully canonical form.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 6 & 8 & 10 & 12 & 15 \\ 3 & 6 & 9 & 11 & 15 & 18 & 21 \\ 4 & 8 & 12 & 1000 & 20 & 24 & 2000 \end{bmatrix}$$

f. (5 pts.) Find the dimension of the kernel of

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 6 & 8 & 10 & 12 & 15 \\ 3 & 6 & 9 & 11 & 15 & 18 & 21 \\ 4 & 8 & 12 & 1000 & 20 & 24 & 2000 \end{bmatrix}$$

g. (5 pts.) Find the dimension of the subspace of $\mathbb{R}^7$ spanned by

$$(1,2,3,4,5,6,7), \quad (2,4,6,8,10,12,15),$$
$$(3,6,9,11,15,18,21) \quad \text{and} \quad (4,8,12,1000,20,24,2000).$$

h. (5 pts.) Compute

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (e^{2x} - e^4)_+ e^{-x^2/2}\, dx.$$