FM 5002 Spring 2009, Midterm #1
Handout date: Wednesday 4 March 2009

PRINT NAME:

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.
Show work; a correct answer, by itself, may be insufficient for credit.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN NAME:

1. Definitions: Complete the following sentences.

1a. (5 pts.) Let X and Y be PCRVs. Then the **covariance**, $\mathrm{Covar}[X, Y]$, is defined by $\mathrm{Covar}[X, Y] = \cdots$

1b. (5 pts.) Let X and Y be PCRVs. Then the **correlation**, $\mathrm{Corr}[X, Y]$, is defined by $\mathrm{Corr}[X, Y] = \cdots$

1c. (5 pts.) The **third order Maclaurin approximation** to a function f is the polynomial P that has the following properties: $\cdots$

1d. (5 pts.) A series $a_1 + a_2 + a_3 + \cdots$ is said to **converge** if ...

1e. (5 pts.) Bayes' formula reads $\Pr[A|B] = (\Pr[B|A])\left(\frac{\cdots}{\cdots}\right)$.

2. True or False. (No partial credit.)

2a. (5 pts.) The series $1 + (1/2) + (1/3) + (1/4) + \cdots$ diverges.

2b. (5 pts.) If every term of a series is nonnegative, then it has a sum (possibly equal to infinity).

2c. (5 pts.) Let A and B be events of positive probability. If $\Pr[A|B] = \Pr[A]$, then $\Pr[B|A] = \Pr[B]$.

2d. (5 pts.) If X and Y are independent PCRVs, then $\mathrm{Covar}[X, Y] = 0$.

2e. (5 pts.) If X and Y are uncorrelated PCRVs, then X and Y are independent.

2f. (5 pts.) Let M be a positive semidefinite symmetric matrix all of whose entries along the main diagonal are zero. Then all entries of M are zero.

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1.

2.

3a.

3bc.

3def.

3gh.

3i.

3. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Answers typically must be exactly correct. No partial credit, except in unusual situations. No need to simplify arithmetic.)

3a. (5 pts.) Let X be the PCRV defined by

$$X(\omega) = \begin{cases} 2, & \text{if } 0 \leq \omega < 0.1 \\ 0, & \text{if } 0.1 \leq \omega \leq 1 \end{cases}$$

Compute $\mathrm{E}[X]$ and $\mathrm{SD}[X]$.

3b. (5 pts.) Compute $\int_{-\infty}^{\infty} e^{8x} e^{-x^2/2}\, dx$.

3c. (5 pts.) Let $C_1, C_2, C_3, \ldots$ be a sequence of pairwise-independent standard binary PCRVs, as described in class. For all integers $n > 1$, let $X_n := (C_1 + \cdots + C_n)/\sqrt{n}$. Compute $\lim_{n\to\infty} \mathrm{E}[e^{8X_n}]$.

3d. (5 pts.) Let

$$M := \begin{bmatrix} 9 & 18 \\ 18 & 52 \end{bmatrix}.$$

Find a lower triangular matrix A such that $AA^t = M$.

3e. (5 pts.) Let

$$M := \begin{bmatrix} 9 & 18 \\ 18 & 52 \end{bmatrix}.$$

Find an upper triangular matrix B such that $B^t B = M$.

3f. (5 pts.) Let Z_1 and Z_2 be independent standard normal PCRVs. Find real numbers a, b, c such that $\mathrm{Var}[aZ_1] = 9$, $\mathrm{Var}[bZ_1 + cZ_2] = 52$ and $\mathrm{Covar}[aZ_1, bZ_1 + cZ_2] = 18$.

3g. (5 pts.) Find the fourth order Maclaurin approximation to e^{3x^2}.

3h. (5 pts.) Suppose $f(0) = 0$, $f'(0) = 0$ and $f''(0) = 0$. Suppose also that, for all $x \in [-1, 0]$, we have $f'''(x) \leq 1$. Among all such functions, assume that f is chosen so that $f(-1)$ is as negative as possible. What is that most negative possible value for $f(-1)$?

3i. (5 pts.) Give an example of two binary PCRVs X and Y such that $\mathrm{E}[X|Y]$ is deterministic, but such that neither X nor Y is deterministic. Draw graphs of X and Y.