FM 5011 Fall 2007, Midterm #1
Handout date: Thursday 11 October 2007

1. Definitions: Complete the following sentences.

a. (5 pts.) The holder of a **put option** has the right ...

(NOTE: Your next words must be either "but not the obligation" or "and the obligation". You should continue from there.)

b. (5 pts.) The holder of a **forward** contract has the right ...

(NOTE: Your next words must be either "but not the obligation" or "and the obligation". You should continue from there.)

c. (5 pts.) The **risk-neutral world** is an imaginary probabilistic world in which all assets have the same ....

d. (5 pts.) The $\mathbf{\Delta}$ of an option is the following difference quotient: the ... difference over the ... difference.

2. True or False. (No partial credit.)

a. (5 pts.) Assuming a binary price model for the underlying, to hedge the sale of a put option on a risky asset, one holds a certain amount of the risky asset and shorts a certain amount the risk-free asset, but these amounts may vary over time.

b. (5 pts.) Assuming a binary price model for the underlying, to hedge the sale of a call option on a risky asset, one holds a certain amount of the risky asset and shorts a certain amount of the risk-free asset, but these amounts may vary over time.

c. (5 pts.) To hedge the sale of a forward on a risky financial asset (with no money chaning hands at the start, and no carrying costs), one holds the risky asset and shorts the risk-free asset. The amounts do not vary over time.

d. (5 pts.) In a risk-averse world, risky assets have a lower expected return than risk-free assets.

e. (5 pts.) The Black-Scholes formula does not depend on the volatility of the underlying asset, but it does depend on its drift.

3. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Answers typically must be exactly correct. No partial credit, except in unusual situations.)

a. (5 pts.) Suppose we wish to hedge the sale of a derivative with an underlying of Euros. We assume a binary model for the exchange rate with only one subperiod. Suppose our model is that the Euro will end at either \$1.10 or at \$0.90. Suppose that the derivative will end with values of either \$10 or \$8, respectively. According to the principle of Δ-hedging, how many Euros should the option sell hold in her hedging portfolio.

b. (5 pts.) Assume the uptick and downtick factors in a model for an underlying asset are 1.07 and 0.97, respectively. Suppose the risk-free factor, over the same time period, is 1.06. Compute the risk-neutral uptick probability.

c. (5 pts.) Compute $\displaystyle\int_{-\infty}^{\infty} e^{-x^2/2}e^{3x+4}\,dx$.

d. (5 pts.) Compute $\displaystyle\int_{-\infty}^{\infty} x^2e^{-x^2/2}\,dx$.

e. (5 pts.) Compute $\displaystyle\int_{-\infty}^{\infty} (4e^x-4)_+e^{-x^2/2}\,dx$.

f. (5 pts.) Let p and q be positive real numbers such that $p+q=1$. Let a and b be real numbers satisfying $pa+qb=0$ and $pa^2+qb^2=1$. Let $i:=\sqrt{-1}$. Let $f(t)=pe^{-iat}+qe^{-ibt}$. Compute $\displaystyle\lim_{n\to\infty}[f(7/\sqrt{n})]^n$. (Hint: Find the second order MacLaurin approximation to $f(t)$.)

4. Miscellaneous.

a. (5 pts.) Let Z be a standard normal variable. (That is, the amount of probability at x is $[e^{-x^2/2}/\sqrt{2\pi}]\,dx$.) Compute the expected value of e^{3Z+4}.

b. (5 pts.) Let Z be a standard normal variable. (That is, the amount of probability at x is $[e^{-x^2/2}/\sqrt{2\pi}]\,dx$.) Compute the expected value of Z^2.

c. (5 pts.) Let Z be a standard normal variable. (That is, the amount of probability at x is $[e^{-x^2/2}/\sqrt{2\pi}]\,dx$.) Compute the expected value of $4e^Z-4]_+$.

d. (Note: CLT = Central Limit Theorem.) Imagine a coin-flipping game, with 10^{10} flips of a fair coin, in which we are paid $P:=\left[\dfrac{6H-4T-10^{10}}{10^5}\right]_+$, where H is the number of heads and T is the number of tails. Let $X:=(H-T)/10^5$, so that, according to the CLT, X is very close in distribution to a standard normal random variable.

(1) (5 pts.) Find constants a and b such that $P=[aX+b]_+$.

(2) (5 pts.) Using the CLT, approximate the expected value of P.