

FM 5011 Fall 2008, Midterm #1  
Handout date: Thursday 16 October 2008

PRINT NAME:

1. Definitions: Complete the following sentences.

a. (5 pts.) Let  $X$  be a PCRV. Then the Fourier transform of the distribution of  $X$  is the expression of  $t$  given by ...

b. (5 pts.) The **risk-neutral world** is an imaginary probabilistic world in which all assets have the same ....

c. (5 pts.) The  $\Delta$  of an option is the following difference quotient: the ... difference over the ... difference.

d. (5 pts.) An algebra on a set  $X$  is a nonempty collection  $\mathcal{B}$  of subsets of  $X$  which is closed under ...

e. (5 pts.) Let  $\mathcal{B}$  be a  $\sigma$ -algebra on a set  $X$  and let  $W \subseteq X$ . Then the **restriction** of  $\mathcal{B}$  to  $W$ , written  $\mathcal{B}|_W$ , is defined to be ...

2. True or False. (No partial credit.)

a. (5 pts.) If  $\mathcal{B}$  and  $\mathcal{C}$  are both  $\sigma$ -algebras on a set  $X$ , then their union  $\mathcal{B} \cup \mathcal{C}$  is also a  $\sigma$ -algebra on  $X$ .

b. (5 pts.) Any  $\sigma$ -algebra is an algebra.

c. (5 pts.) Under the CRR model, the risk-neutral drift tends to the real-world drift, as number of subperiods tends to infinity.

d. (5 pts.) Under the CRR model, the risk-neutral volatility tends to the real-world volatility, as number of subperiods tends to infinity.

e. (5 pts.) In a risk-averse world, risky assets have a higher expected return than risk-free assets.

f. (5 pts.) Any two standard binary PCRVs must have the same distribution.

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1.

2.

3ab.

3cd.

3efg.

3h.

3i.

3. Computations. Some of your answers may involve  $\Phi$ , the cumulative distribution function of the standard normal distribution. (Answers typically must be exactly correct, and can be left in any form easily calculated on a calculator. No partial credit, except in unusual situations.)

a. (5 pts.) A put option has price 5. The initial price of the underlying is \$10, and the strike price has present value equal to \$9.95. Using put-call parity, find the price of the corresponding call option (with the same term, the same underlying and the same strike).

b. (5 pts.) Let  $X$  be a sum of 25 independent binary PCRVs, all in  $\mathcal{B}_{0.6,3}^{0.4,6}$ . Compute the standard deviation of  $X$ .

c. (5 pts.) Expand and collect terms in  $\left[\frac{2}{n^2} - \frac{1}{n^3} + \frac{o}{n^3}\right] \left[-\frac{7}{n^3} + \frac{8}{n^4} + \frac{o}{n^5}\right]$ . Your answer should have only one term involving  $o$ .

d. (5 pts.) Let  $f(x) = 1 - \frac{x^2}{2} + \sin^3 x$ . Compute  $\lim_{n \rightarrow \infty} (f(5/\sqrt{n}))^n$ .

e. (5 pts.) Let  $X$  be a sum of 1,000,000 independent binary PCRVs, all with the same distribution. Assume that  $X$  is standard. Each of the 1,000,000 PCRVs has the same mean. What is it?

f. (5 pts.) Let  $X$  be a sum of 1,000,000 independent binary PCRVs, all with the same distribution. Assume that  $X$  is standard. Each of the 1,000,000 PCRVs has the same standard deviation. What is it?

g. (5 pts.) For each integer  $n \geq 1$ , let  $X_n$  be a sum of  $n$  independent binary PCRVs, all with the same distribution. Assume, for all integers  $n \geq 1$ , that the uptick and downtick probabilities are between 0.1 and 0.9. Assume, for all integers  $n \geq 1$ , that  $X_n$  is standard. Using the Triangular Central Limit Theorem, compute  $\lim_{n \rightarrow \infty} E[(e^{\bar{X}_n} - e^5)_+]$ .

h. (5 pts.) Compute  $\int_{-\infty}^{\infty} (e^x - e^5)_+ e^{-x^2/2} dx$ .

i. (5 pts.) Compute  $\int_{-4}^{\infty} (7x + 5)e^{-x^2/2} dx$ .