FM 5001 Fall 2009, Midterm #1
Handout date: Wednesday 21 October 2009

PRINT NAME:

Remember to read to the bottom and to SIGN YOUR NAME BELOW!

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.
Show work; a correct answer, by itself, may be insufficient for credit.

I understand the above, and I understand that cheating has **severe consequences**, from a failing grade to expulsion.

REMEMBER TO SIGN YOUR NAME:

I. Definitions: Complete the following sentences.

a. (Topic 0006, 3 pts.) A set in $\mathbb{R}^n$ is **compact** if ...

b. (Topic 0001, 3 pts.) A function: $f : X \to Y$ is **injective** if ...

c. (Topic 0003, 3 pts.) Let $S \subseteq \overline{\mathbb{R}}$. Let $b \in \overline{\mathbb{R}}$. We say that b is the **infimum** of S if ...

d. (Topic 0010, 3 pts.) Let V and W be subspaces of Euclidean spaces. A map $L : V \to W$ is **linear** if ....

e. (Topic 0009, 3 pts.) If $v = (a_1, \ldots, a_n)$ and $w = (b_1, \ldots, b_n)$, then the **dot product** $v \cdot w$ of v and w is defined by the formula $v \cdot w = \cdots$.

II. True or False. (No partial credit.)

a. (Topic 0006, 3 pts.) The only compact open subset of $\mathbb{R}$ is $\emptyset$.

b. (Topic 0006, 3 pts.) If $A \subseteq \mathbb{R}^n$ is open and if $B \subseteq \mathbb{R}^n$ is closed, then $A \cap B$ is both open and closed.

c. (Topic 0009, 3 pts.) If F is a linearly independent set in $\mathbb{R}^n$ and if $v \in \mathbb{R}^n \backslash \langle F \rangle$, then $F \cup \{v\}$ is linearly independent.

d. (Topic 0008, 3 pts.) For any two positive integers n and k: if $k \leq n$, then the binomial coefficients $\binom{n}{k}$ and $\binom{n}{n-k}$ are equal.

e. (Topic 0010, 3 pts.) The function $F : \mathbb{R}^2 \to \mathbb{R}$ defined by $F(x, y) = x^2 + 2y - 1$ is linear.

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PLEASE DO NOT WRITE BELOW THE LINE

I.

II.

III(1,2).

III(3,4).

III(5).

III(6,7).

III(8).

III. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.)

1. (Topic 0002, 10 pts.) Compute $\displaystyle\int_0^\infty (2x-5)e^{-x^2/2}\,dx$.

2. (Topic 0008, 5 pts.) How many monomials are there of degree exactly 10 in 7 variables? (Leave your answer in terms of factorials; no need to compute them. However, **don't leave any binomial coefficients in your answer**.)

3. (Topic 0010, 10 pts.) Find the 4×3 matrix M such that
$L_M(v, w, x) = (0, x, 2v + 3w, 4v - 5x)$, for all $(v, w, x) \in \mathbb{R}^3$.

4. (Topic 0004, 10 pts.) Assume that $6 \leq f'''(x) \leq 12$, for all $x \in \mathbb{R}$. Assume that $f(0) = f'(0) = f''(0) = 0$. Among all functions f satisfying those two conditions,
(a) find the minimum possible value of $f(-1)$; and
(b) find the maximum possible value of $f(-1)$.

5. (Topic 0002, 10 pts.) Compute $\dfrac{d}{dx}[(x^2+1)^x]$.

6. (Topic 0008, 10 pts.) Compute the binomial coefficient $\binom{7}{4}$. Your answer should be an integer in decimal form – **be sure there are no factorials in the answer**.

7. (Topic 0005, 5 pts.) Let $i := \sqrt{-1}$ and let $z := \dfrac{\ln 2}{2} + \dfrac{\pi i}{4}$. Compute e^z.

8. (Topic 0002, 10 pts.) Find a 2×2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $b = c$ and such that, for all $(x, y) \in \mathbb{R}^2$, we have: $(L_M(x, y)) \cdot (x, y) = x^2 + 4xy + y^2$.