FM 5001 Fall 2009, Midterm #2
Handout date: Wednesday 18 November 2009

PRINT NAME:

Remember to read to the bottom and to SIGN YOUR NAME BELOW!

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.
Show work; a correct answer, by itself, may be insufficient for credit.

I understand the above, and I understand that cheating has **severe consequences**, from a failing grade to expulsion.

REMEMBER TO SIGN YOUR NAME:

I. Definitions: Complete the following sentences.

a. (Topic 0011, 3 pts.) Let $A, C \in \mathbb{R}^{n\times n}$. The **left conjugate** of A by C is ...

b. (Topic 0012, 3 pts.) Let $N \in \mathbb{R}^{n\times n}$. We say that N is **nilpotent** if ...

c. (Topic 0014, 3 pts.) The **dimension** of a subspace V of $\mathbb{R}^n$ is ...

d. (Topic 0015, 3 pts.) The **standard oriented parallelogram** in $\mathbb{R}^2$ is...

e. (Topic 0012, 3 pts.) For any $M \in \mathbb{R}^{n\times n}$, $e^M := \cdots$

II. True or False. (No partial credit.)

a. (Topic 0011, 3 pts.) For all $M \in \mathbb{R}^{n\times n}$, for all $v, w \in \mathbb{R}^n$, $(L_M(v)) \cdot w = (L_M(w)) \cdot v$.

b. (Topic 0012, 3 pts.) For any two matrices $A, B \in \mathbb{R}^{3\times 3}$, $e^{AB} = e^A e^B$.

c. (Topic 0012, 3 pts.) Any conjugate of a nilpotent matrix is again nilpotent.

d. (Topic 0016, 3 pts.) If A and B are square matrices, then $\det(A \oplus B) = (\det A) + (\det B)$.

e. (Topic 0016, 3 pts.) For any matrix $A \in \mathbb{R}^{n\times n}$, we have: $\det(A^t) = \det(A)$, where A^t is the transpose of A.

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PLEASE DO NOT WRITE BELOW THE LINE

I.

II.

III(1).

III(2).

III(3).

III(4).

III(5).

III(6).

III(7).

III. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.)

1. Let $P := \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$, $\quad Q := \begin{bmatrix} 3 & 2 \\ -3 & -2 \end{bmatrix}$.

a. (Topic 0011, 2 pts.) Compute $P \oplus Q$.

a. (Topic 0011, 3 pts.) Compute $P \otimes Q$.

2. Let I be the 5×5 identity. Let M be the standard nilpotent 5×5 matrix.

a. (Topic 0012, 2 pts.) Display I, M.

b. (Topic 0012, 3 pts.) Compute $M^2/(2!)$, $M^3/(3!)$ and $M^4/(4!)$.

c. (Topic 0012, 1 pts.) Compute M^5.

d. (Topic 0012, 4 pts.) Compute e^M.

3. (Topic 0014, 10 pts.) Find the dimensions of the image and kernel of

$$\begin{bmatrix} 1 & 2 & 4 & 2 & 0 \\ 1 & 1 & 2 & 2 & 0 \\ 2 & 3 & 6 & 4 & 0 \\ 3 & 4 & 8 & 6 & 0 \end{bmatrix}.$$

4. (Topic 0016, 5 pts.) Compute the determinant of

$$\begin{bmatrix} 1 & 2 & 4 & 2 \\ 1 & 1 & 2 & 2 \\ 2 & 3 & 6 & 4 \\ 3 & 4 & 8 & 6 \end{bmatrix}.$$

5. Let $M := \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

a. (Topic 0016, 5 pts.) Compute the matrix of minors of M. Your answer should be a matrix whose entries are all expressions of a, b, c, d.

b. (Topic 0016, 5 pts.) Compute the cofactor matrix of M. Your answer should be a matrix whose entries are all expressions of a, b, c, d.

c. (Topic 0016, 10 pts.) Let C be the cofactor matrix of M, with transpose C^t. Compute $C^t M$. Your answer should be a matrix whose entries are all expressions of a, b, c, d.

6. (Topic 0016, 10 pts.) Compute the signed volume of the oriented 3-parallelpiped

$$(\quad (0,1,0) \quad , \quad (2,3,8) \quad , \quad (4,7,2) \quad).$$

7. (Topic 0014, 10 pts.) Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 11 \\ 2 & 5 & 3 \end{bmatrix}$.