FM 5001 Fall 2011, Midterm #2
Handout date: Wednesday 16 November 2011
Time for exam: ONE HOUR

For PROCTORS of online students:
Email scan to: adams@math.umn.edu
Preferred FAX: 612-624-6702 Alternate FAX: 612-626-2017
Exam must be received by 24 hours after the ending time for in-person students. Thank you.
Time to take exam: 1 hour

STUDENT, PLEASE PRINT NAME:

Remember to read to the bottom and to SIGN YOUR NAME BELOW!

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.
Show work; a correct answer, by itself, may be insufficient for credit.

I understand the above, and I understand that cheating has **severe consequences**, from a failing grade to expulsion.

STUDENT, PLEASE REMEMBER TO SIGN YOUR NAME:

I. Definitions: Complete the following sentences.

a. (Topic 0027(25), 3 pts.) The **determinant** of an $n \times n$ matrix M is denoted $\det(M)$ and is defined by: For all oriented n-parallelpipeds P, we have ...

b. (Topic 0026(14), 3 pts.) Let S be a subspace of a Euclidean space. The **dimension** of S is defined by $\dim(S)$ is the cardinality of ...

c. (Topic 0024(20), 3 pts.) An $n \times n$ matrix M is said to be **orthogonal** if ...

d. (Topic 0024(6), 3 pts.) An $n \times n$ matrix M is said to be **nilpotent** if ...

e. (Topic 0023(45), 3 pts.) Let A and B be $n \times n$ matrices. We say that B is the **inverse** of A if ...

II. True or False. (No partial credit.)

a. (Topic 0026(33), 3 pts.) If A and B are two matrices, and if AB is an identity matrix, then both A and B are square matrices.

b. (Topic 0027(22), 3 pts.) Let M be an $n \times n$ shearing matrix, *i.e.*, a matrix which is equal to the $n \times n$ identity, except that a single off diagonal entry is nonzero. Then $\det(M) = 1$.

c. (Topic 0026(7), 3 pts.) If there is an injective linear map $\mathbb{R}^p \to \mathbb{R}^q$, then $p \geq q$.

d. (Topic 0025(25-26), 3 pts.) Every elementary matrix is invertible.

e. (Topic 0024(6), 3 pts.) If a square matrix is both diagonal and nilpotent, then all of its entries are equal to zero.

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PLEASE DO NOT WRITE BELOW THE LINE

I.

II.

III(1,2).

III(3).

III(4).

III(5abc).

III(6).

III. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.)

1. (Topic 0006(24), 10 pts.) How many subsets of $\{1, 2, 3, \ldots, 10\}$ have five elements? (Express your answer as a product of positive integers.)

2. (Topic 0028(43), 10 pts.) Find the signed volume of the oriented parallelpiped

$$P \quad := \quad (\ (1,3,4)\ ,\ (0,1,-2)\ ,\ (0,0,-1)\ ) \quad .$$

3. (Topic 0028(44), 15 pts.) Recall that $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$. Let $M := \begin{bmatrix} -2 & 4 \\ 3 & -3 \end{bmatrix}$. Let $I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ be the 2×2 identity matrix. Find a number $\lambda \in \mathbb{R}$ such that $M - \lambda I$ is not invertible.

4. (Topic 0025(44), 10 pts.) Show all fully canonical 3×5 matrices.

5. Let $C := \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, let $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and let $M := CDC^{-1}$.

a. (Topic 0025(26), 5 pts.) Compute C^{-1}.

b. (Topic 0023(15), 5 pts.) Compute M.

c. (Topic 0024(17), 5 pts.) Compute e^M.

6. (Topic 0026(41), 10 pts.) Find the inverse of $M := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 13 \\ 2 & 5 & 3 \end{bmatrix}$.