Financial Mathematics

Polynomial approximation, bilinear forms and quadratic forms

0029-1. Let $B : \mathbb{R}^5 \times \mathbb{R}^5 \to \mathbb{R}$ be the bilinear form defined by

$$[B] = \begin{bmatrix} 4 & 7 & 9 & 0 & 6 \\ 5 & 0 & 8 & 0 & 2 \\ 1 & -2 & -4 & 1 & -8 \\ 4 & -2 & 3 & 3 & 1 \\ 0 & 4 & 0 & 9 & 0 \end{bmatrix}.$$

a. Let $v := (-1, 3, 1, 0, 0)$, $w := (1, -1, 0, 1, 0)$. Compute $B(v, w)$.

b. Define $Q : \mathbb{R}^5 \to \mathbb{R}$ by $Q(v) = B(v, v)$. Write out $Q(p, q, r, s, t)$.

c. Find a symm. matrix $M \in \mathbb{R}^{5\times 5}$ s.t., if S is the SBF def'd by $[S] = M$, then $S(v, v) = B(v, v)$.

0029-2.

Let $Q : \mathbb{R}^5 \to \mathbb{R}$ be the quadratic form def'd by

$$\begin{aligned} Q(p,q,r,s,t) = 2p^2 + 4q^2 - 7r^2 - 9s^2 \\ +4pq - 8pr - 6ps - 2pt \\ -6qr - 8qs + 4qt \\ -6rs + 2rt \\ +100st. \end{aligned}$$

Let $B : \mathbb{R}^5 \times \mathbb{R}^5 \to \mathbb{R}$ be the polarization of Q.

Write out the matrix $[B]$ of B.

0029-3. Let $Q : \mathbb{R}^2 \to \mathbb{R}$ be the quadratic form defined by $Q(x,y) = 2x^2 + 6xy + y^2$.
Determine whether Q is positive semidefinite.
Hint: $Q(x,1) = 2x^2 + 6x + 1$
Is this always positive?
Then check $Q(x,2)$.
Then check $Q(x,y)$, in general.

0029-4. Let $P : \mathbb{R}^2 \to \mathbb{R}$ be the quadratic form defined by $P(x,y) = (x^2/25) + (y^2/16)$.

a. Graph $\{(x,y) \,|\, P(x,y) = 1\}$.

b. Let $v := (2,8)$.
Let B be the polarization of P.
Find a vector $w \in \mathbb{Z}^2 \backslash \{(0,0)\}$ such that $B(v,w) = 0$.