Financial Mathematics

The multivariable chain rule

0039-1. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by

$$f(s,t) = (s-t , \; 2s+t , \; s-3).$$

Let $g : \mathbb{R}^3 \to \mathbb{R}^4$ be defined by

$$g(x,y,z) = (e^{x-y} , \; x-4y+z , \; x^3+y^2 , \; xyz).$$

a. Compute $f'(1,1) \in \mathbb{R}^{3\times 2}$.

b. Compute $g'(f(1,1)) \in \mathbb{R}^{4\times 3}$.

c. Compute $[g'(f(1,1))][f'(1,1)] \in \mathbb{R}^{4\times 2}$.

d. Compute $(g \circ f)(s,t)$.

e. Compute $(g \circ f)'(1,1) \in \mathbb{R}^{4\times 2}$.

0039-2. Let $f : \mathbb{R}^2 \to \mathbb{R}^4$, $g : \mathbb{R}^4 \to \mathbb{R}^3$ be smooth and let $h := g \circ f : \mathbb{R}^2 \to \mathbb{R}^3$.

Let $f_1, f_2, f_3, f_4 : \mathbb{R}^2 \to \mathbb{R}$ be the components of f, so $f = (f_1, f_2, f_3, f_4)$.
Let $g_1, g_2, g_3 : \mathbb{R}^4 \to \mathbb{R}$ be the components of g, so $g = (g_1, g_2, g_3)$.
Let $h_1, h_2, h_3 : \mathbb{R}^2 \to \mathbb{R}$ be the components of h, so $h = (h_1, h_2, h_3)$.

Let $(r, s) \in \mathbb{R}^2$, let $(t, u, v, w) := f(r, s) \in \mathbb{R}^4$, and let $(x, y, z) := g(t, u, v, w) \in \mathbb{R}^3$.

Write out a detailed four-term formula for $(\partial_2 h_3)(r, s)$
in terms of $(\partial_k g_j)(t, u, v, w)$, $j \in \{1, 2, 3\}$, $k \in \{1, 2, 3, 4\}$,
and $(\partial_l f_k)(r, s)$, $k \in \{1, 2, 3, 4\}$, $l \in \{1, 2\}$.