Financial Mathematics

Multivariable change of variables

0041-1. Fix $r > 0$.

$D := (0, r) \times (0, 2\pi)$.

$E := (\{(x, y) \mid \frac{x^2}{4} + \frac{y^2}{9} < r^2\}) \backslash ([0, r) \times \{0\})$.

Define $f : D \to E$ by $f(s, t) = (2s \cos t, 3s \sin t)$.

$f : D \to E$ is a smooth bijection.

$$f'(s, t) = \begin{bmatrix} 2\cos t & -2s \sin t \\ 3 \sin t & 3s \cos t \end{bmatrix}$$

$$\text{Area}(E) = \int\int_D |\det f'(s, t)| \, ds \, dt$$

$$= \int_0^{2\pi} \int_0^r |\det f'(s, t)| \, ds \, dt$$

a. Finish this computation.

b. Graph E.

0041-2. Let $D := (0,4) \times (0,\pi) \times (0,\pi/3)$.

Define $\psi : D \to \mathbb{R}^3$ by

$$\psi(r,\theta,\phi) = \left(r(\cos\phi)(\cos\theta) \, , \, r(\cos\phi)(\sin\theta) \, , \, 2r(\sin\phi) \right)$$

Let $E := \psi(D)$.

Then $\psi : D \to E$ is bijective and smooth, and has contin. extension to the closure of D.

Compute the volume of E by computing

$$\int_E 1 \, dx \, dy \, dz$$

via the change of variables formula.