Financial Mathematics

Conditional probability, independence and the Central Limit Theorem

0047-1. Suppose $\Pr[A|B] = 0.6$,

$\Pr[A] = 0.3$ and $\Pr[B] = 0.5$.

a. Find $\Pr[A \text{ and } B]$.

b. Find $\Pr[B|A]$.

0047-2. a. Find two PCRVs X and Y s.t.

$\Pr[(X = 1)|(Y = 2)] = 0.6$,

$\Pr[X = 1] = 0.3$ and $\Pr[Y = 2] = 0.5$.

b. Compute $\Pr[(Y = 2)|(X = 1)]$.

0047-3. Let $C_1, C_2, C_3, \ldots$ be our standard sequence of coin-flipping PCRVs.

$\forall$integers $n \geq 1$, let $D_n := (C_1 + \cdots + C_n)/\sqrt{n}$.

a. Compute $\lim_{n \to \infty} \mathrm{E}[D_n^6]$.

b. Compute $\lim_{n \to \infty} \mathrm{E}[80(e^{4D_n - 3} - e)_+]$.

0047-4. Let X and Y be PCRVs

s.t. $\Pr[X = 4] = 0.8$,

s.t. $\Pr[(X = 4)\&(Y = 9)] = 0.35$

and s.t. $\Pr[(X = 4)\&(Y = 2)] = 0.45$.

a. Find $\Pr[(Y = 2)|(X = 4)]$.

b. Find $E[Y|(X = 4)]$.

0047-5. Find two PCRVs

which are uncorrelated,

but not independent.

Sketch the graphs

of the two PCRVs.

WARNING: Neither can be deterministic.

WARNING: At least one must have

three values of positive probability.

0047-6. Let $X_1, X_2, X_3, \ldots, X_{40}$ be iid.

Suppose $\mathrm{E}[X_1 + X_2 + X_3 + \cdots + X_{40}] = 60$

and $\mathrm{SD}[X_1 + X_2 + X_3 + \cdots + X_{40}] = 10$.

Let $\mu := \mathrm{E}[X_1] = \mathrm{E}[X_2] = \cdots = \mathrm{E}[X_{40}]$

and $\sigma := \mathrm{SD}[X_1] = \mathrm{SD}[X_2] = \cdots = \mathrm{SD}[X_{40}]$.

Compute μ and σ.