Financial Mathematics

Functional analysis

0053-1. $L^2(\mathbb{R})^{\mathbb{C}} := \left\{ f : \mathbb{R} \to \mathbb{C} \,\middle|\, \int_{-\infty}^{\infty} |f|^2 < \infty \right\}$

$L_C : L^2(\mathbb{R})^{\mathbb{C}} \to L^2(\mathbb{R})^{\mathbb{C}}$ defined by

$$(L_C f)(x) = \int_{-\infty}^{\infty} [e^{-ixy}][f(y)]\, dy.$$

$T^{\mathbb{C}} : L^2(\mathbb{R})^{\mathbb{C}} \to L^2(\mathbb{R})^{\mathbb{C}}$ defined by

$$(T^{\mathbb{C}} f)(x) = 7[f(x+3)] - 4[f(x+9)].$$

Find $r : \mathbb{R} \to \mathbb{C}$ s.t. $\forall f \in L^2(\mathbb{R})^{\mathbb{C}}$,

$$(L_C^{-1} T^{\mathbb{C}} L_C f)(x) = [r(x)][f(x)].$$