Financial Mathematics

Foundational material:
logic and set theory

Standard Notation

$\forall$ stands for for all
or, sometimes, for any

$\exists$ stands for there exists
or, sometimes, there exist

s.t. stands for such that

$\Rightarrow$ stands for implies

"$A \Rightarrow B$" is equivalent to "if A then B".

iff and $\Leftrightarrow$ both stand for if and only if

"$A \Leftrightarrow B$" is equivalent to "both $A \Rightarrow B$ and $B \Rightarrow A$".

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e.g.:

$\forall$

iff and $\Leftrightarrow$ both stand for if and only if

if $0 < |x - a| < \delta$, then $|[f(x)] - L| < \varepsilon$.

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$\forall \varepsilon > 0, \exists \delta > 0$ s.t.

if $0 < |x - a| < \delta$, then $|[f(x)] - L| < \varepsilon$.

$\Updownarrow$

$$0 < |x - a| < \delta \quad \Rightarrow \quad |[f(x)] - L| < \varepsilon$$

Standard Notation

QED marks the end of a proof

■ marks the end of a problem

e.g. means for example

i.e. means that is

$\Rightarrow$ stands for implies

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$0 < |x - a| < \delta \quad \Rightarrow \quad |[f(x)] - L| < \varepsilon$

The **size** (a.k.a. **cardinality**) of a set S, denoted $\#S$, is the number of elements in S.

e.g.: The size of $\{2,7,8\}$ is 3. $\#\{2,7,8\}=3$

$\emptyset = \{\quad\}$ is the set with no elements	$\#\emptyset = 0$

union: $\{4,5,6\}\cup\{5,6,7,8\}=\{4,5,6,7,8\}$

intersection: $\{4,5,6\}\cap\{5,6,7,8\}=\{5,6\}$

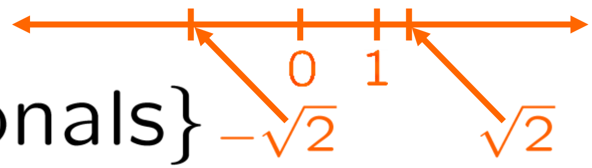
complement: $\{4,5,6\}\backslash\{5,6,7,8\}=\{4\}$

is not an elt of

$\in$ stands for is an element of	$7\in\{7,8,9\}$	$6\notin\{7,8,9\}$

$\mathbb{Z} := \{\text{integers}\} = \{\ldots,-2,-1,0,1,2,\ldots\}$

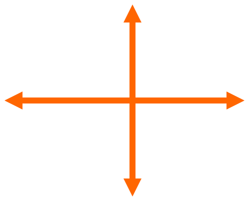
$\mathbb{R} := \{\text{real numbers}\}$ "="



$= \{\text{rationals}\}\cup\{\text{irrationals}\}$

$\mathbb{Q} := \{\text{rational numbers}\}$

$\mathbb{C} := \{\text{complex numbers}\}$ "="



disjoint union: $\{4,5,6\} \amalg \{1,2,3\} = \{1,2,3,4,5,6\}$

meaning: both $\{4,5,6\} \cup \{1,2,3\} = \{1,2,3,4,5,6\}$

and $\{4,5,6\} \cap \{1,2,3\} = \emptyset$

disjoint := empty intersection

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complement: $\{4,5,6\} \setminus \{5,6,7,8\} = \{4\}$

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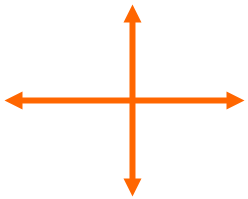

$A \subseteq B$ means: $\forall x \in A,\ x \in B$.
read: A "is a subset of" B

$B \supseteq A$ means: $\forall x \in A,\ x \in B$.
read: B "is a superset of" A

$A \subseteq B \quad \Leftrightarrow \quad B \supseteq A$

e.g.: $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$
$\mathbb{C} \supseteq \mathbb{R} \supseteq \mathbb{Q} \supseteq \mathbb{Z}$

$\in$ stands for is an element of	$7 \in \{7,8,9\}$	$6 \notin \{7,8,9\}$

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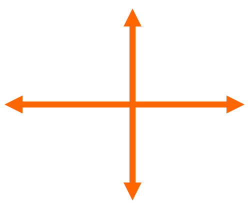
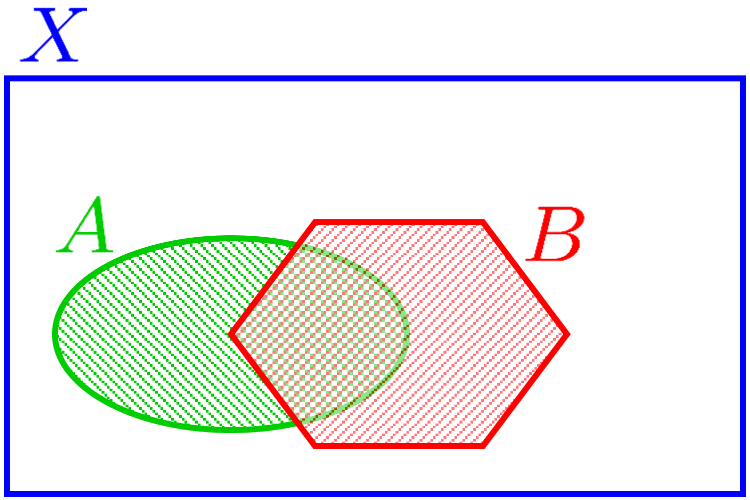
$\mathbb{C} := \{\text{complex numbers}\}$ "=" 

Set theoretic visualization: Venn diagrams

Union of two sets

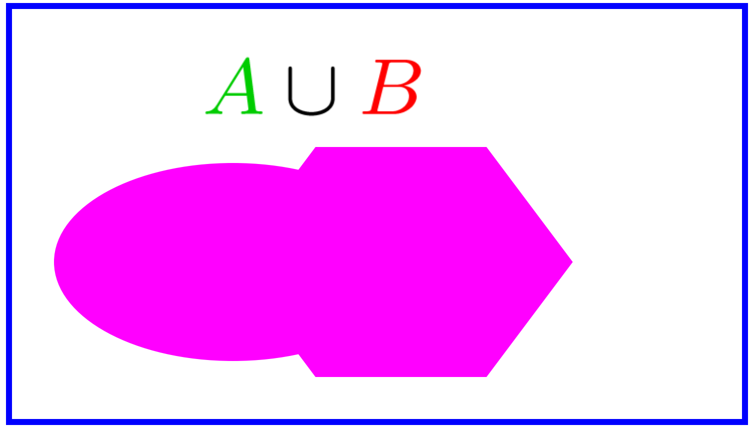
points in
either set?



Union of two sets

X A "union" B

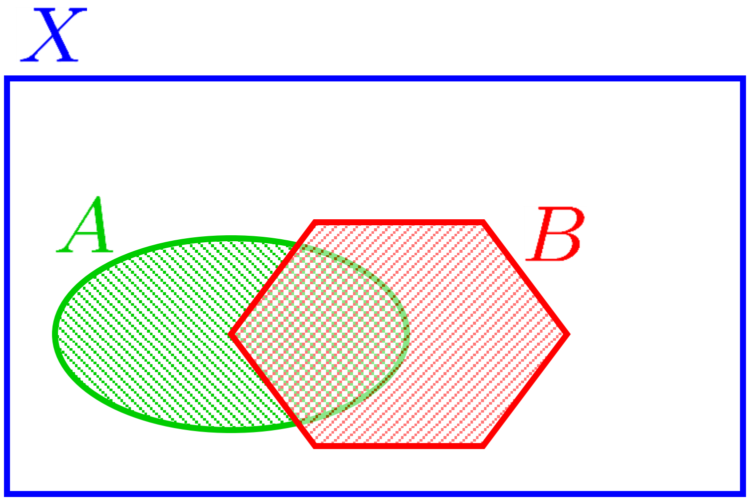
points in either set?



The "union" of A and B

Intersection of two sets

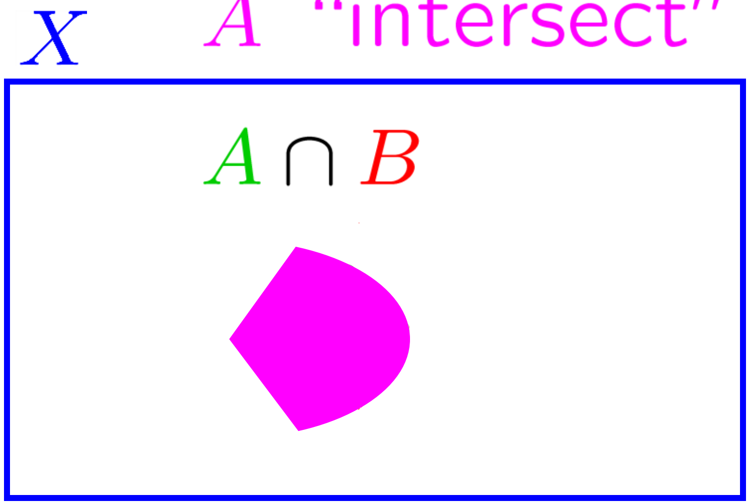
common points?



Intersection of two sets

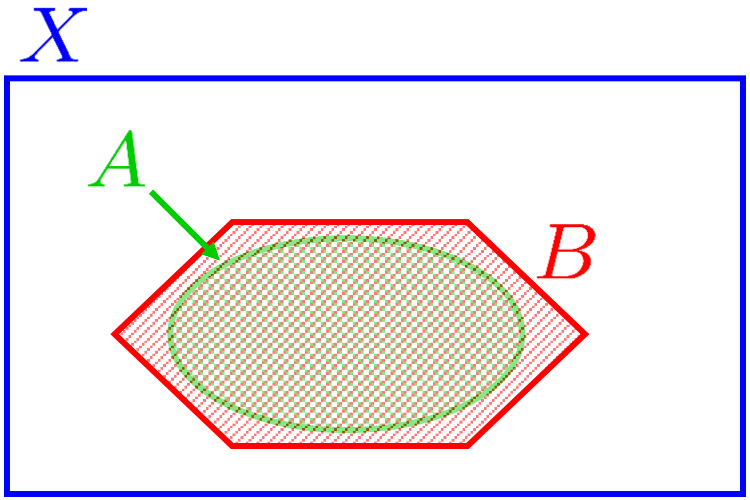
A “intersect” B

common points?



The “intersection” of A and B

Intersection of two sets

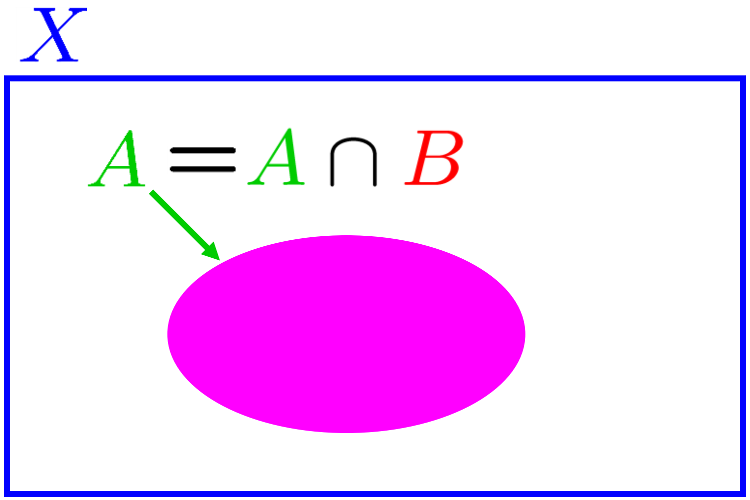


$A \subseteq B$

intersection?

Intersection of two sets

intersection?



If

$A \subseteq B$

then

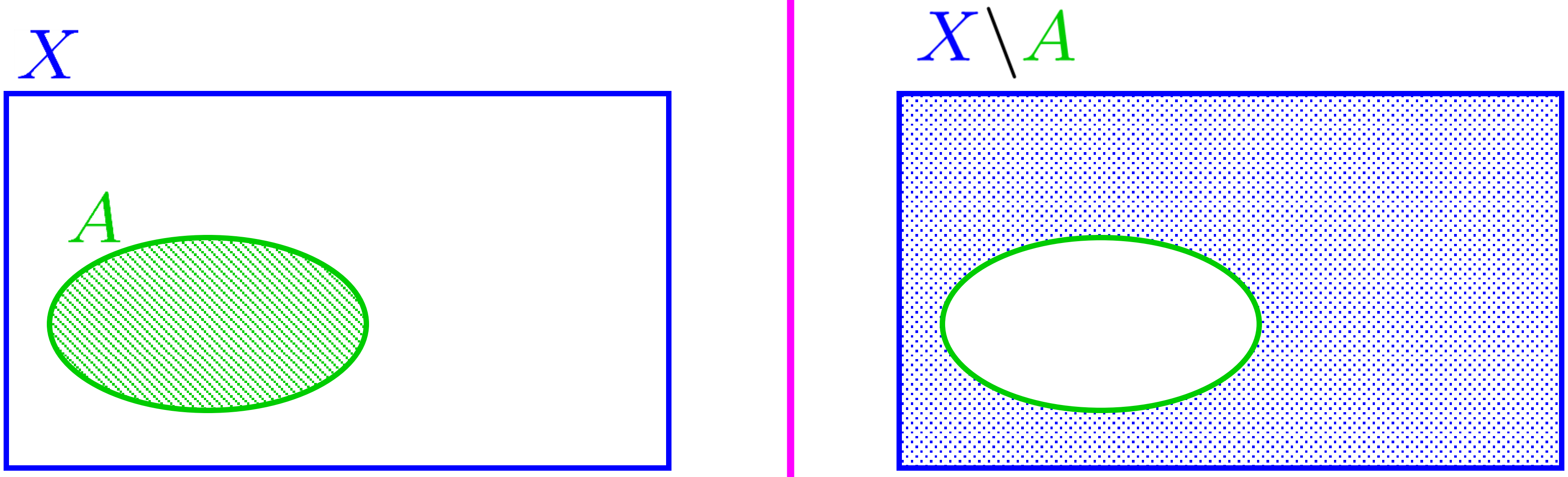
$A \cap B = A$.

Complement of a set

the complement of A (in X), or
X minus A

Def'n:

$$\boxed{X\backslash A} := \{x \in X \,|\, x \notin A\}$$



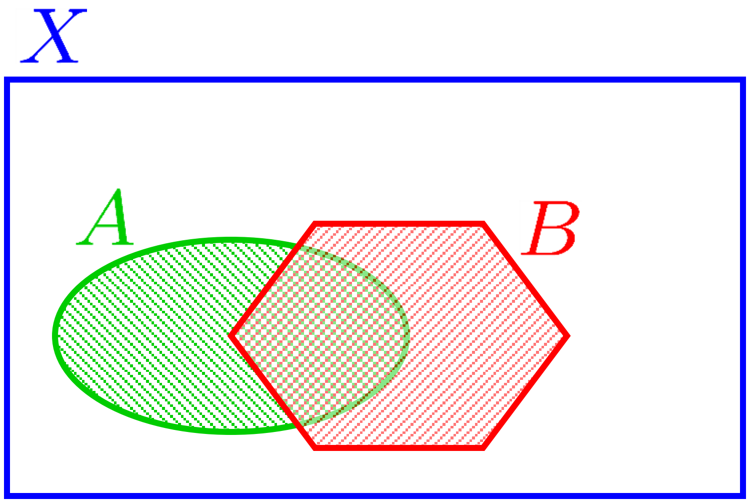
All sets under discussion are inside X;
X is the "universe".

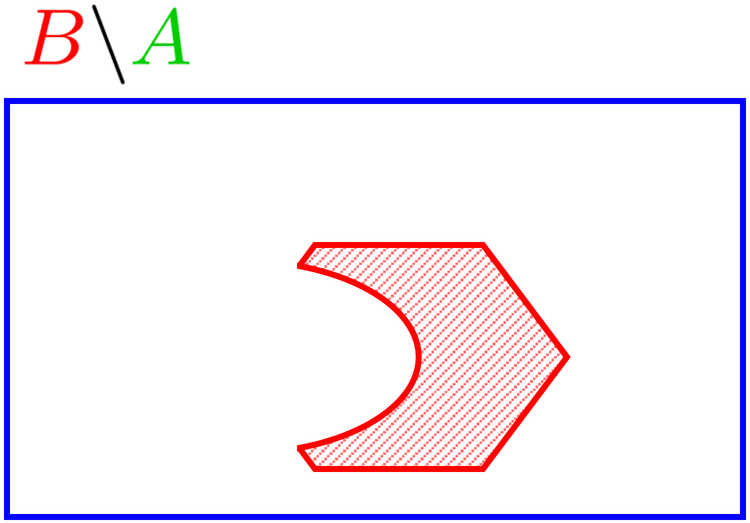
Set-theoretic difference

the complement of B in A, or
B minus A

Def'n:

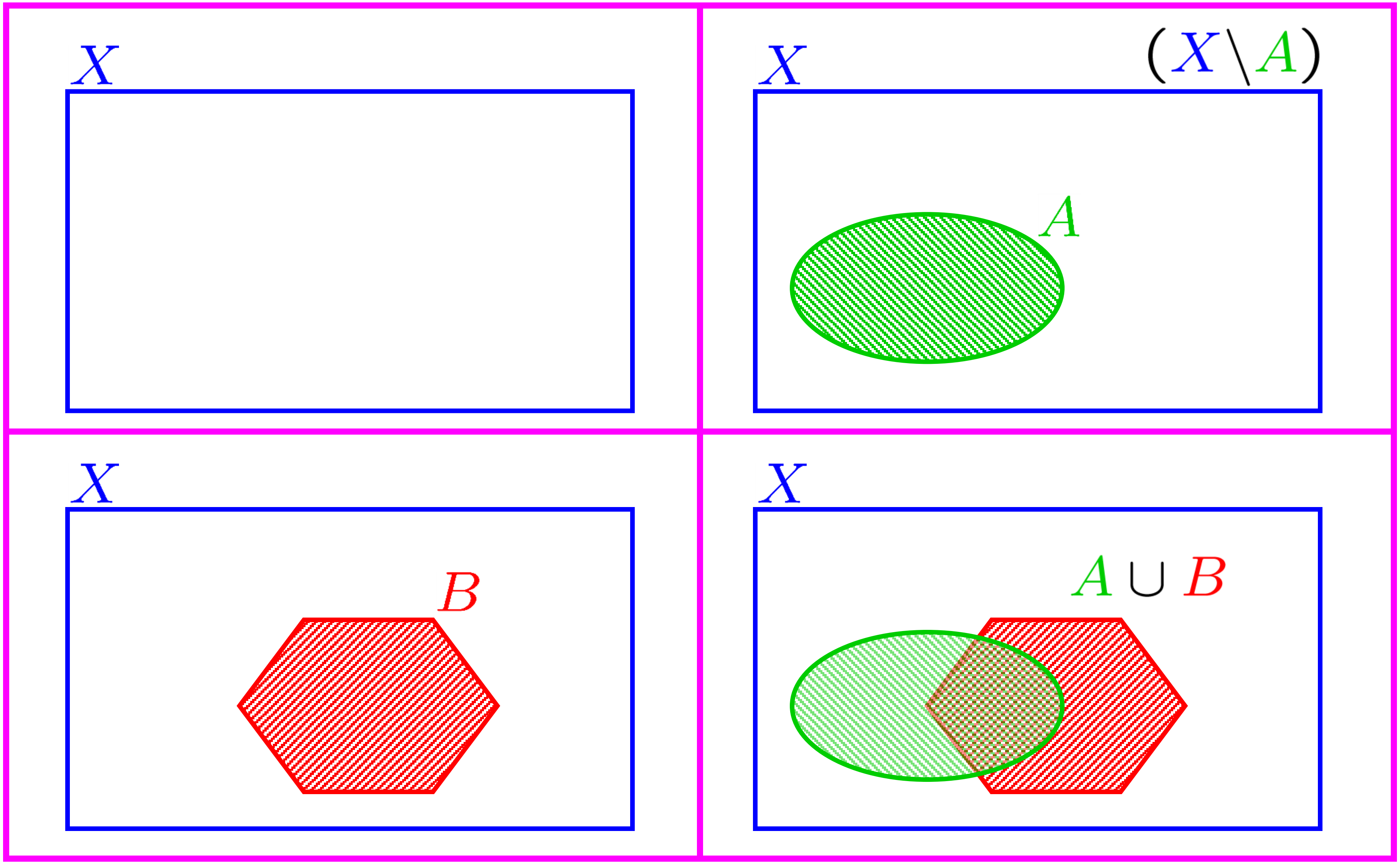
$$\boxed{B\backslash A} := \{x \in B \,|\, x \notin A\}$$





Set theoretic visualization: Venn diagrams

Complement of the union



Set theoretic visualization: Venn diagrams

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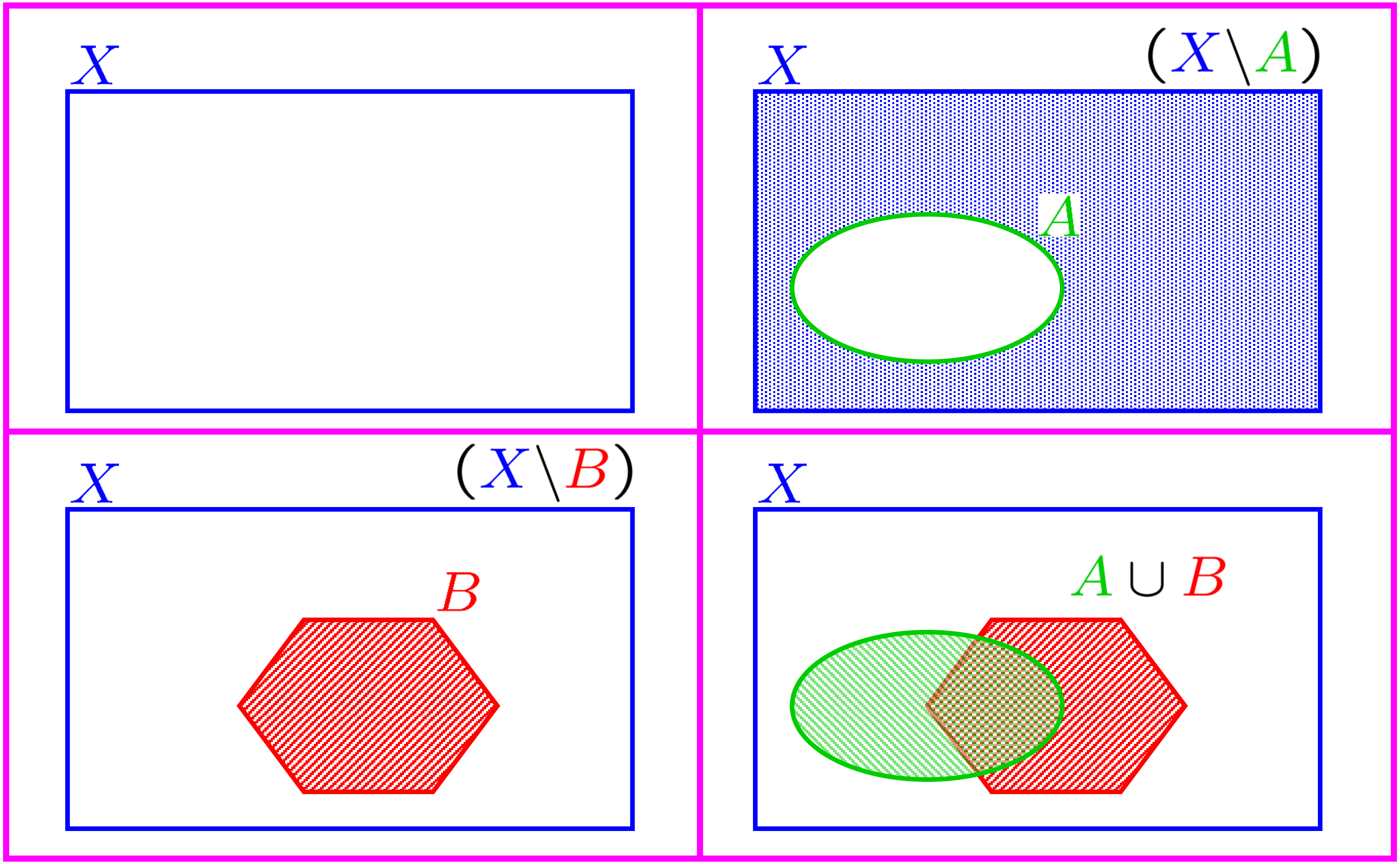

Set theoretic visualization: Venn diagrams

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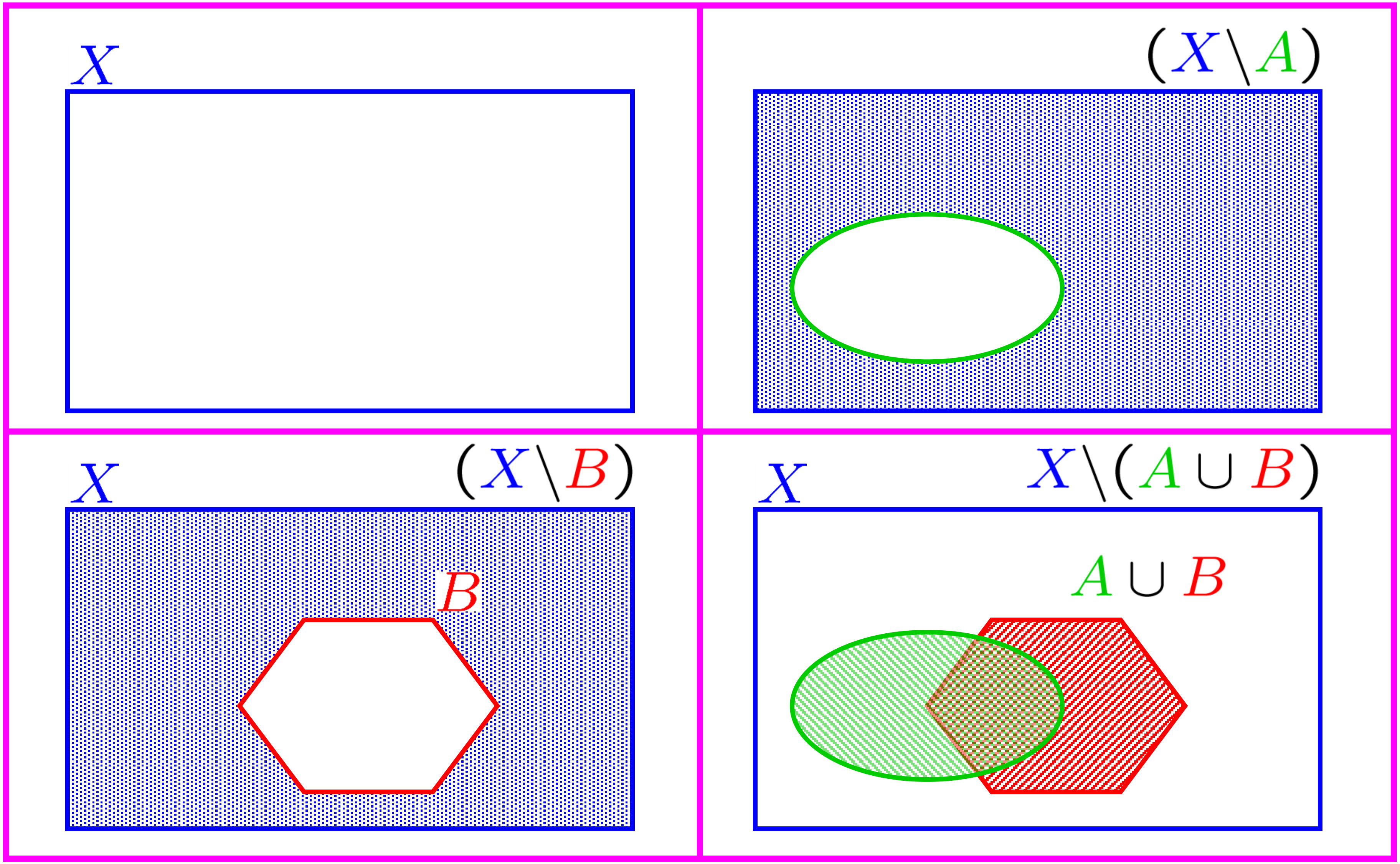

Set theoretic visualization: Venn diagrams

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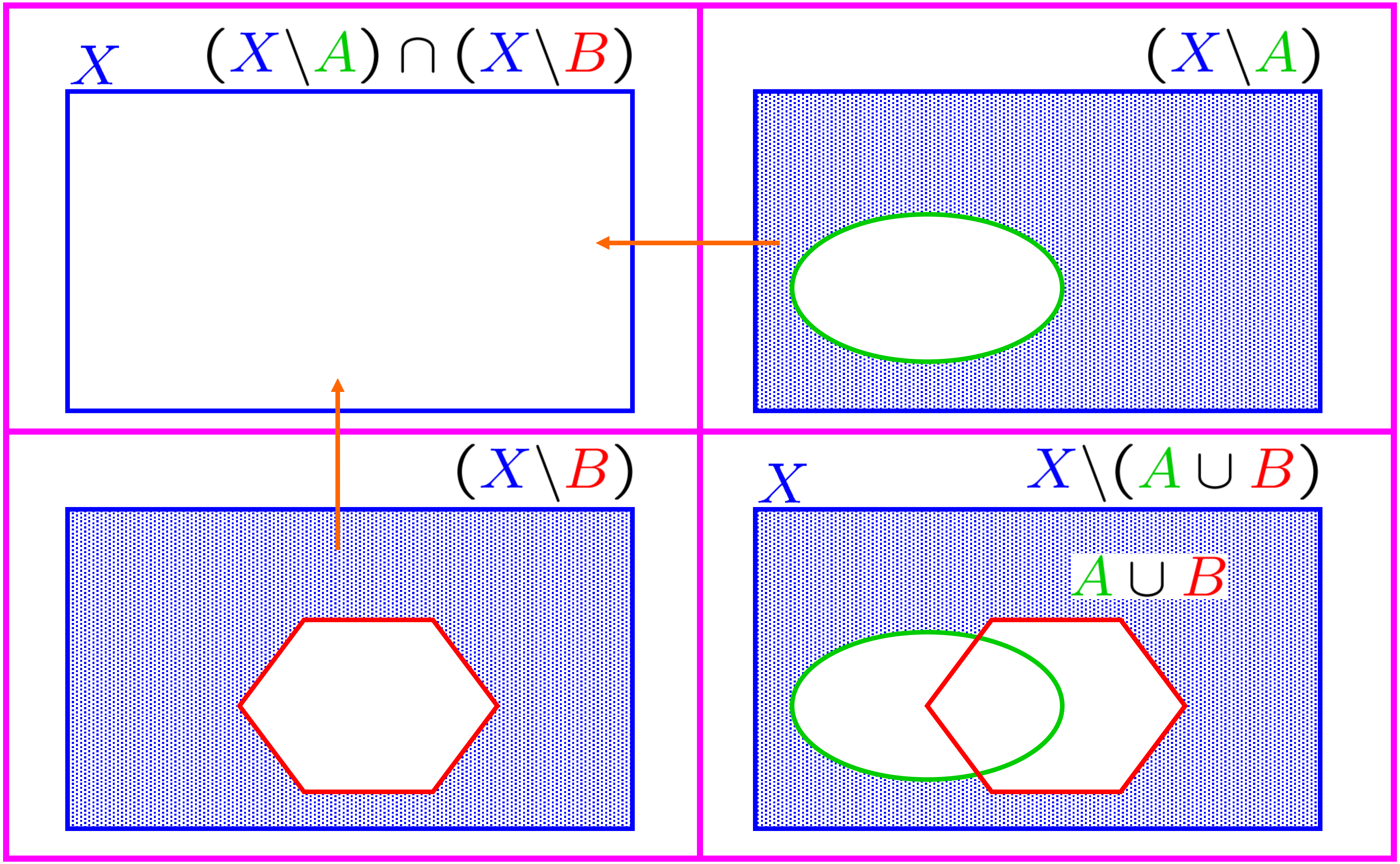
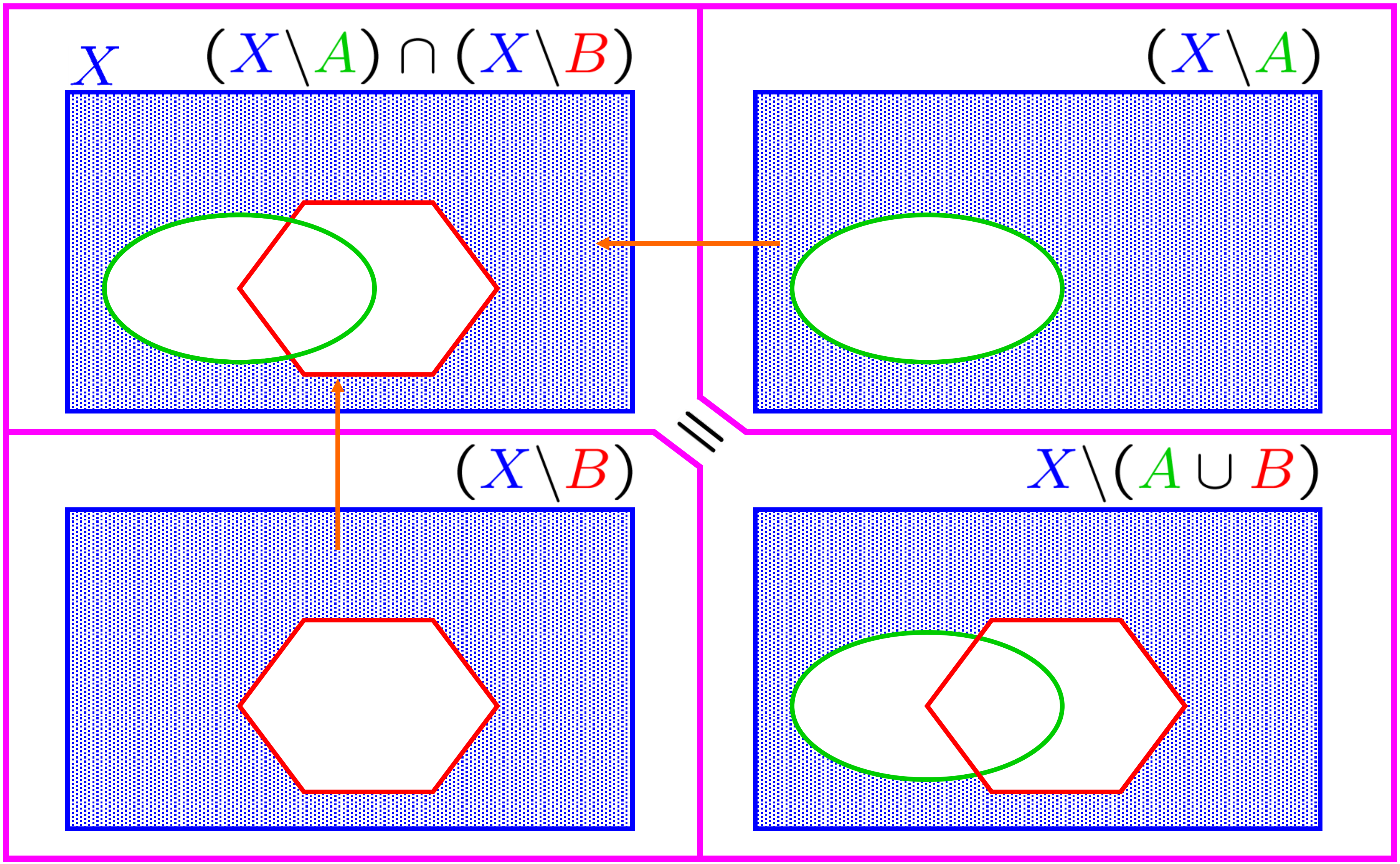


Set theoretic visualization: Venn diagrams

Fact: $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$



Definition: A **partition** of a set S is a set $\mathcal{P}$ of subsets of S such that

both $\displaystyle\bigcup_{P\in\mathcal{P}} P = S$

pairwise disjoint

and $\forall P, Q \in \mathcal{P}, \quad (P \neq Q \quad \Rightarrow \quad P \cap Q = \emptyset)$.

e.g.: $\{ \quad \{1\} \quad , \quad \{2,3,5\} \quad , \quad \{4,6\} \quad \}$
is a partition of $\{1,2,3,4,5,6\}$.

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is a partition of $\{1,2,3\}$.

non-e.g.: $\{ \quad \{1,2\} \quad , \quad \{2,3\} \quad \}$
$\{1,2\} \cap \{2,3\} \neq \emptyset$ is not a partition of $\{1,2,3\}$.

non-e.g.: $\{ \quad \{1\} \quad , \quad \{3\} \quad \}$ $2 \notin \{1\} \cup \{3\}$
is not a partition of $\{1,2,3\}$.

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$\Updownarrow$

$\coprod_{P \in \mathcal{P}} P = S$

e.g.: $\{1\} \coprod \{2,3,5\} \coprod \{4,6\} = \{1,2,3,4,5,6\}$

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$\displaystyle\coprod_{P\in\mathcal{P}} P = S$

$\forall$set S, $\displaystyle\coprod_{s\in S} \{s\} = S$

e.g.: $\{1\} \amalg \{2,3,5\} \amalg \{4,6\} = \{1,2,3,4,5,6\}$

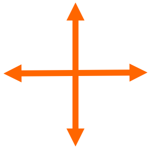
e.g.: $\cdots \amalg \{-2\} \amalg \{-1\} \amalg \{0\} \amalg \{1\} \amalg \{2\} \amalg \cdots = \mathbb{Z}$

i.e.: $\displaystyle\coprod_{n\in\mathbb{Z}} \{n\} = \mathbb{Z}$

e.g.: $\displaystyle\coprod_{a\in\mathbb{R}} \{a\} = \mathbb{R}$

$$\boxed{A \times B} := \{(a,b) \mid a \in A, b \in B\}$$

$$\boxed{A^n} := \{(a_1, \ldots, a_n) \mid a_1, \ldots, a_n \in A\}$$

e.g.: $\mathbb{R}^2 := \{(x,y) \mid x, y \in \mathbb{R}\}$ "=" 

$$\mathbb{R} \times \{3\} = \{(x,3) \in \mathbb{R}^2 \mid x \in \mathbb{R}\}.$$

(a,b) is an ordered pair,

i.e., $(a,b) \neq (b,a)$.

By contrast, $\{a,b\}$ is a set, and so is **unordered**,

i.e., $\{a,b\} = \{b,a\}$.

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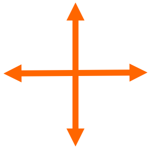
e.g.: $\mathbb{R}^2 := \{(x,y) \mid x, y \in \mathbb{R}\}$ "=" 

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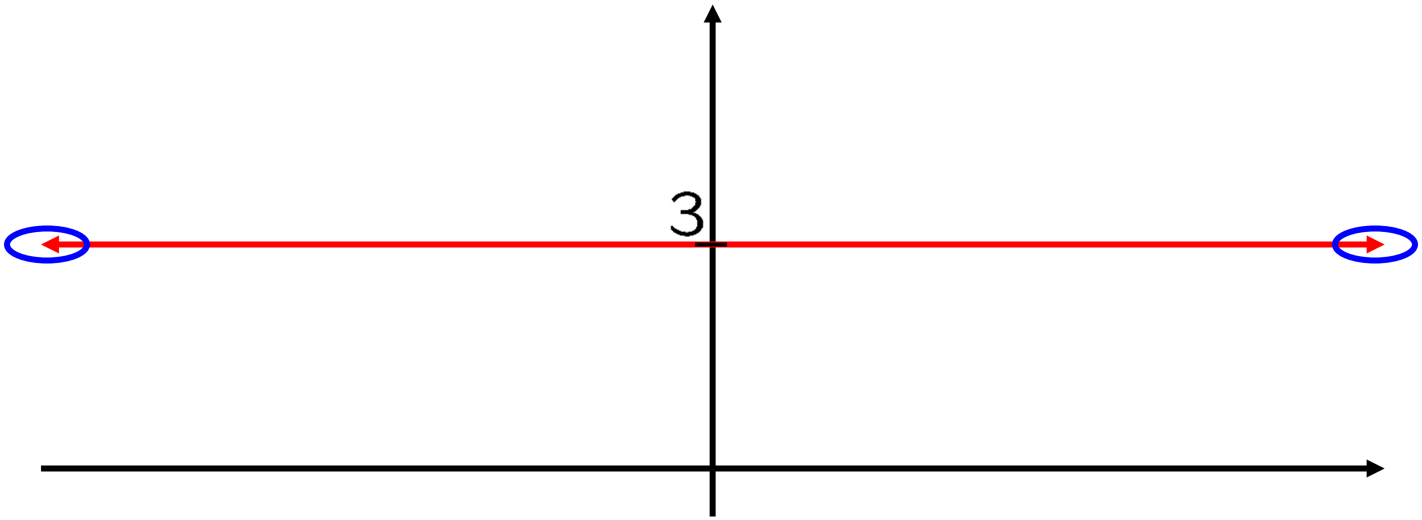

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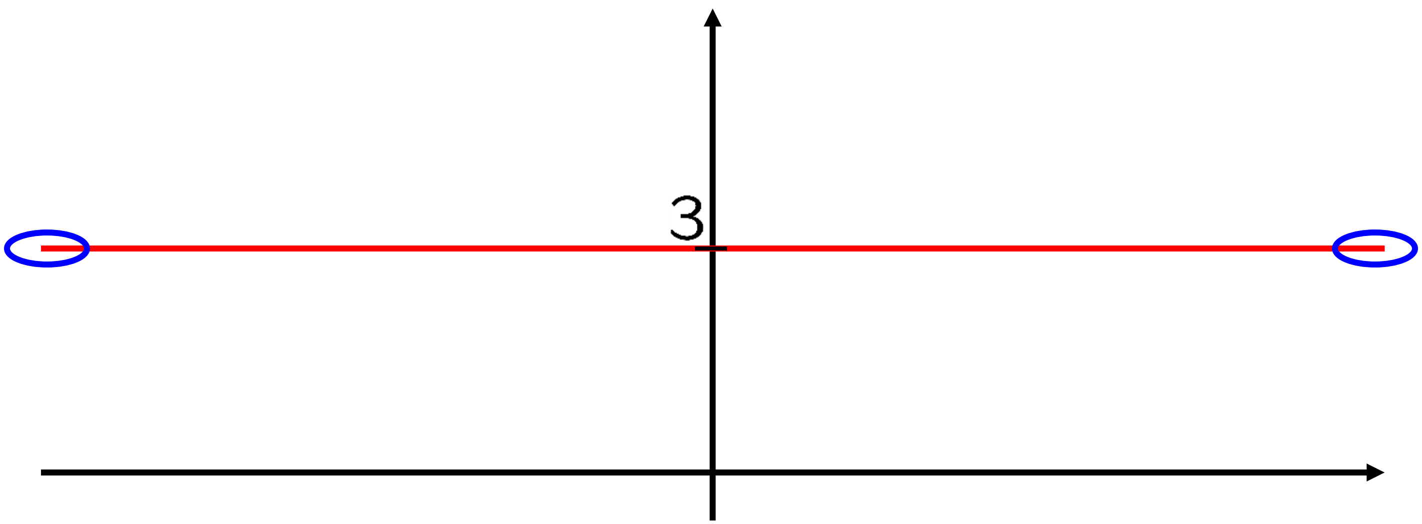

Def'n: $\forall$integers $n \geq 1$,

Euclidean n**-space** $:= \mathbb{R}^n$