Financial Mathematics

Functions and expressions

domain target

Let A and B be sets.

A **function** $f : A \to B$ is a rule that assigns, to each element of A, an element of B.

$\forall a \in A$, the element of B assigned to a by f is denoted $\boxed{f(a)}$.

---

e.g.:

Define $f : \{1, 2, 3\} \to \{4, 5, 6, 7\}$ by

$$f(1) = 5,$$
$$f(2) = 5,$$
$$f(3) = 6.$$

When f is clear, we sometimes use:

$$1 \mapsto 5,$$
$$2 \mapsto 5,$$
$$3 \mapsto 6.$$

domain target

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e.g.:

all must be used exactly once

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---

e.g.:

all must be used exactly once

can remove them. . . unused, and that's OK

Define $f : \{1, 2, 3\} \to \{4, 5, 6, 7\}$ by

used twice, and that's OK

$1 \mapsto 5,$

$2 \mapsto 5,$

$3 \mapsto 6.$

domain target

Let A and B be sets.

A **function** $f : A \to B$ is a rule that assigns, to each element of A, an element of B.

$\forall a \in A$, the element of B assigned to a by f is denoted $\boxed{f(a)}$.

---

e.g.: function unchanged

Define $f : \{1, 2, 3\} \to \{5, 6\}$ by

$$1 \mapsto 5,$$
$$2 \mapsto 5,$$
$$3 \mapsto 6.$$

---

e.g.: $(-\infty, 0)$ unused can remove...

Define $f : \mathbb{R} \to \mathbb{R}$ by

$$\forall x \in \mathbb{R}, \quad f(x) = x^2.$$

domain target

Let A and B be sets.

A **function** $f : A \to B$ is a rule that assigns, to each element of A, an element of B.

$\forall a \in A$, the element of B assigned to a by f is denoted $\boxed{f(a)}$.

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e.g.:

Define $f : \{1, 2, 3\} \to \{5, 6\}$ by

$$1 \mapsto 5,$$
$$2 \mapsto 5,$$
$$3 \mapsto 6.$$

---

e.g.:

function unchanged

Define $f : \mathbb{R} \to [0, \infty)$ by

$$f(x) = x^2.$$

domain → A, target → B

Let A and B be sets.

A **function** $f : A \to B$ is a rule that assigns, to each element of A, an element of B.

$\forall a \in A$, the element of B assigned to a by f is denoted $\boxed{f(a)}$.

Notational Convention

Suppose $G : \mathbb{R}^3 \to \mathbb{R}$ is some function.
To plug $(2,3,4) \in \mathbb{R}^3$ into G,
we do not usually write $G((2,3,4))$,
but rather $G(2,3,4)$.

---

e.g.:

Define $f : \mathbb{R} \to [0,\infty)$ by

$$f(x) = x^2.$$

domain → A, target → B

Let A and B be sets.

A **function** $f : A \to B$ is a rule that assigns, to each element of A, an element of B.

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Suppose $G : \mathbb{R}^3 \to \mathbb{R}$ is some function.

To plug $(2, 3, 4) \in \mathbb{R}^3$ into G,
we do not usually write $G((2, 3, 4))$,
but rather $G(2, 3, 4)$.

Either is correct,
but $G(2, 3, 4)$ is strongly preferred.

The **image** of a function $f : A \to B$ is

$$f(A) := \{f(a) \mid a \in A\}.$$

function

A fn $f : A \to B$ is **onto** C if $f(A) = C$.

"$f : A \to B$ **is onto**" or
"$f : A \to B$ **is surjective**"
means: "$f : A \to B$ is onto B"

A function $f : A \to B$ is **one-to-one** (a.k.a. **1-1**,**injective**) if, $\forall a, a' \in A$,

$$a \neq a' \quad \Rightarrow \quad f(a) \neq f(a')$$

Define $g : \{1, 2, 3\} \to \{1, 2\}$ by

two numbers map to one number

$1 \mapsto 2$,
$2 \mapsto 2$,
$3 \mapsto 1$.

means "onto $\{1, 2\}$"

Then $g : \{1, 2, 3\} \to \{1, 2\}$ is onto,
but not 1-1.

The **image** of a function $f : A \to B$ is

$$\boxed{f(A)} := \{f(a) \,|\, a \in A\}.$$

function

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unused

Define $g : \{1, 2\} \to \{\boxed{1}, 2, 3\}$ by

$$1 \mapsto 2,$$
$$2 \mapsto 3.$$

means "onto $\{1, 2, 3\}$"

Then $g : \{1, 2\} \to \{1, 2, 3\}$ is 1-1, but not onto.

The **image** of a function $f : A \to B$ is

$$\boxed{f(A)} := \{f(a) \mid a \in A\}.$$

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unused

Define $g : \{1, 2, 3\} \to \{7, 8, \boxed{9}\}$ by

two numbers map to one number

$1 \mapsto 8$,
$2 \mapsto 8$,
$3 \mapsto 7$.

means "onto $\{7, 8, 9\}$"

Then $g : \{1, 2, 3\} \to \{7, 8, 9\}$ is neither 1-1, nor onto.

Define $g : \{1, 2, 3\} \to \{4, 5, 6\}$ by

$$1 \mapsto 5,$$
$$2 \mapsto 6,$$
$$3 \mapsto 4.$$

Then $g : \{1, 2, 3\} \to \{4, 5, 6\}$ is bijective.

Define $h : \{4, 5, 6\} \to \{1, 2, 3\}$ by

$$5 \mapsto 1,$$
$$6 \mapsto 2,$$
$$4 \mapsto 3.$$

out of order

We say $f : A \to B$ is **bijective**
if $f : A \to B$ is both 1-1 and onto.

Define $g : \{1,2,3\} \to \{4,5,6\}$ by

$$1 \mapsto 5,$$
$$2 \mapsto 6,$$
$$3 \mapsto 4.$$

Then $g : \{1,2,3\} \to \{4,5,6\}$ is bijective.

Define $h : \{4,5,6\} \to \{1,2,3\}$ by

$$4 \mapsto 3,$$
$$5 \mapsto 1,$$
$$6 \mapsto 2.$$

Then, $\forall x \in \{1,2,3\}$, $h(g(x)) = x$

and, $\forall y \in \{4,5,6\}$, $g(h(y)) = y$.

We say $f : A \to B$ is **bijective**

if $f : A \to B$ is both 1-1 and onto.

Define $g : \{1, 2, 3\} \to \{4, 5, 6\}$ by

$$\begin{aligned} 1 &\mapsto 5, \\ 2 &\mapsto 6, \\ 3 &\mapsto 4. \end{aligned}$$

Then $g : \{1, 2, 3\} \to \{4, 5, 6\}$ is bijective.

Define $h : \{4, 5, 6\} \to \{1, 2, 3\}$ by

$$\begin{aligned} 4 &\mapsto 3, \\ 5 &\mapsto 1, \\ 6 &\mapsto 2. \end{aligned}$$

Then, $\forall x \in \{1, 2, 3\}$, $h(g(x)) = x$

and, $\forall y \in \{4, 5, 6\}$, $g(h(y)) = y$.

We say that g and h are **inverses**.

A function $f : A \to B$ is bijective

iff f has an inverse $B \to A$.

If $\phi : A \to B$ and $\psi : B \to C$,

"ϕ then ψ"

then $\boxed{\psi \circ \phi} : A \to C$ is defined by

"the **composite** **of ψ and ϕ**"

$$(\psi \circ \phi)(a) = \psi(\phi(a)).$$

Then, $\forall x \in \{1, 2, 3\}$, $\boxed{h(g(x))} = x$

and, $\forall y \in \{4, 5, 6\}$, $\boxed{g(h(y))} = y$.

We say that g and h are **inverses**.

A function $f : A \to B$ is bijective iff f has an inverse $B \to A$.

If $\phi : A \to B$ and $\psi : B \to C$,

"ϕ then ψ"

then $\psi \circ \phi : A \to C$ is defined by

"the **composite** of ψ and ϕ"

$$(\psi \circ \phi)(a) = \psi(\phi(a)).$$

Def'n: For any set A,

"the **identity** on A"

$\mathrm{id}_A : A \to A$ is defined by: $\mathrm{id}_A(x) = x$.

Then, $\forall x \in \{1, 2, 3\}$, $(h \circ g)(x) = x$ $\quad h \circ g = \mathrm{id}_{\{1,2,3\}}$

and, $\forall y \in \{4, 5, 6\}$, $(g \circ h)(y) = y$. $\quad g \circ h = \mathrm{id}_{\{4,5,6\}}$

We say that g and h are **inverses**.

A function $f : A \to B$ is bijective

iff f has an inverse $B \to A$.

If $\phi : A \longrightarrow B$ and $\psi : B \longrightarrow C$,

"ϕ then ψ"

then $\boxed{\psi \circ \phi} : A \longrightarrow C$ is defined by

"the **composite of ψ and ϕ**"

$$(\psi \circ \phi)(a) = \psi(\phi(a)).$$

Fact: If $\phi : A \to B$, if $\psi : B \to C$, and if $\chi : C \to D$,

then $(\chi \circ \psi) \circ \phi = \chi \circ (\psi \circ \phi)$

composition is associative

$$\alpha : \{1,2,3\} \to \{1,2,3\}$$
$$\begin{aligned} 1 &\mapsto 2 \\ 2 &\mapsto 3 \\ 3 &\mapsto 1 \end{aligned}$$

$$\beta : \{1,2,3\} \to \{1,2,3\}$$
$$\begin{aligned} 1 &\mapsto 2 \\ 2 &\mapsto 1 \\ 3 &\mapsto 3 \end{aligned}$$

Exercise: Check that $(\alpha \circ \beta)(1) \neq (\beta \circ \alpha)(1)$.

composition is not commutative

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composition is associative

$$f(x) = x^2$$

function

expression

expression

$$[x^2]_{x:\to 3} = 9$$

$$[x^2]_{x:\to 3}^{x:\to 4} = 16 - 9$$

$$f(t) = t^2$$

$$[t^2]_{t:\to 3} = 9$$

$$[t^2]_{t:\to 3}^{t:\to 4} = 16 - 9$$

$$f = (\ )^2$$

$$f(3) = 9$$

$$f|_3^4 = 16 - 9$$

Def'n: Let $b \in \overline{\mathbb{R}}$, $S \subseteq \overline{\mathbb{R}}$.

We say b is the **inf** (or **glb**) **of** S, and write $b = \inf S$,

infimum — **greatest lower bound**

$\forall$lower bound a for S, we have $a \leq b$.

If, in addition, $b \in S$,

then we say that b is the **min** **of** S, and write $b = \min S$.

minimum

Def'n: Let $b \in \overline{\mathbb{R}}$, $S \subseteq \overline{\mathbb{R}}$.

We say b is the **sup** (or **lub**) **of** S, and write $b = \sup S$,

supremum — **least upper bound**

$\forall$upper bound a for S, we have $a \geq b$.

If, in addition, $b \in S$,

then we say that b is the **max** **of** S, and write $b = \max S$.

maximum

Say f is real-valued and defined on A.

I.e., $A \subseteq \operatorname{dom} f$ and $\operatorname{im} f \subseteq \mathbb{R}$.

Def'n: $\sup_A f := \sup \underbrace{f(A)}_{\{f(a) \,|\, a \in A\}}$

$\sup_A f$ = $\sup_{x \in A} f(x) =: \sup_{s \in A} f(s)$ *etc.*

Def'n: Let $b \in \overline{\mathbb{R}}$, $S \subseteq \overline{\mathbb{R}}$.

We say b is the **sup** (supremum) (or **lub** (least upper bound)) **of** S,

and write $b = \sup S$,

$\forall$upper bound a for S, we have $a \geq b$.

If, in addition, $b \in S$,

then we say that b is the **max** (maximum) **of** S,

and write $b = \max S$.

Say f is real-valued and defined on A.

Def'n: $\sup_A f := \sup f(A)$

$$\sup_A f = \sup_{x\in A} f(x) =: \sup_{s\in A} f(s) \quad \textit{etc.}$$

e.g.: $\sup_{(-2,5)} (\bullet)^3 = \sup(-8, 125) = 125$

sup :→ inf

$$\sup_{(-2,5)} (\bullet)^3 = \sup_{x\in(-2,5)} x^3 = \sup_{s\in(-2,5)} s^3 \quad \textit{etc.}$$

$$\sup_{x\in(-2,5)} x^3 = \sup_{-2<x<5} x^3, \qquad \sup_{s\in(-2,5)} s^3 = \sup_{-2<s<5} s^3 \quad \textit{etc.}$$

Say f is real-valued and defined on A.

Def'n: $\inf_A f := \inf f(A)$

$$\inf_A f \; = \; \inf_{x\in A} f(x) =: \inf_{s\in A} f(s) \quad \textit{etc.}$$

e.g.: $\inf_{(-2,5)} (\bullet)^3 = \inf\,(-8, 125) = -8$

inf :→ max

$$\inf_{(-2,5)} (\bullet)^3 = \inf_{x\in(-2,5)} x^3 = \inf_{s\in(-2,5)} s^3 \quad \textit{etc.}$$

$$\inf_{x\in(-2,5)} x^3 = \inf_{-2<x<5} x^3, \qquad \inf_{s\in(-2,5)} s^3 = \inf_{-2<s<5} s^3 \quad \textit{etc.}$$

Say f is real-valued and defined on A.

Def'n: $\max\limits_A f := \max f(A)$

$$\max_A f = \max_{x\in A} f(x) =: \max_{s\in A} f(s) \quad \textit{etc.}$$

e.g.: $\max\limits_{(-2,5)} (\bullet)^3$, $\max(-8, 125)$,

$(-2, 5) :\rightarrow [-2, 5]$

$\max\limits_{x\in(-2,5)} x^3$, $\max\limits_{s\in(-2,5)} s^3$, *etc.* ,

$\max\limits_{-2<x<5} x^3$, $\max\limits_{-2<s<5} s^3$, *etc.*

do not exist

Say f is real-valued and defined on A.

Def'n: $\max_A f := \max f(A)$

$$\max_A f = \max_{x \in A} f(x) =: \max_{s \in A} f(s) \quad \textit{etc.}$$

e.g.: $\max_{[-2,5]} (\bullet)^3 = \max[-8, 125] = 125$

max :→ min

$$\max_{[-2,5]} (\bullet)^3 = \max_{x \in [-2,5]} x^3 = \max_{s \in [-2,5]} s^3 \quad \textit{etc.}$$

$$\max_{x \in [-2,5]} x^3 = \max_{-2 \leq x \leq 5} x^3, \qquad \max_{s \in [-2,5]} s^3 = \max_{-2 \leq s \leq 5} s^3 \quad \textit{etc.}$$

Say f is real-valued and defined on A.

Def'n: $\min\limits_A f := \min f(A)$

$\|$

$\min\limits_{x\in A} f(x) =: \min\limits_{s\in A} f(s)$ *etc.*

e.g.: $\min\limits_{[-2,5]} (\bullet)^3 = \min\,[-8, 125] = -8$

$\|$

$\min\limits_{x\in[-2,5]} x^3 = \min\limits_{s\in[-2,5]} s^3$ *etc.*

$\|$ $\|$

$\min\limits_{-2\le x\le 5} x^3$ $\min\limits_{-2\le s\le 5} s^3$ *etc.*

Say f is real-valued and defined on A.

Def'n: $\min_A f := \min f(A)$

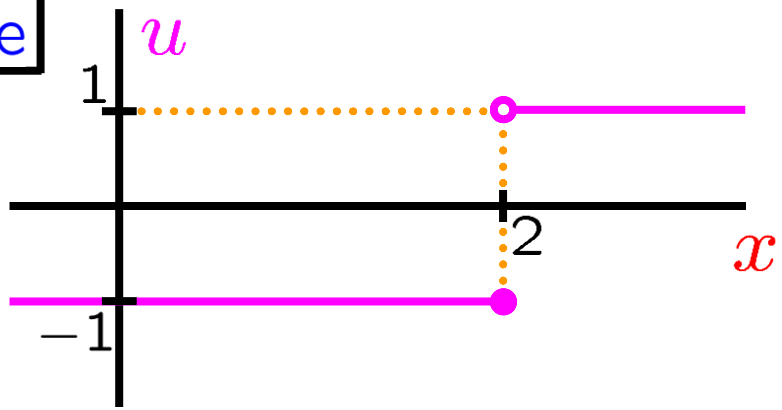
$$\min_A f = \min_{x \in A} f(x) =: \min_{s \in A} f(s) \quad \textit{etc.}$$

Th'm (Extreme Value Theorem):

If f is continuous on a compact interval I, then both $\max_I f$ and $\min_I f$ exist.

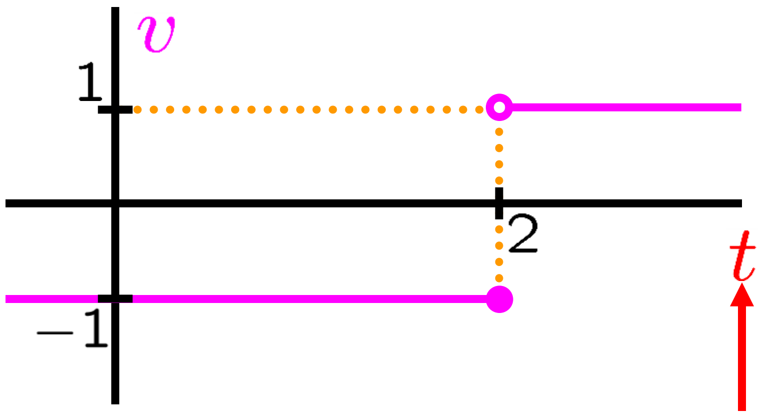
continue à gauche, limite à droite

$$u = \begin{cases} -1, & \text{if } x \le 2 \\ 1, & \text{if } x > 2 \end{cases}$$



u is **caglad** in x

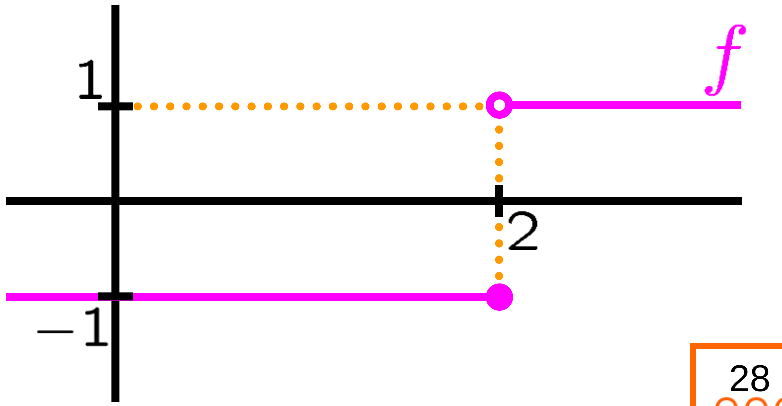
$$v = \begin{cases} -1, & \text{if } t \le 2 \\ 1, & \text{if } t > 2 \end{cases}$$



v is **caglad** in t

unusual notation

$$f = \begin{cases} -1, & \text{if } \bullet \le 2 \\ 1, & \text{if } \bullet > 2 \end{cases}$$



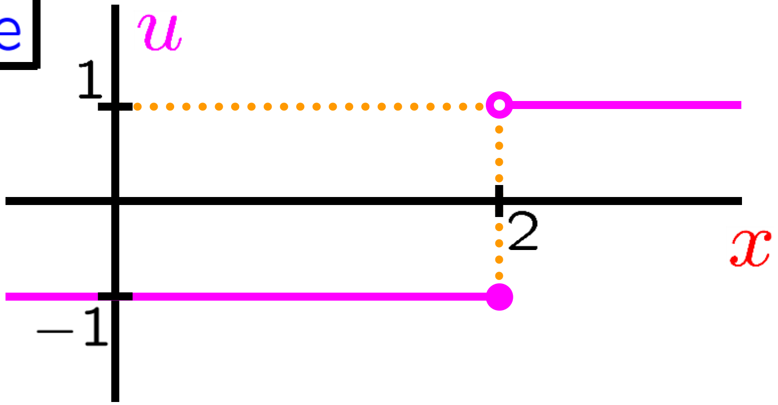
f is **caglad**

continue à gauche, limite à droite

$$u = \begin{cases} -1, & \text{if } x \le 2 \\ 1, & \text{if } x > 2 \end{cases}$$

// $f(x)$

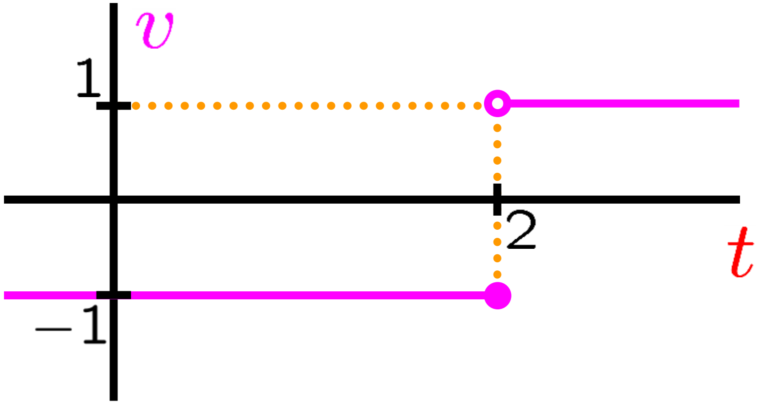
u is **caglad** in x



$$v = \begin{cases} -1, & \text{if } t \le 2 \\ 1, & \text{if } t > 2 \end{cases}$$

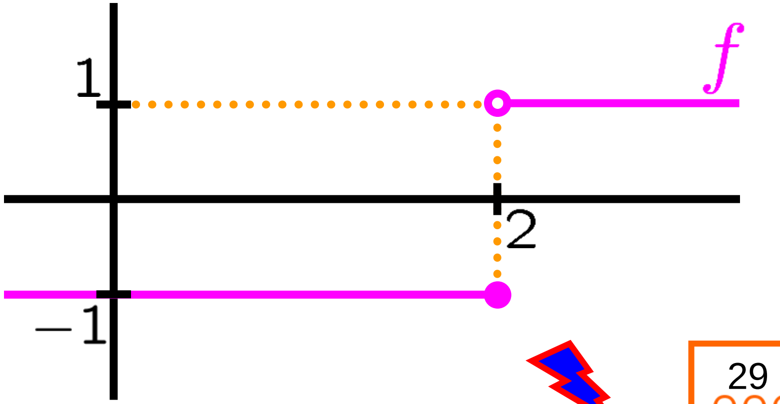
// $f(t)$

v is **caglad** in t



$$f(x) = \begin{cases} -1, & \text{if } \boxed{x} \le 2 \\ 1, & \text{if } \boxed{x} > 2 \end{cases}$$

f is **caglad**



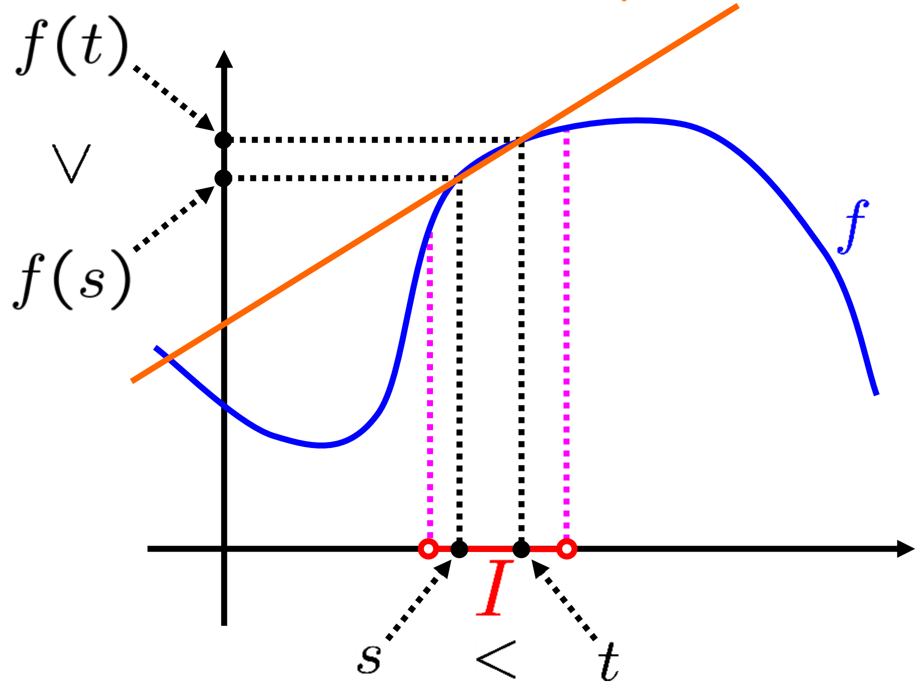
DEFINITION: Let I be an interval.

A function f is called **increasing on** I if

$$f(s) < f(t) \quad \text{whenever } s, t \in I \text{ and } s < t.$$

e.g.:

$$\text{slope} = \frac{[f(t)] - [f(s)]}{t - s} > 0$$



"secant lines run uphill" (slopes > 0)

DEFINITION: Let I be an interval.

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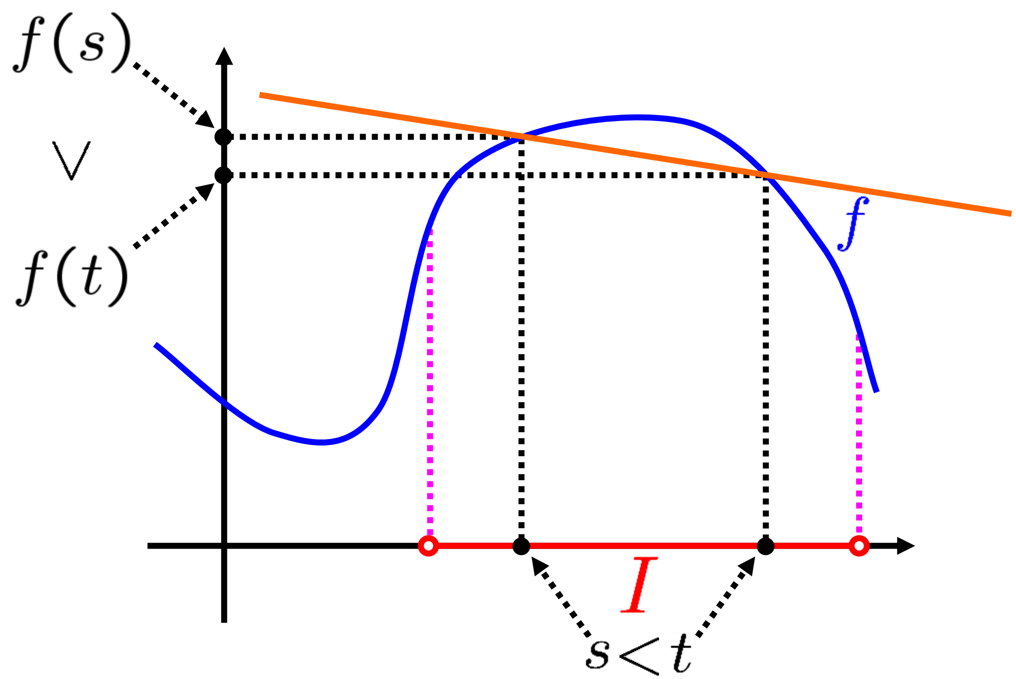
non-*e.g.*:



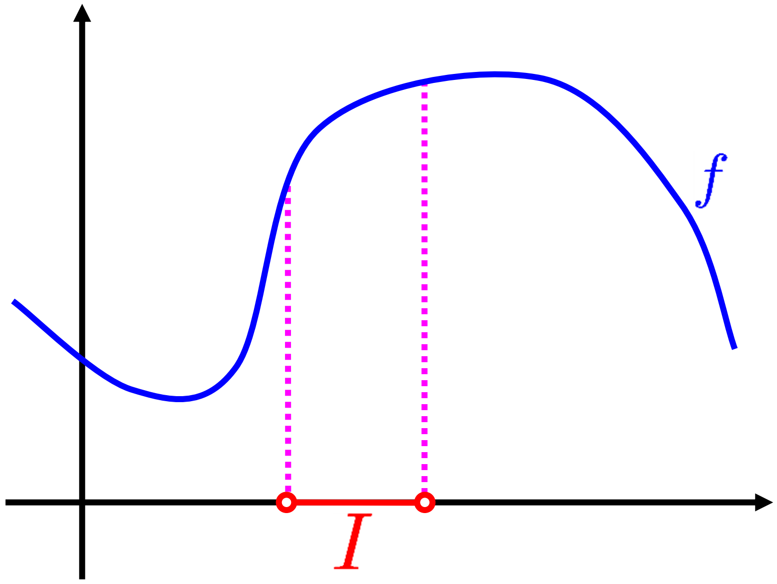
"there's a secant line that does not run uphill"

DEFINITION: Let I be an interval.

A function f is called **increasing on** I if

$f(s) < f(t)$ whenever $s, t \in I$ and $s < t$.

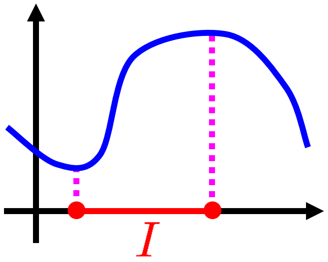
e.g.:



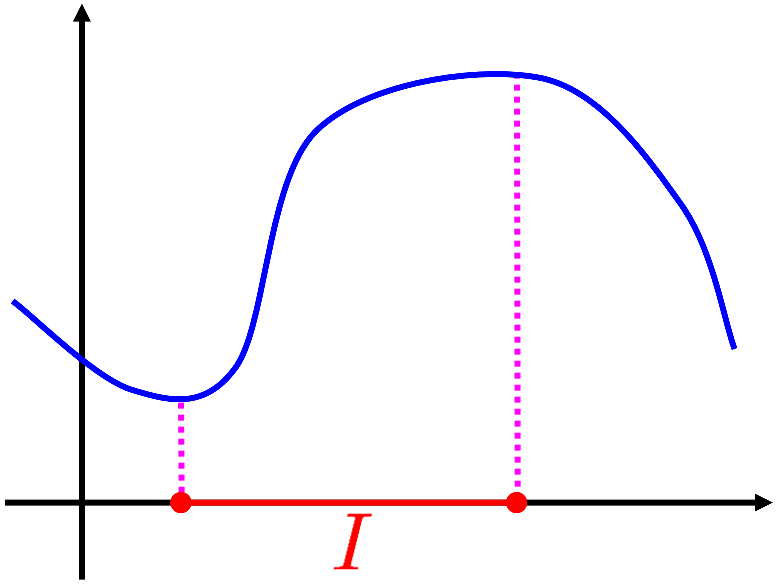
Typical to make the interval as large as possible. . .

DEFINITION: Let I be an interval.

A function f is called **increasing on** I if
$f(s) < f(t)$ whenever $s, t \in I$ and $s < t$.



e.g.:

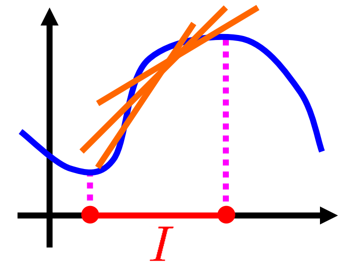


DEFINITION: Let I be an interval.

A function f is called **increasing on** I if

$f(s) < f(t)$ whenever $s, t \in I$ and $s < t$.

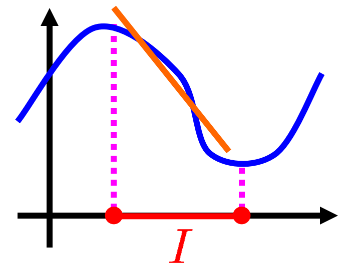
"secant lines run uphill" (slopes > 0)



A function f is called **decreasing on** I if

$f(s) > f(t)$ whenever $s, t \in I$ and $s < t$.

"secant lines run downhill" (slopes < 0)

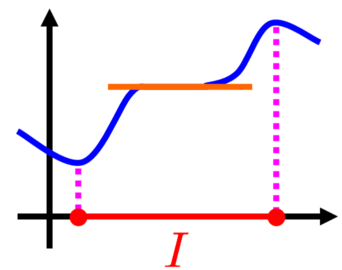


(semi-increasing)

A function f is called **nondecreasing on** I if

$f(s) \leq f(t)$ whenever $s, t \in I$ and $s \leq t$.

"secant lines don't run downhill" (slopes ≥ 0)

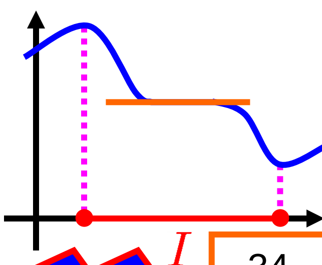


(semi-decreasing)

A function f is called **nonincreasing on** I if

$f(s) \geq f(t)$ whenever $s, t \in I$ and $s \leq t$.

"secant lines don't run uphill" (slopes ≤ 0)



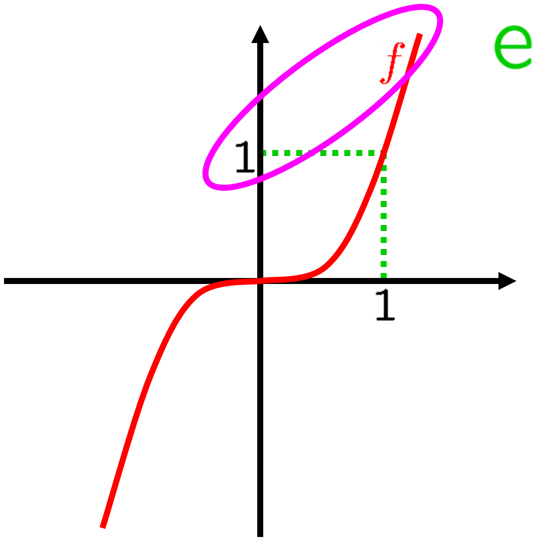
Rescaling functions and expressions (a.k.a. scalar multiplication)

If $f : A \to \mathbb{R}$ is a function,

and if $c \in \mathbb{R}$ is a number,

then $\boxed{cf} : A \to \mathbb{R}$ is the function

defined by $(cf)(x) = c[f(x)]$.

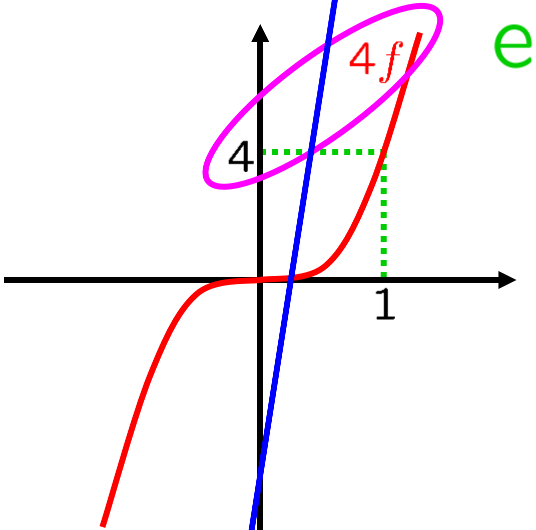


e.g.: Let $F := f(x) = x^3$.

$4F = (4f)(x) = 4x^3$

Rescaling functions and expressions (a.k.a. scalar multiplication)

If $f : A \to \mathbb{R}$ is a function,
and if $c \in \mathbb{R}$ is a number,
then $cf : A \to \mathbb{R}$ is the function
defined by $(cf)(x) = c[f(x)]$.



e.g.: Let $F := f(x) = x^3$.
$4F = (4f)(x) = 4x^3$

In these lectures, typically,
scalar := real number

A **scalar** is a number.
Context: real number, complex number,
p-adic number, *etc.*

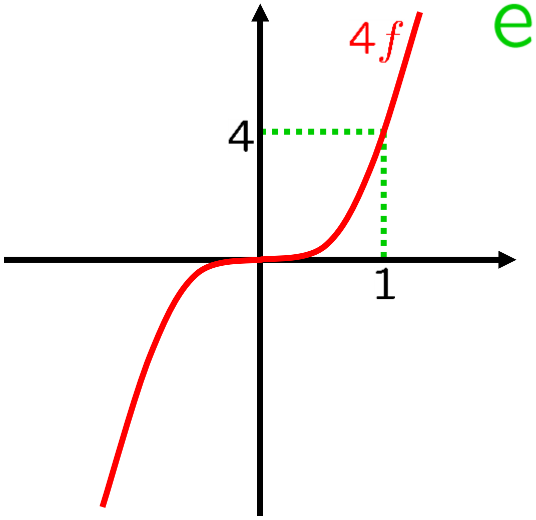
Rescaling functions and expressions (a.k.a. scalar multiplication)

If $f : A \to \mathbb{R}$ is a function,

and if $c \in \mathbb{R}$ is a scalar, could also say "constant"

then $cf : A \to \mathbb{R}$ is the function

defined by $(cf)(x) = c[f(x)]$.



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Addition of functions

If $f : A \to \mathbb{R}$ and $g : B \to \mathbb{R}$ are functions, then $\boxed{f + g} : \underline{A \cap B} \to \mathbb{R}$ is the function defined by $(f+g)(x) = [f(x)] + [g(x)]$.

e.g.: Let $F := f(x) = x^3 + 2x$
and $G := g(x) = 2x^3 - 6x$.

$$F + G = (f+g)(x) = 3x^3 - 4x$$

e.g.: Let $Q := q(x) = \ln x$
and $R := r(x) = \sqrt{1-x}$.

$$Q + R = (q+r)(x) = [\ln x] + \sqrt{1-x}$$

$$q : (0, \infty) \to \mathbb{R}$$

$$r : (-\infty, 1] \to \mathbb{R}$$

$$q + r : (0, 1] \to \mathbb{R}$$

Linear operations and linear combinations

Def'n: The **linear operations** are
scalar multiplication and addition.

Def'n: $\forall$integers $j \in [1, n]$,
let $f_j : A_j \to \mathbb{R}$ be a function.
Let $c_1, \ldots, c_n \in \mathbb{R}$.
The **linear combination of** $f_1, \ldots, f_n$
with coefficients $c_1, \ldots, c_n$
is $c_1 f_1 + \cdots + c_n f_n$.

e.g.: The linear combination of sin and cos
with coefficients 2 and $-\sqrt{2}$
is the function $2\sin - \sqrt{2}\cos : \mathbb{R} \to \mathbb{R}$ def'd by
$(2\sin - \sqrt{2}\cos)(x) = 2(\sin x) - \sqrt{2}(\cos x)$.

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Let $c_1, \ldots, c_n \in \mathbb{R}$. $x :\to t$

The **linear combination of** $f_1(x), \ldots, f_n(x)$ **with coefficients** $c_1, \ldots, c_n$ is $c_1[f_1(x)] + \cdots + c_n[f_n(x)]$.

e.g.: The linear combination of $\sqrt{1-x}$ and $\ln x$ with coefficients -1 and 4 is $H := -\sqrt{1-x} + 4 \ln x$.

Domain of H: $x \in (-\infty, 1] \cap (0, \infty) = (0, 1]$

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Domain of H: $t \in (-\infty, 1] \cap (0, \infty) = (0, 1]$

Polynomials in one variable

Def'n: A **polynomial in** x is a finite linear combination of $1, x, x^2, x^3, x^4, \ldots$.

e.g.: $4 + 7x + 8x^2$ degree $= 2$

$2 - 6x + 3x^2 + \pi x^3 - ex^4$ degree $= 4$

8 degree $= 0$

$x^{1000000}$ degree $= 1000000$

$2 - 7x^{\text{googol plex}}$ degree $=$ googol plex $= 10^{10^{100}}$

The **degree** of a polynomial in x is the maximum of the exponents on x.

$x :\to t$

Polynomials in one variable

Def'n: A **polynomial in** t is a finite linear combination of $1, t, t^2, t^3, t^4, \ldots$.

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$t :\to r$

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Def'n: A **polynomial in** r is a finite linear combination of $1, r, r^2, r^3, r^4, \ldots$.

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expressions :$\longrightarrow$ functions

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The **degree** of a polynomial is the maximum of the exponents.

Polynomials in one variable

Let $P(x)$ be a polynomial in x.

SKILL
Find degree of poly

degree of $P(x)$:=
highest power of x appearing in $P(x)$

e.g.: $3x + 4x^5 - 2x + 7$ has degree 5

Constant means degree 0
Constant polynomials: 2, 7, -8, π, *etc.*

Linear means degree 1
Linear polynomials: $2x + 5$, $ex - \sqrt{2}$, *etc.*

Quadratic means degree 2
Quadratic polynomials: $-7x^2 - 4x + 8$, *etc.*

Cubic means degree 3
Cubic polynomials: $2x^3 - \pi x^2 + 6x + 1$, *etc.*

Quartic means degree 4
Quartic polynomials: $8x^4 - 4x^3 + 2x^2 + 4x + 6$, *etc.*

Quintic means degree 5
Quintic polynomials: $4x^5 - \pi x^4 + 2x^3 - ex^2 + 5x - 8$, *etc.*

Polynomials in one variable

A degree six (sextic) polynomial:

$$9x^6 - 8x^5 + 7x^4 - 6x^3 + 5x^2 - 4x + 3$$

Constant term

Linear term

Quadratic term

Cubic term

Quartic term

Quintic term

Degree six term

SKILL
Identify terms of poly

The **coefficients** are the numbers. . .

Polynomials in one variable

A degree six (sextic) polynomial:

$$9x^6 - 8x^5 + 7x^4 - 6x^3 + 5x^2 - 4x + 3$$

Constant coefficient

Linear coefficient

Quadratic coefficient

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Quartic coefficient

Quintic coefficient

Degree six coefficient

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Leading coefficient := the coefficient on the highest degree term.

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Polynomials in one variable

A degree six (sextic) polynomial:

$$\boxed{9x^6} - 8x^5 + 7x^4 - 6x^3 + 5x^2 - 4x + 3$$

SKILL
Identify leading term of poly

Leading term := the highest degree term

Definition: A function $f : \mathbb{R} \to \mathbb{R}$ is **exponentially bounded** if

exp-bdd

$\exists A, B > 0$ s.t., $\forall x \in \mathbb{R}$,

$$|f(x)| < Ae^{B|x|}.$$

same

$p :\to f(x)$ $\qquad$ $q :\to Ae^{B|x|}$

$|p| < q$ iff $-q < p < q$ $\qquad$ $|p| = \text{dist}(p, 0)$

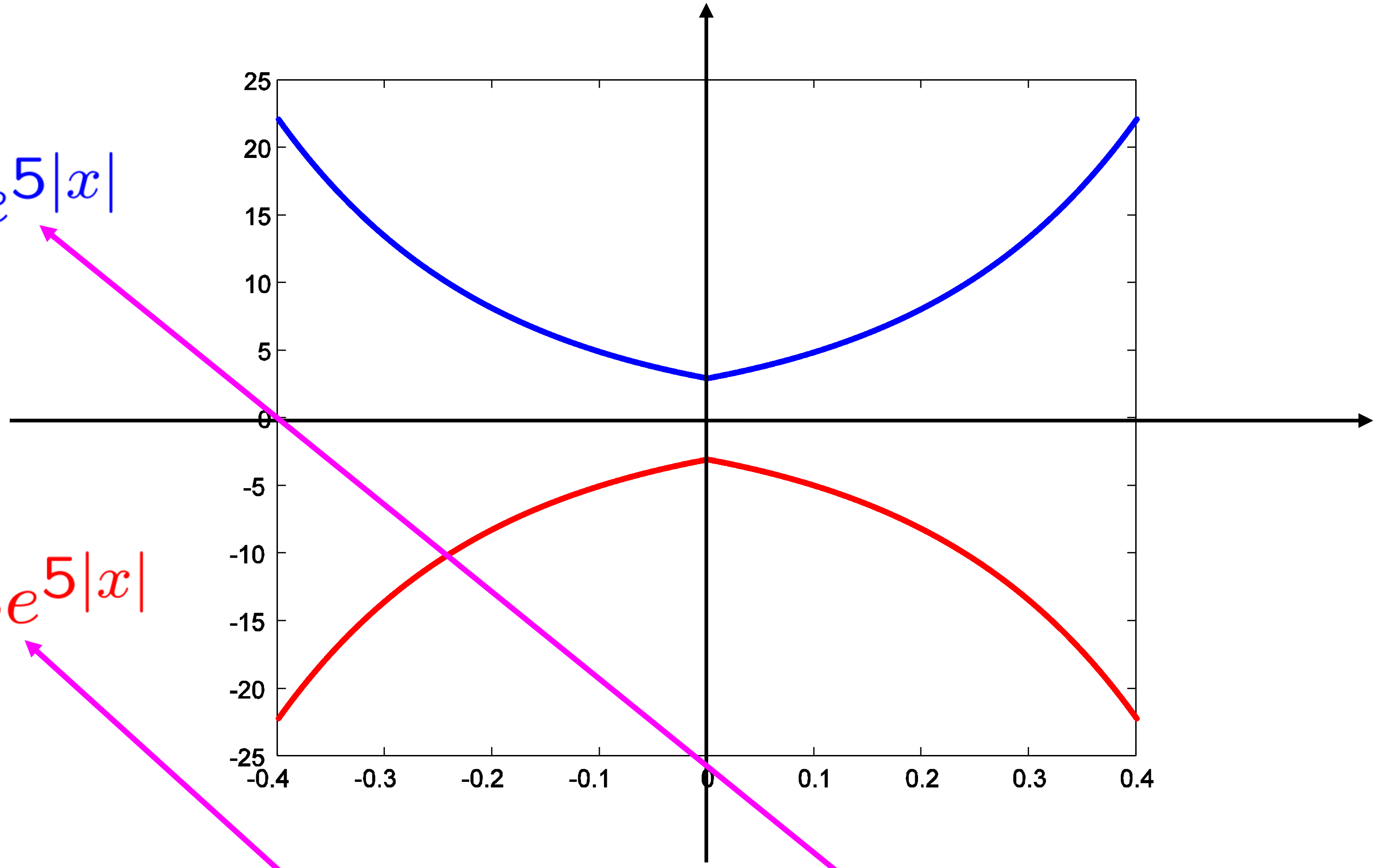
$|f(x)| < Ae^{B|x|}$ iff $-Ae^{B|x|} < f(x) < Ae^{B|x|}$

e.g.: $\quad -3e^{5|x|} < f(x) < 3e^{5|x|}$

implies f is exp-bdd.

$y = 3e^{5|x|}$

$y = -3e^{5|x|}$



e.g.: $-3e^{5|x|} < f(x) < 3e^{5|x|}$

$$-3e^{5|x|} < f(x) < 3e^{5|x|}$$

$y = 3e^{5|x|}$

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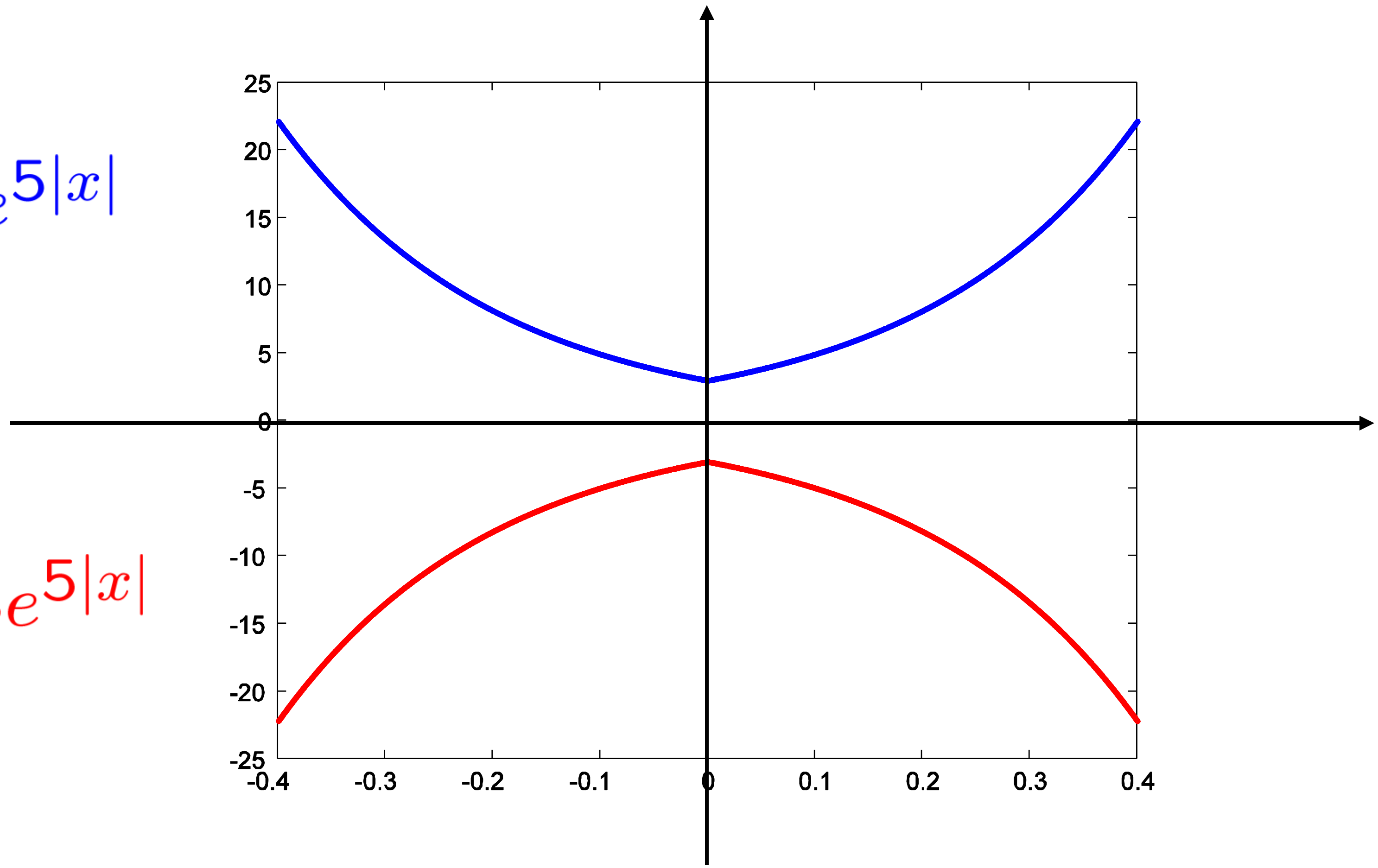

e.g.: $-3e^{5|x|} < f(x) < 3e^{5|x|}$

means that the graph of f stays between the red and blue graphs above.

Definition: A function $f : \mathbb{R} \to \mathbb{R}$ is **exponentially bounded** if

exp-bdd

$$\exists A, B > 0 \text{ s.t., } \forall x \in \mathbb{R},$$
$$|f(x)| < Ae^{B|x|}.$$

Some exp-bdd expressions of x:

ANY polynomial in x

$$3x^7e^{4x} + \cos x + 10^{10^{100}}$$

$$(5e^{2x+7} - 8)_+$$

e^{x^2} is not exp-bdd in x.