Financial Mathematics

Pricing/hedging in three subperiods

Harry is the CEO of XYZ corporation.

Harry wants to buy 1000 shares of XYZ stock for \$970, three months from now.

The right, but not the obligation.

Gail sells (and hedges) this option. Price?

Current: 1 share of XYZ = \$1

Strike price: \$0.97/share (a.k.a. Exercise price)

Underlying market: Shares of XYZ stock

Derivative market: Options on shares of XYZ

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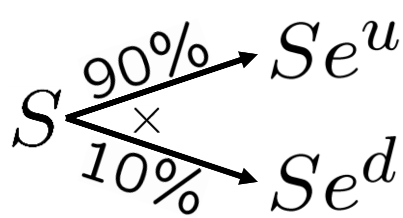
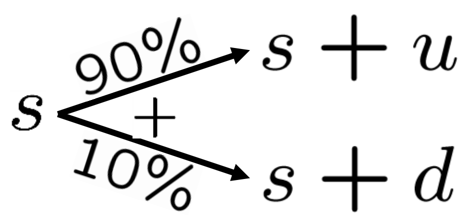
Gail selects:

A three-subperiod $90-10$ CRR model.

stock price: 

(each month, independently)

ln(stock price): 

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Gail selects:

A three-subperiod 90 − 10 CRR model.

Market analyst:

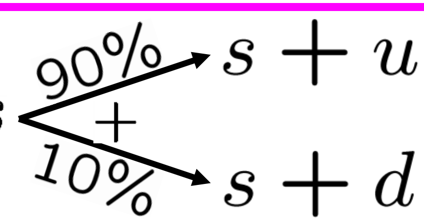
unrealistically high

$$\text{drift} = \overset{\text{annual}}{0.29499181536} / 12 \text{ per month} = 0.024582651$$

unrealistically low

$$\text{volatility} = \overset{\text{annual}}{0.05171367815} / \sqrt{12} \text{ per month} = 0.014928453$$

ln(stock price): s



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A three-subperiod $90 - 10$ CRR model.

Market analyst:

lesser drift   lesser drift $= 0.024582651$

per month $= 0.024582651$

volatility   volatility $= 0.014928453$

per month $= 0.014928453$

change in ln(stock price): $\cancel{s}$ — 90% → $\cancel{s} + u$; + ; 10% → $\cancel{s} + d$

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A three-subperiod $90 - 10$ CRR model.

Market analyst:

$$(0.9)u + (0.1)d = \text{lesser drift} = 0.024582651$$

per month

$$\sqrt{(0.9)(0.1)}(u - d) = \text{volatility} = 0.014928453$$

per month

change in ln(stock price): s → 90% → $s + u$; + ; s → 10% → $s + d$

$u = 0.029558802$

$d = -0.020202707$

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A three-subperiod 90 – 10 CRR model.

stock price: S → (90%) Se^u, (10%) Se^d (×)

$e^u = 1.0300$

$e^d = 0.9800$

change in ln(stock price): s → (90%) $s + u$, (10%) $s + d$ (+)

$u = 0.029558802$

$d = -0.020202707$

S → 90% → 1.03 S

S → 10% → 0.98 S

stock price: $S \xrightarrow{90\%} Se^u$, $S \xrightarrow{10\%} Se^d$ (×)

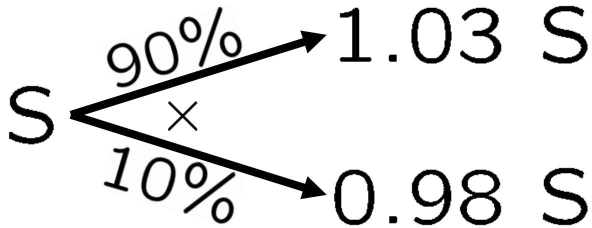
$e^u = 1.0300$

$e^d = 0.9800$

change in ln(stock price): $\cancel{s} \xrightarrow{90\%} \cancel{s} + u$, $\cancel{s} \xrightarrow{10\%} \cancel{s} + d$ (+)

$u = 0.029558802$

$d = -0.020202707$

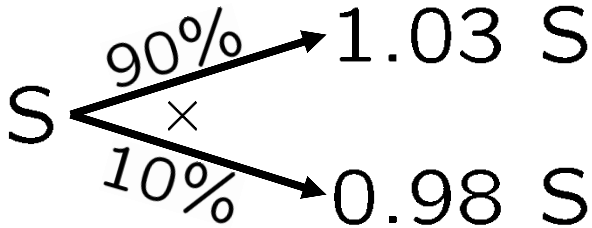


Banker:

continuous compounding nominal rate

$$= 0.23763152755 / 12 \text{ per month}$$

$$= 0.019802627$$



Banker:

$$r = \text{continuous compounding nominal rate}$$
$$= 0.019802627 \text{ per montper month}$$
$$= 0.019802627$$

Change measure to the risk-neutral world . . .

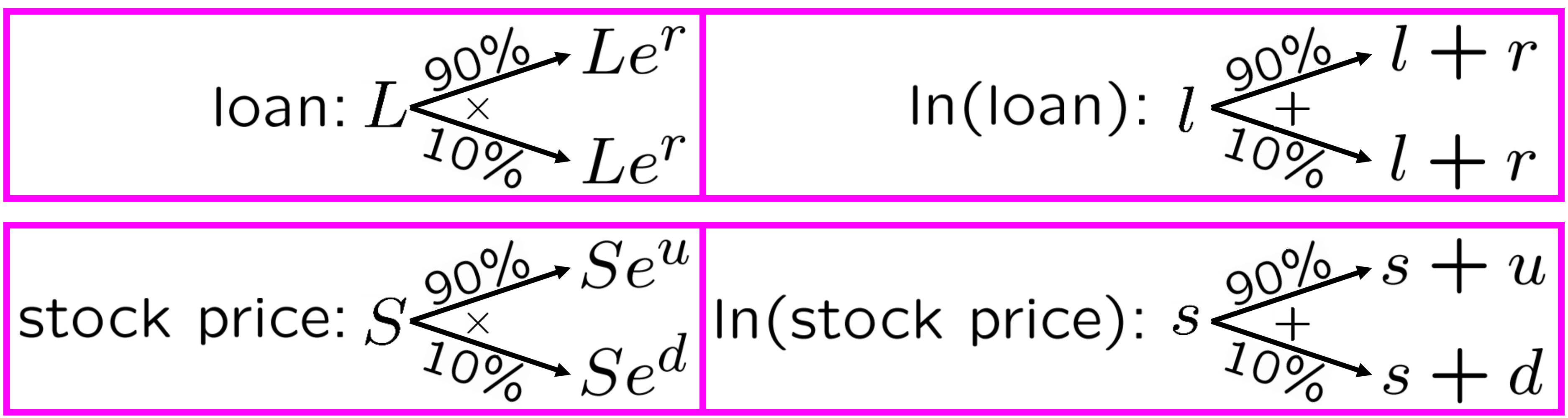
Banker:

(or logarithmic risk-free factor)

r = continuous compounding nominal rate

$= 0.019802627$ per month

$e^r = 1.0200$



Change measure to the risk-neutral world . . .

S → 90% → 1.03 S
S → 10% → 0.98 S
×

L → 90% → 1.02 L
L → 10% → 1.02 L
×

Banker:

r = continuous compounding nominal rate (or logarithmic risk-free factor)
$= 0.019802627$ per month

$$e^r = 1.0200$$

loan: L → 90% → Le^r; L → 10% → Le^r (×)	ln(loan): l → 90% → $l + r$; l → 10% → $l + r$ (+)
stock price: S → 90% → Se^u; S → 10% → Se^d (×)	ln(stock price): s → 90% → $s + u$; s → 10% → $s + d$ (+)

Change measure to the risk-neutral world . . .

S → 80% → 1.03 S
S → 20% → 0.98 S

Expected value: 1.02 S
Expected return: 2%

L → 80% → 1.02 L
L → 20% → 1.02 L

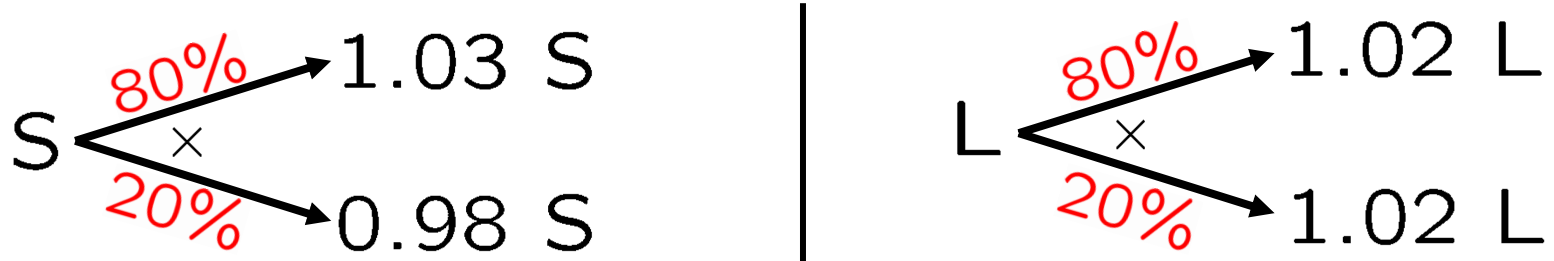
Expected value: 1.02 L
Expected return: 2%

downtick factor	risk-free factor	uptick factor
0.98	1.02	1.03

80% risk-neutral uptick probability

20% risk-neutral downtick probability

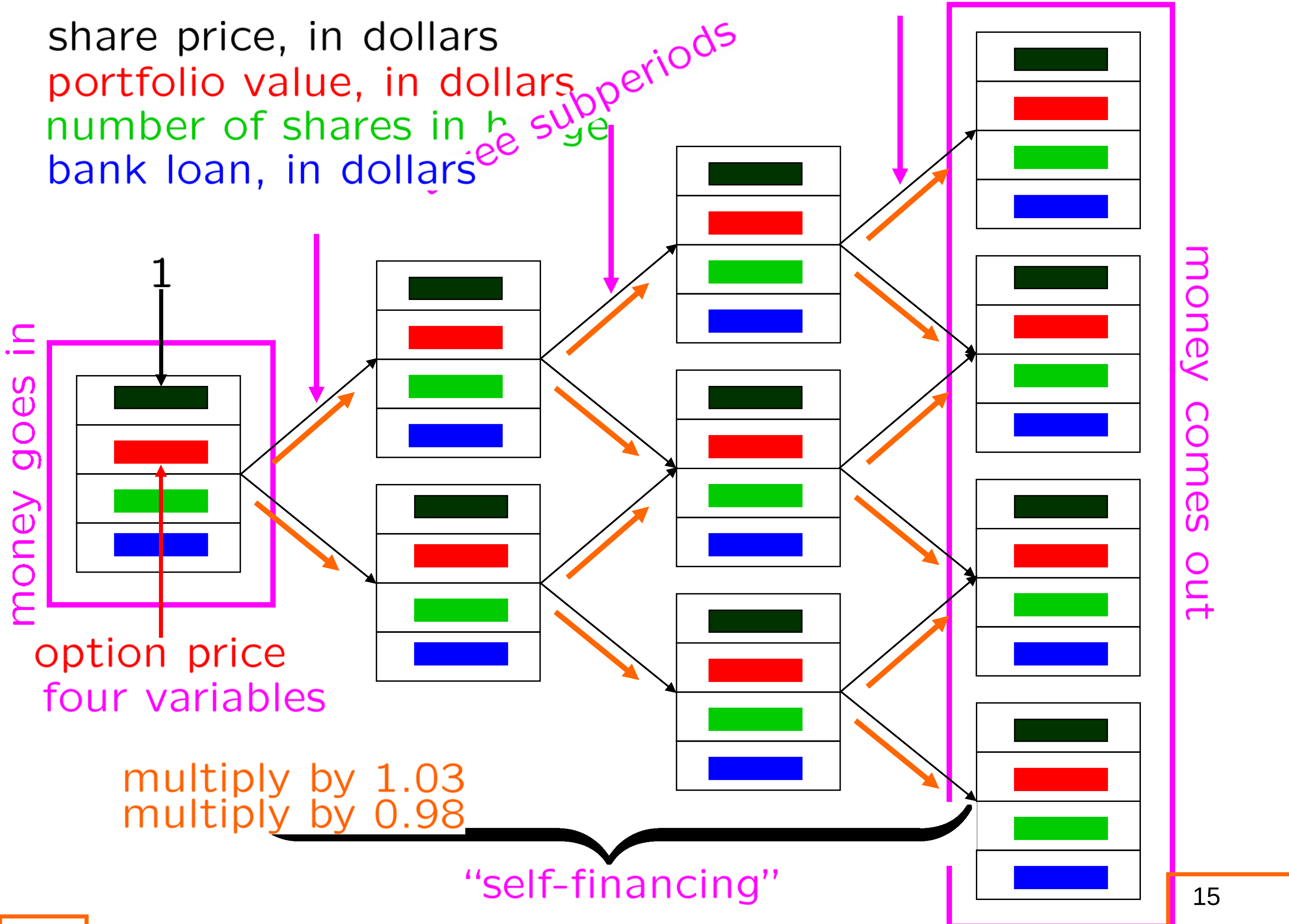
Change measure to the risk-neutral world . . .



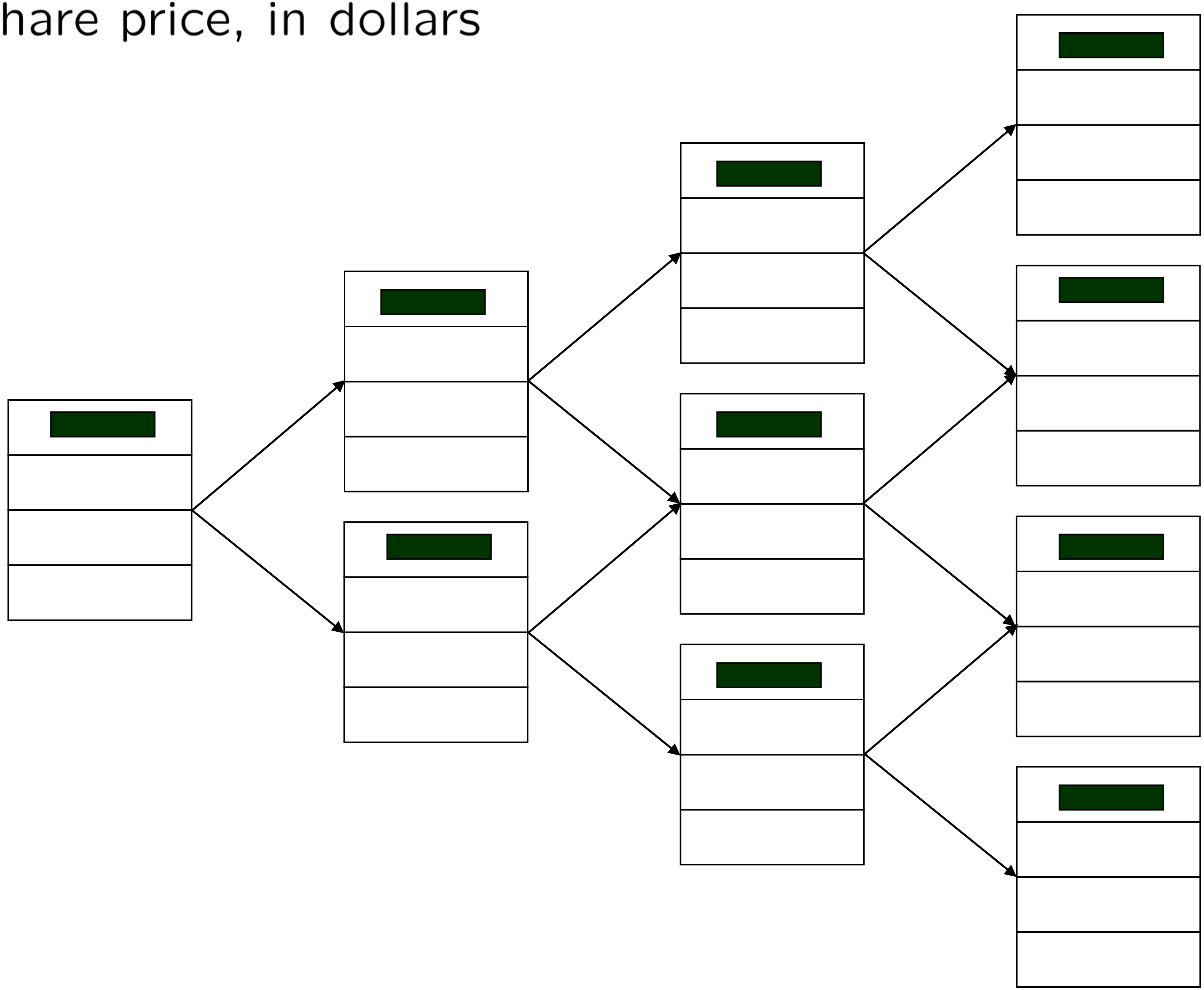
Expected value: 1.02 S	Expected value: 1.02 L
Expected return: 2%	Expected return: 2%

equal!

The 80-20 world is risk-neutral, and, here, ANY portfolio in stock and bank will have an expected growth of 2% per month.

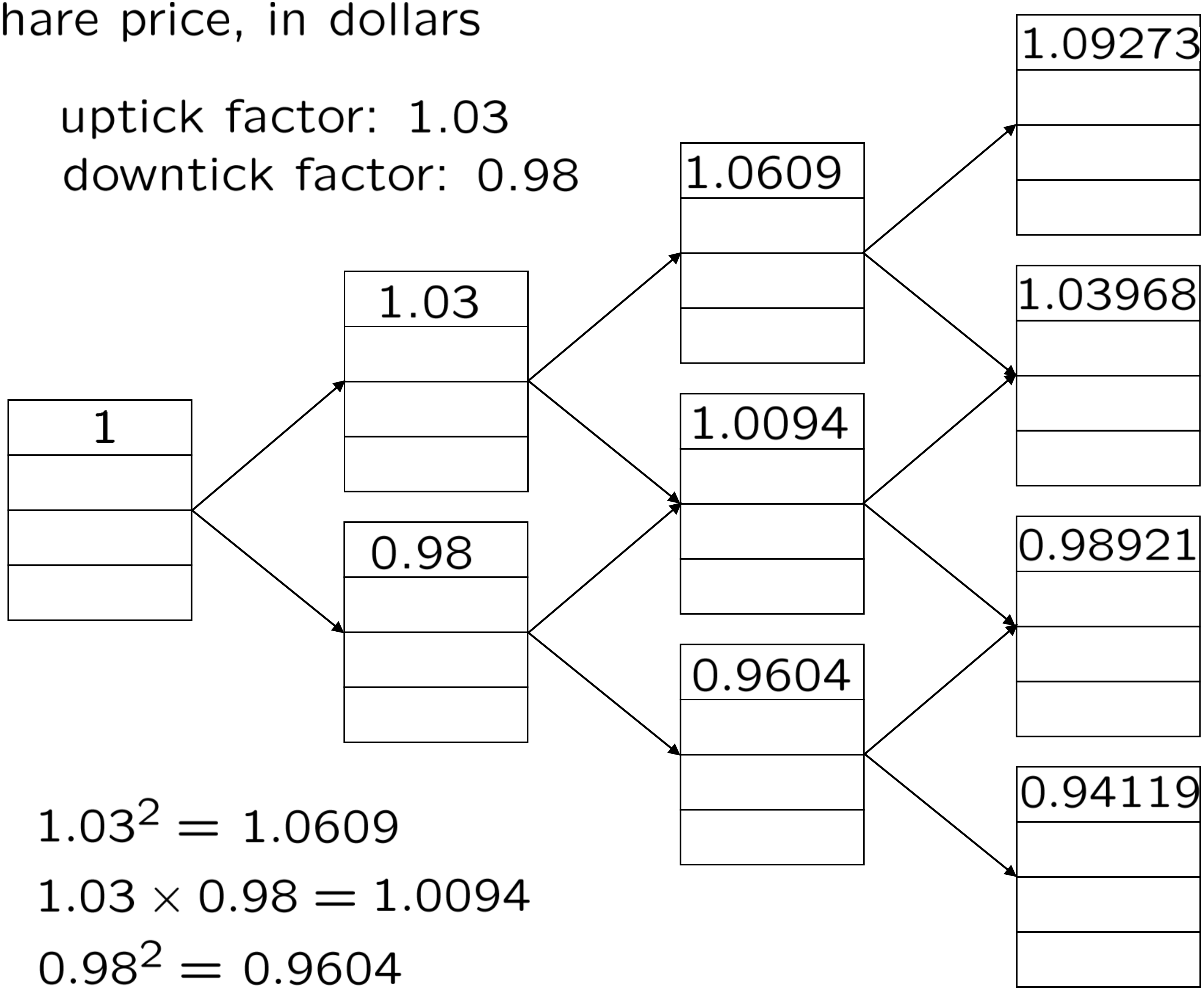

share price, in dollars
portfolio value, in dollars
number of shares in h ge
bank loan, in dollars
ee subperiods
1
money goes in
option price
four variables
multiply by 1.03
multiply by 0.98
"self-financing"
money comes out

share price, in dollars



share price, in dollars

uptick factor: 1.03
downtick factor: 0.98



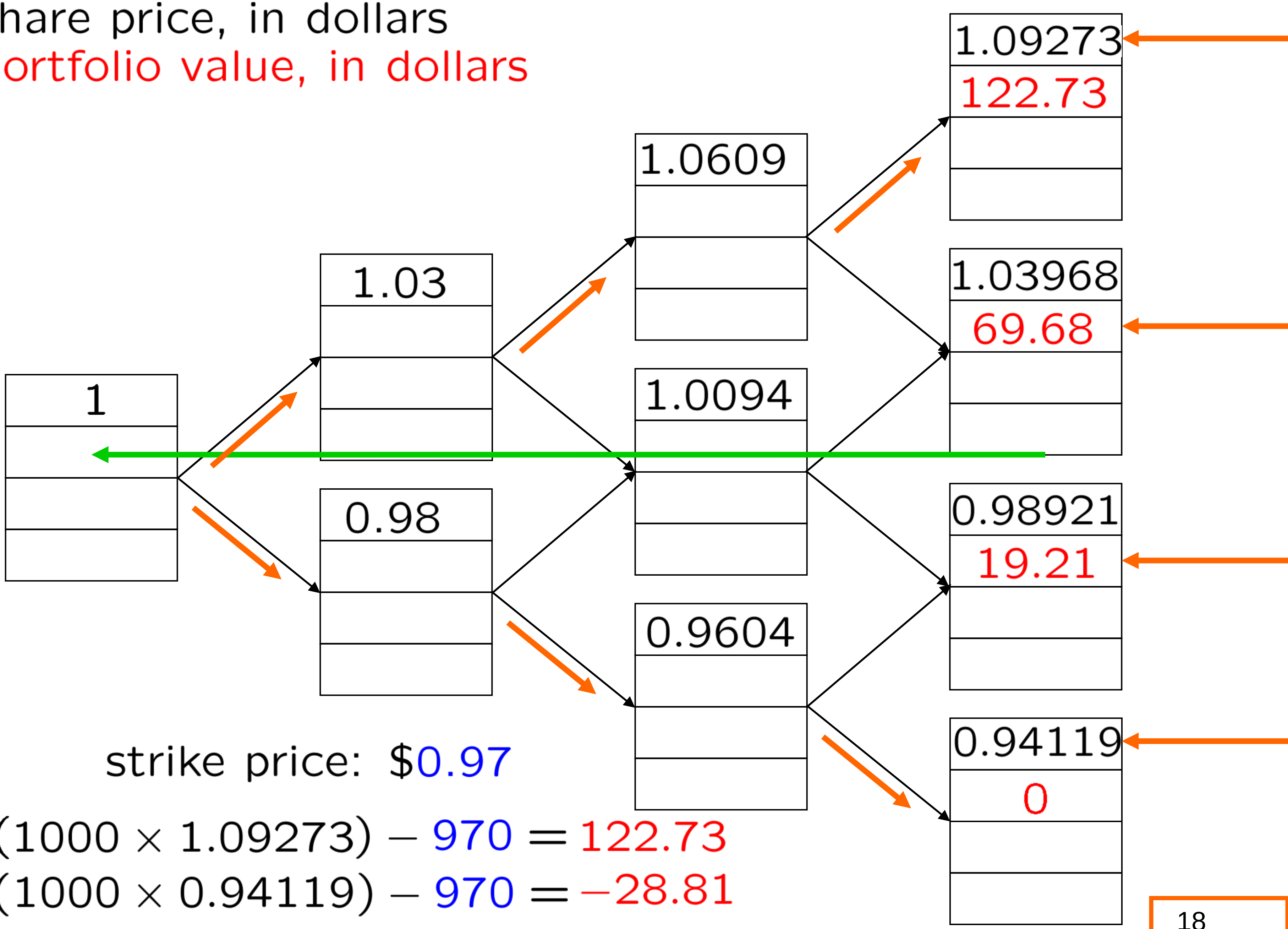
$1.03^2 = 1.0609$

$1.03 \times 0.98 = 1.0094$

$0.98^2 = 0.9604$

share price, in dollars

portfolio value, in dollars

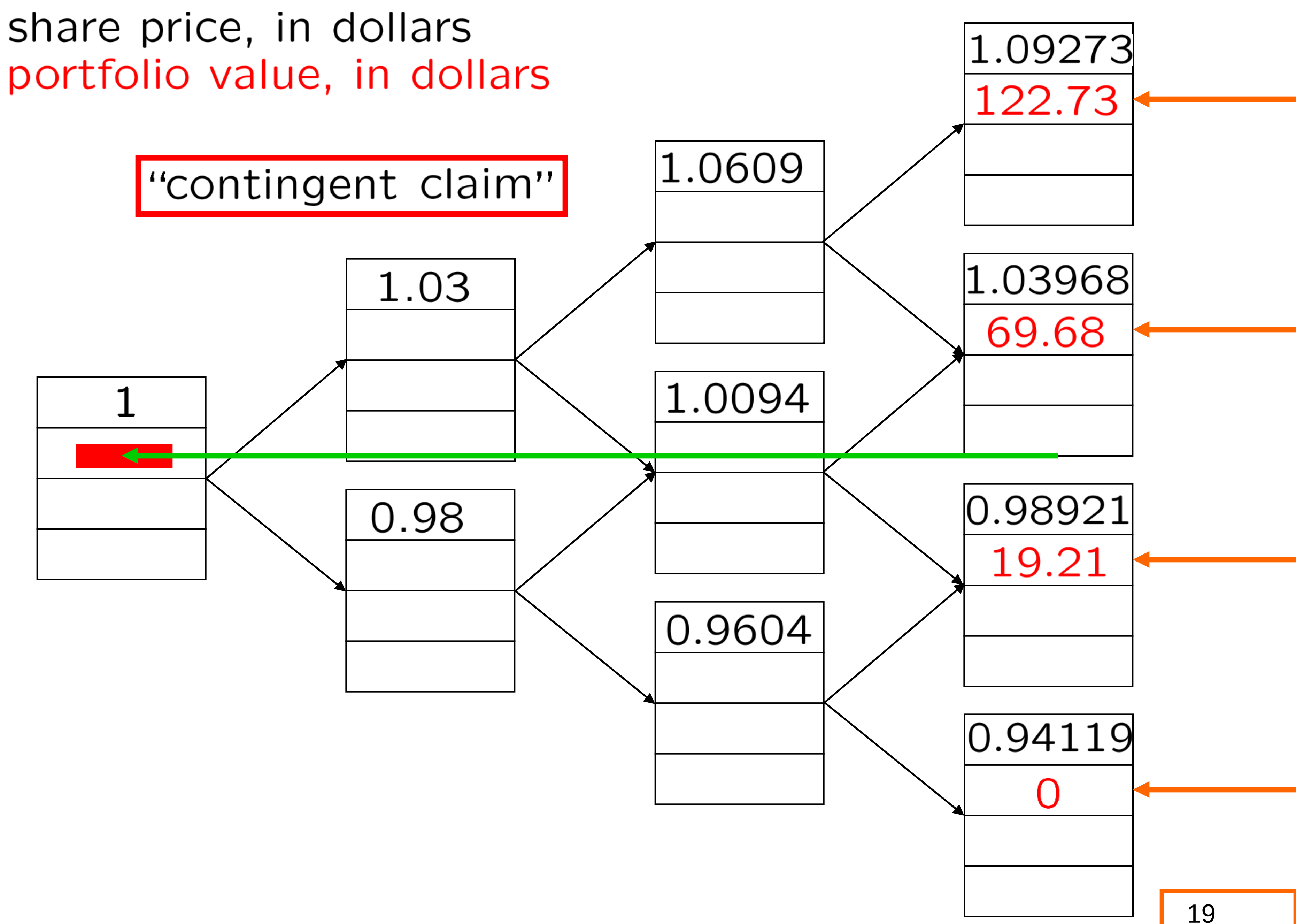


strike price: \$0.97

$$(1000 \times 1.09273) - 970 = 122.73$$

$$(1000 \times 0.94119) - 970 = -28.81$$

Payoff: $f(S) = (1000S - 970)_+ = 1000(S - 0.97)_+$

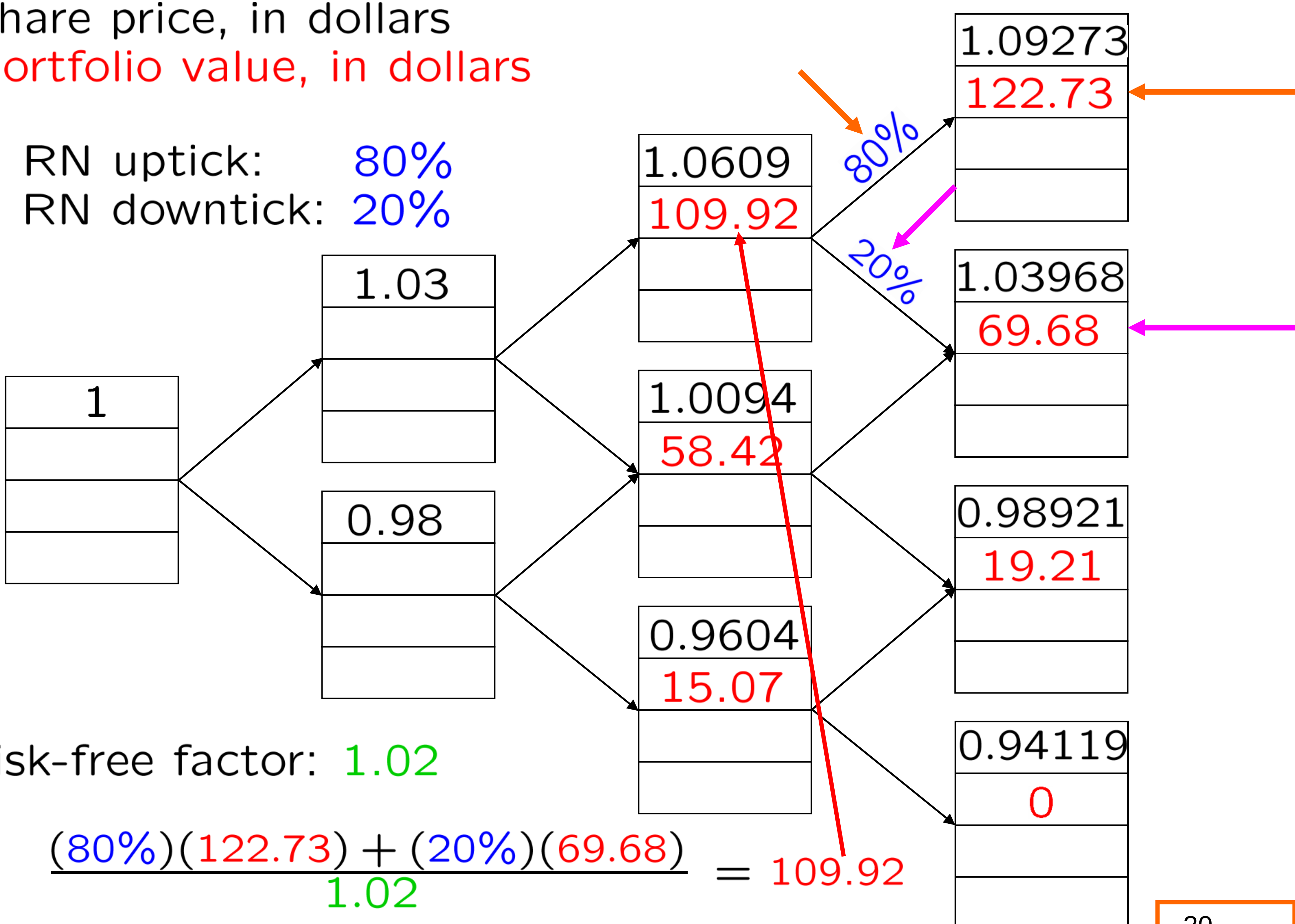

share price, in dollars
portfolio value, in dollars
"contingent claim"
1
1.03
0.98
1.0609
1.0094
0.9604
1.09273
122.73
1.03968
69.68
0.98921
19.21
0.94119
0

share price, in dollars

portfolio value, in dollars

RN uptick: 80%

RN downtick: 20%

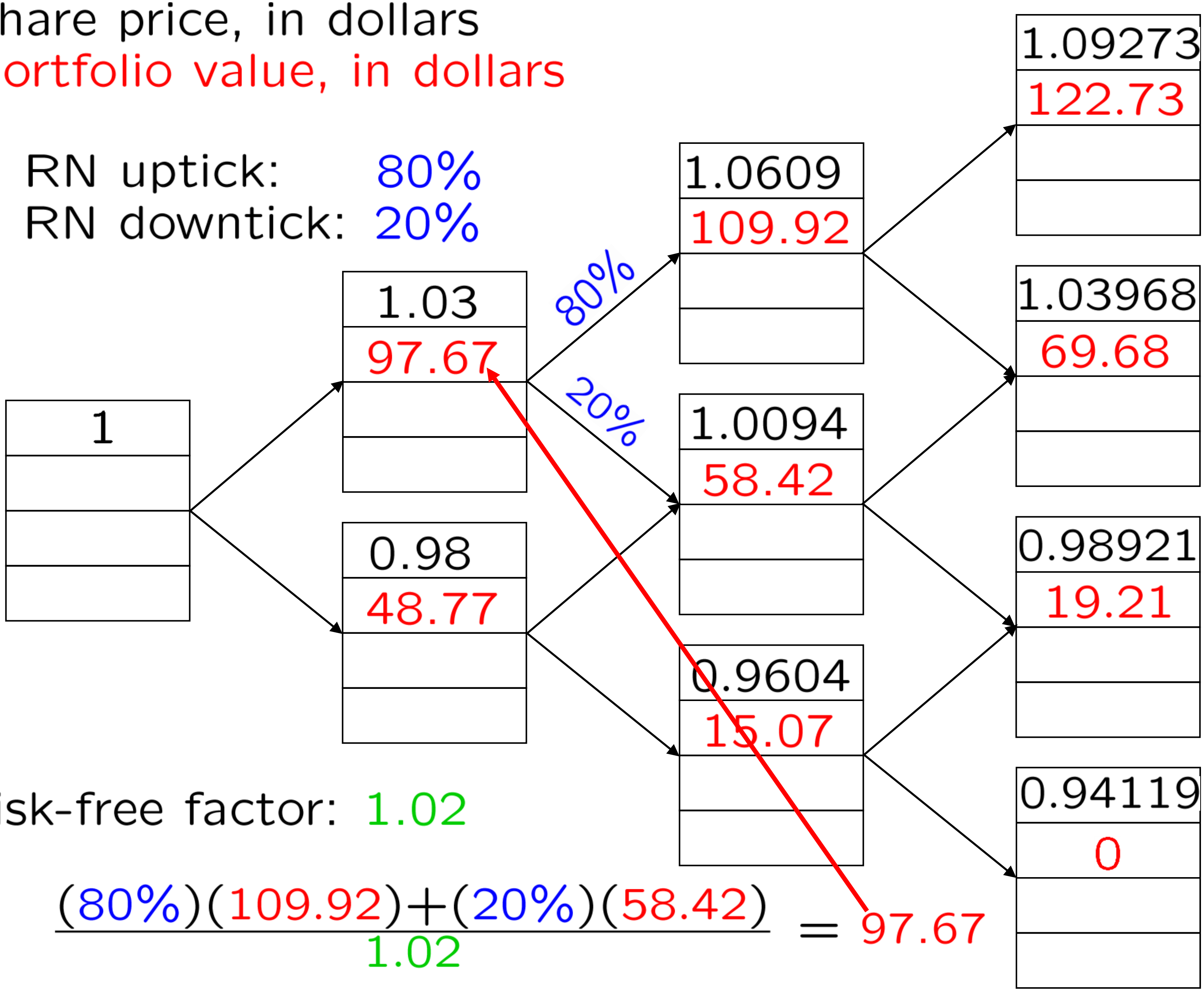


risk-free factor: 1.02

$$\frac{(80\%)(122.73) + (20\%)(69.68)}{1.02} = 109.92$$

share price, in dollars
portfolio value, in dollars

RN uptick: 80%
RN downtick: 20%



risk-free factor: 1.02

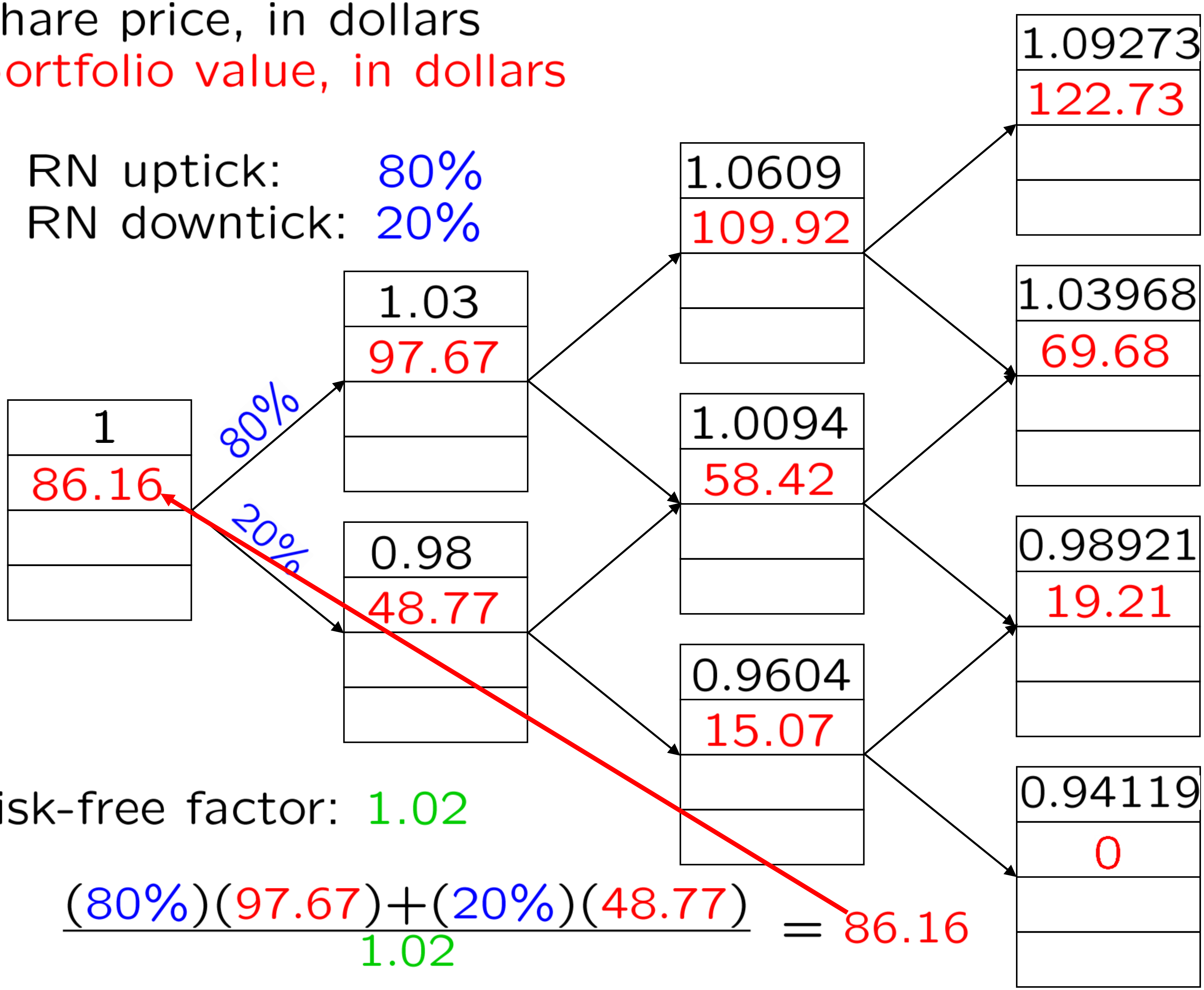
$$\frac{(80\%)(109.92)+(20\%)(58.42)}{1.02} = 97.67$$

share price, in dollars

portfolio value, in dollars

RN uptick: 80%

RN downtick: 20%



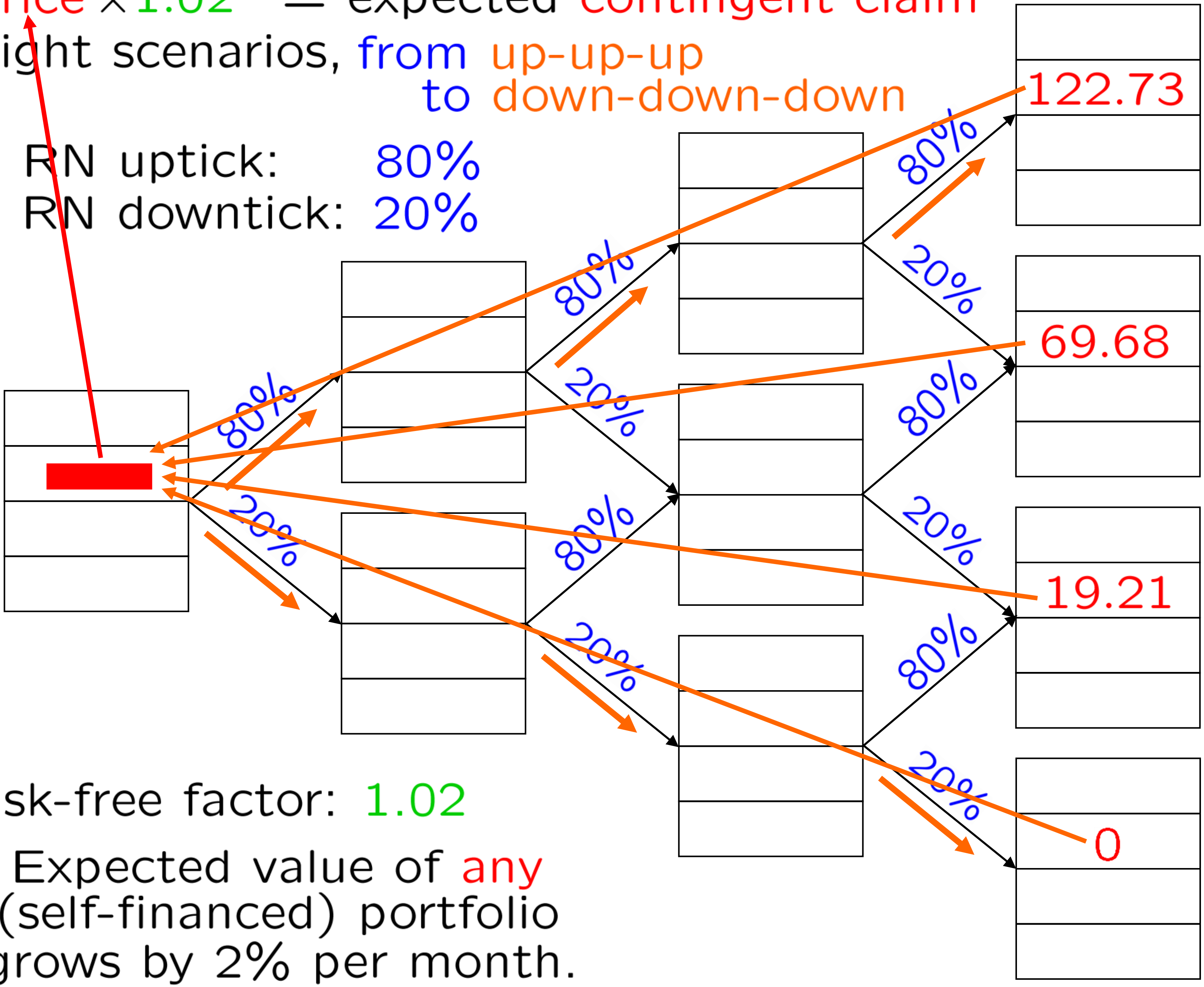
risk-free factor: 1.02

$$\frac{(80\%)(97.67)+(20\%)(48.77)}{1.02} = 86.16$$

$\text{Price} \times 1.02^3 = $ expected contingent claim

Eight scenarios, from up-up-up
to down-down-down

RN uptick: 80%
RN downtick: 20%



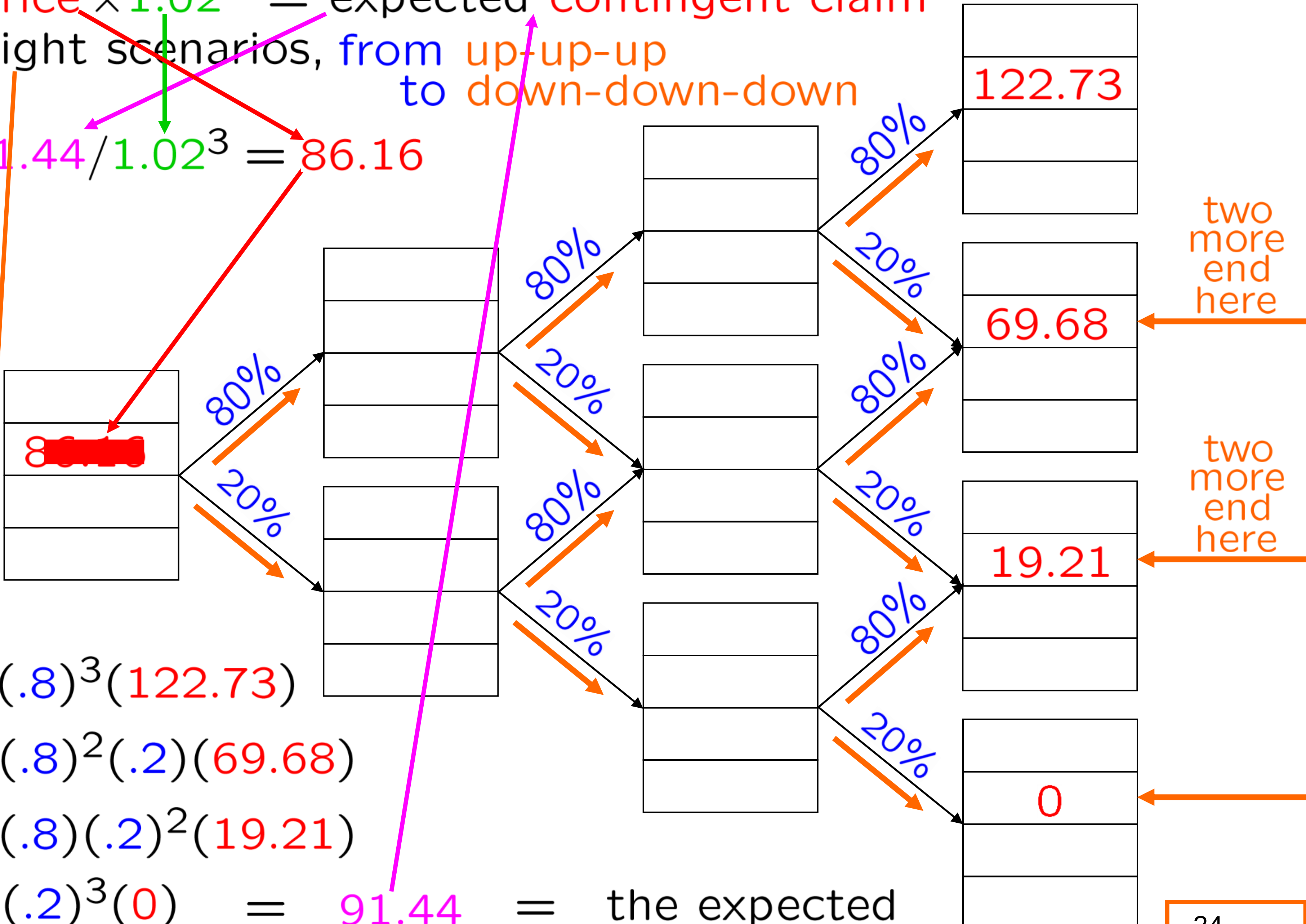
risk-free factor: 1.02

Expected value of any
(self-financed) portfolio
grows by 2% per month.

Price $\times 1.02^3$ = expected contingent claim

Eight scenarios, from up-up-up to down-down-down

$91.44/1.02^3 = 86.16$

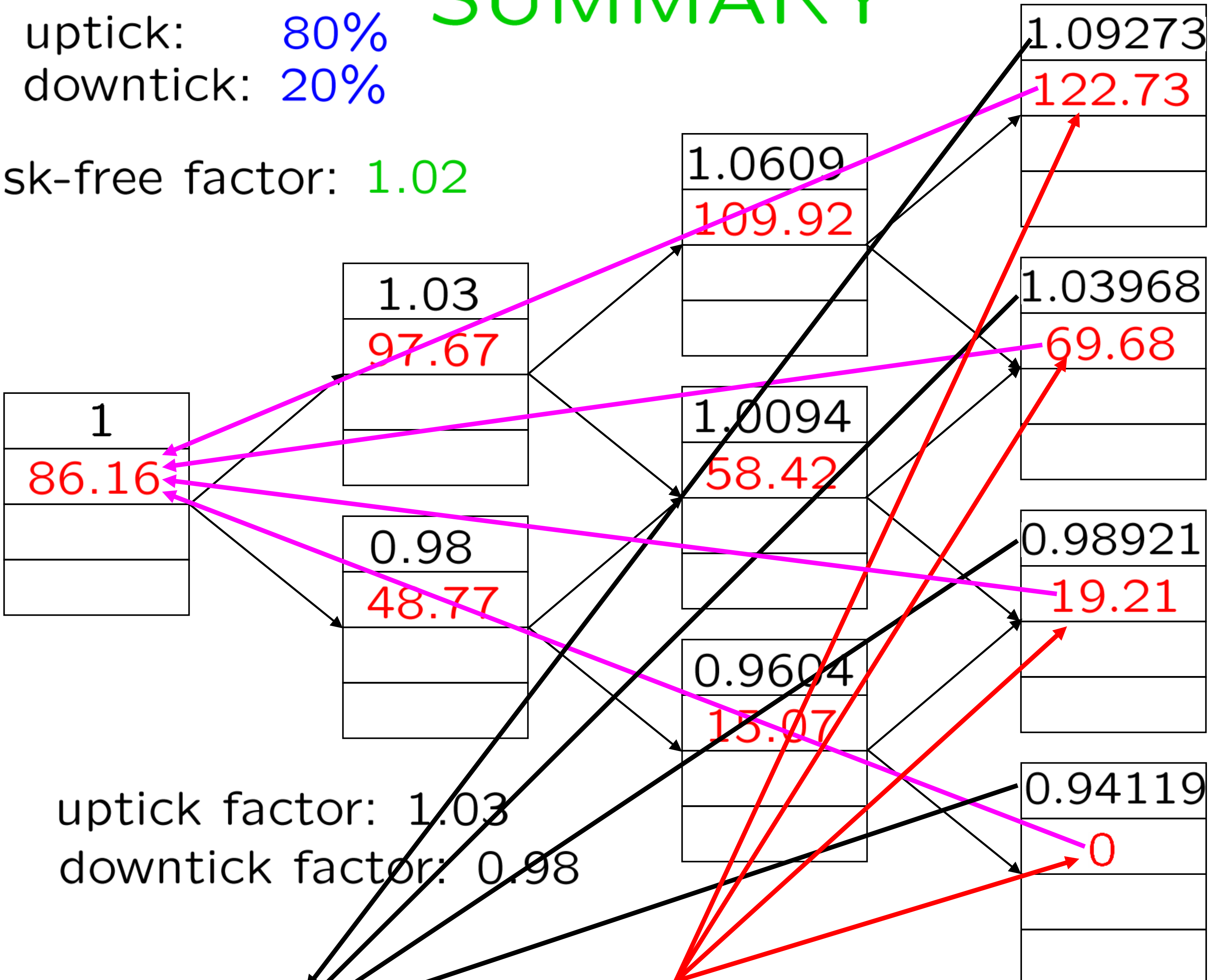


$$
\begin{aligned}
&1(.8)^3(122.73) \\
+&3(.8)^2(.2)(69.68) \\
+&3(.8)(.2)^2(19.21) \\
+&1(.2)^3(0) \quad = \quad 91.44 \quad = \quad \text{the expected contingent claim}
\end{aligned}
$$

SUMMARY

RN uptick: 80%
RN downtick: 20%

risk-free factor: 1.02

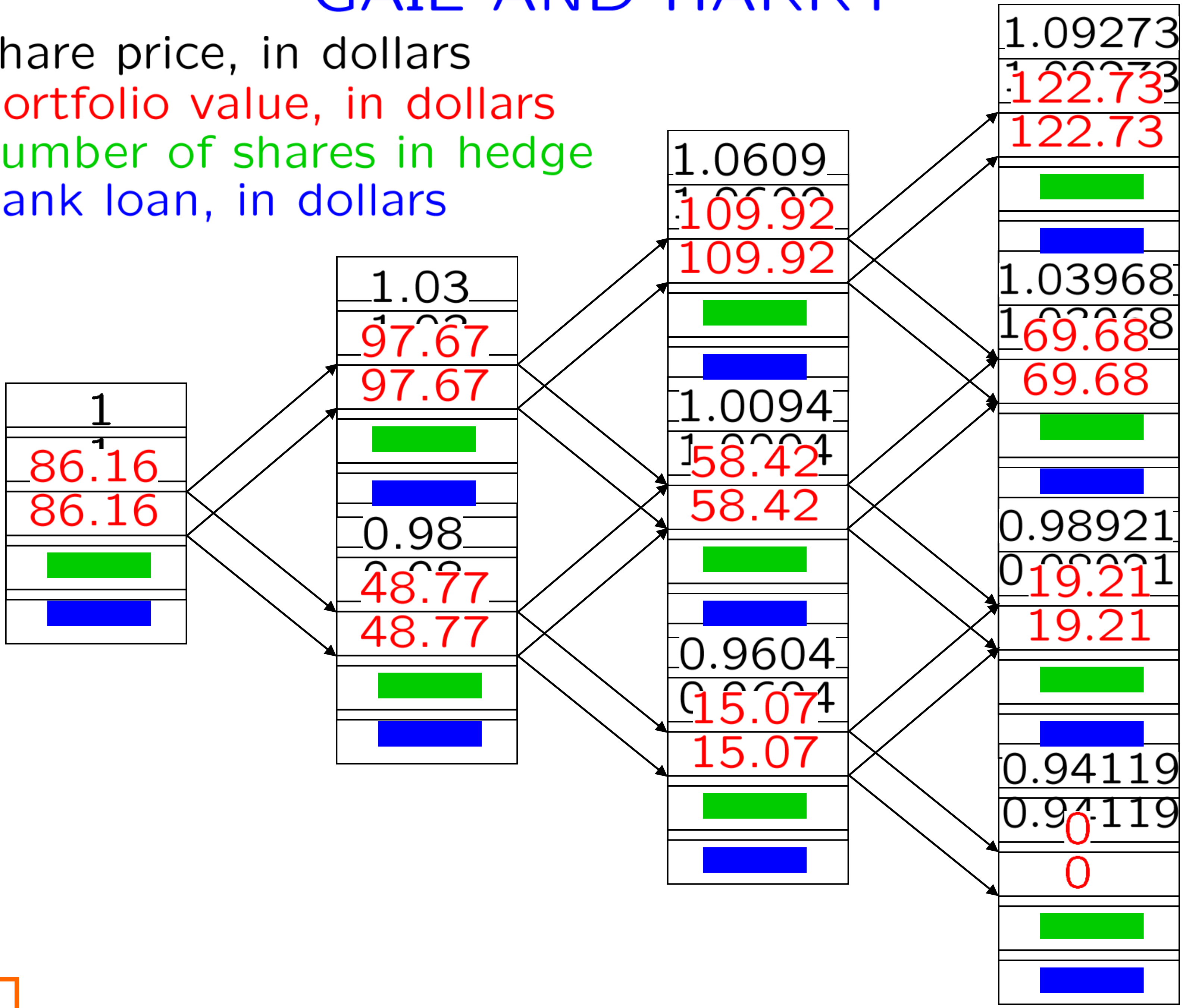


uptick factor: 1.03
downtick factor: 0.98

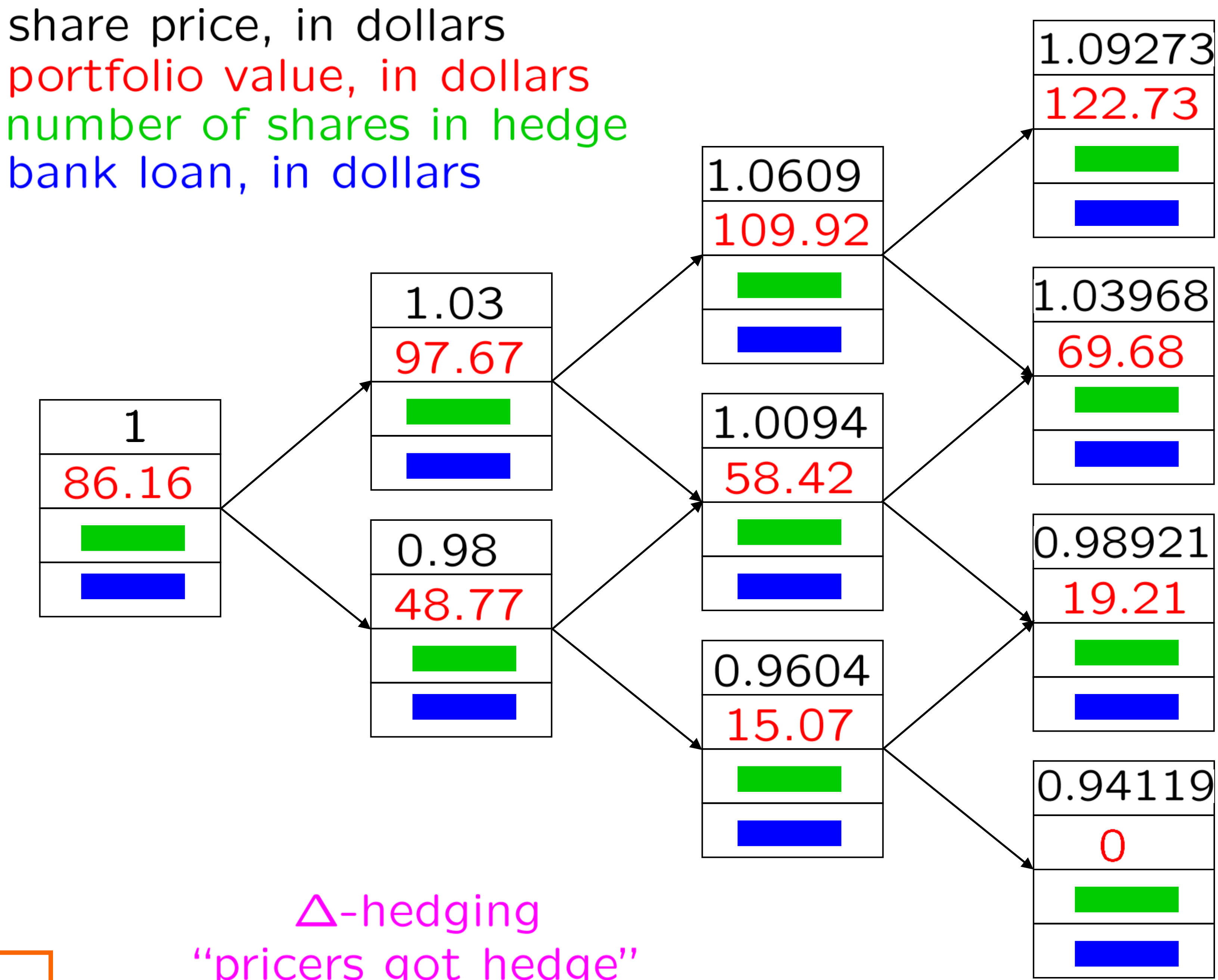
Payoff: $f(S) = (1000S - 970)_+$

GAIL AND HARRY

share price, in dollars
portfolio value, in dollars
number of shares in hedge
bank loan, in dollars

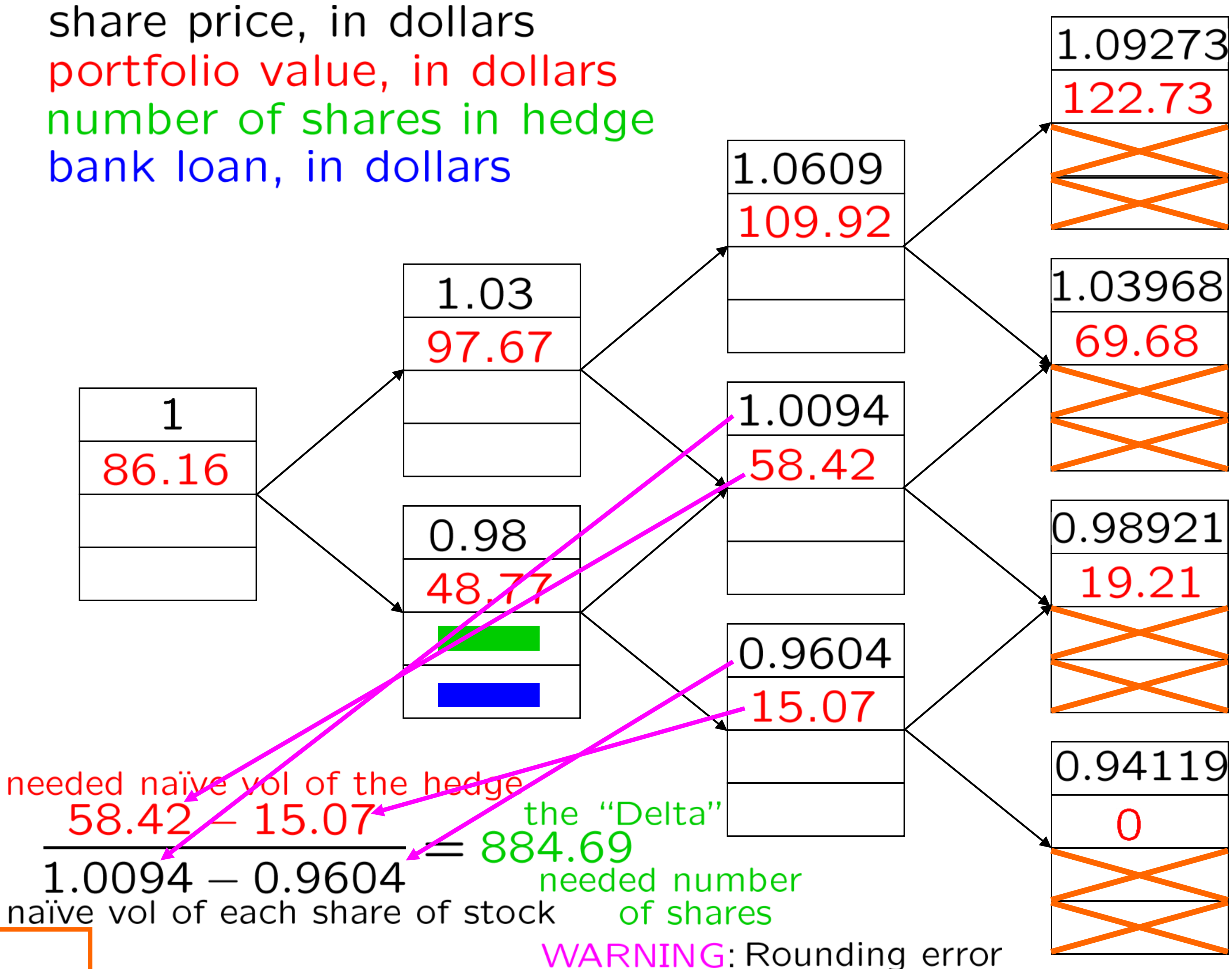


GAIL AND HARRY



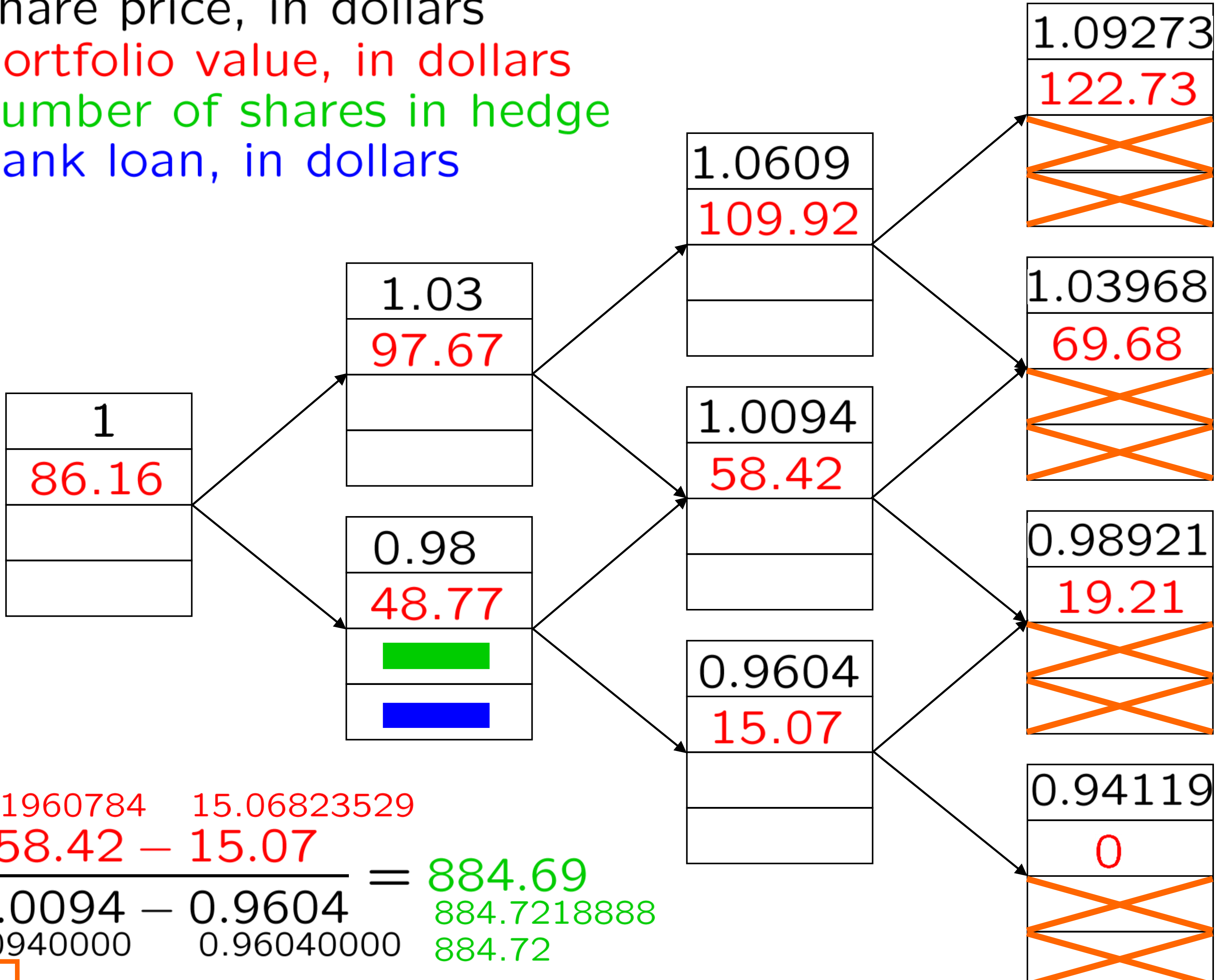
Δ-hedging
"pricers got hedge"

GAIL AND HARRY



GAIL AND HARRY

share price, in dollars
portfolio value, in dollars
number of shares in hedge
bank loan, in dollars



58.41960784 15.06823529

$$\frac{58.42 - 15.07}{1.0094 - 0.9604} = 884.69$$

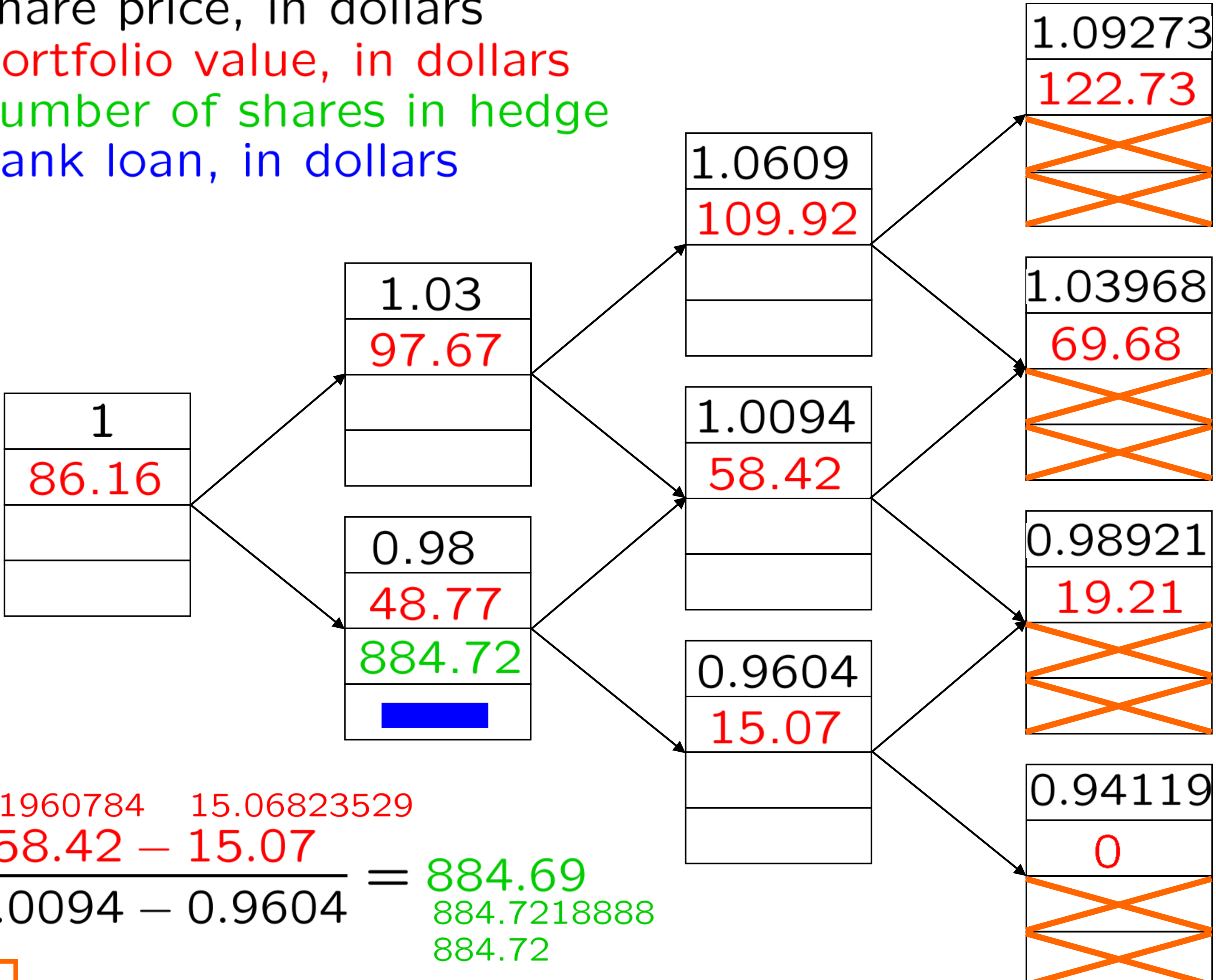
1.00940000 0.96040000

884.7218888

884.72

GAIL AND HARRY

share price, in dollars
portfolio value, in dollars
number of shares in hedge
bank loan, in dollars



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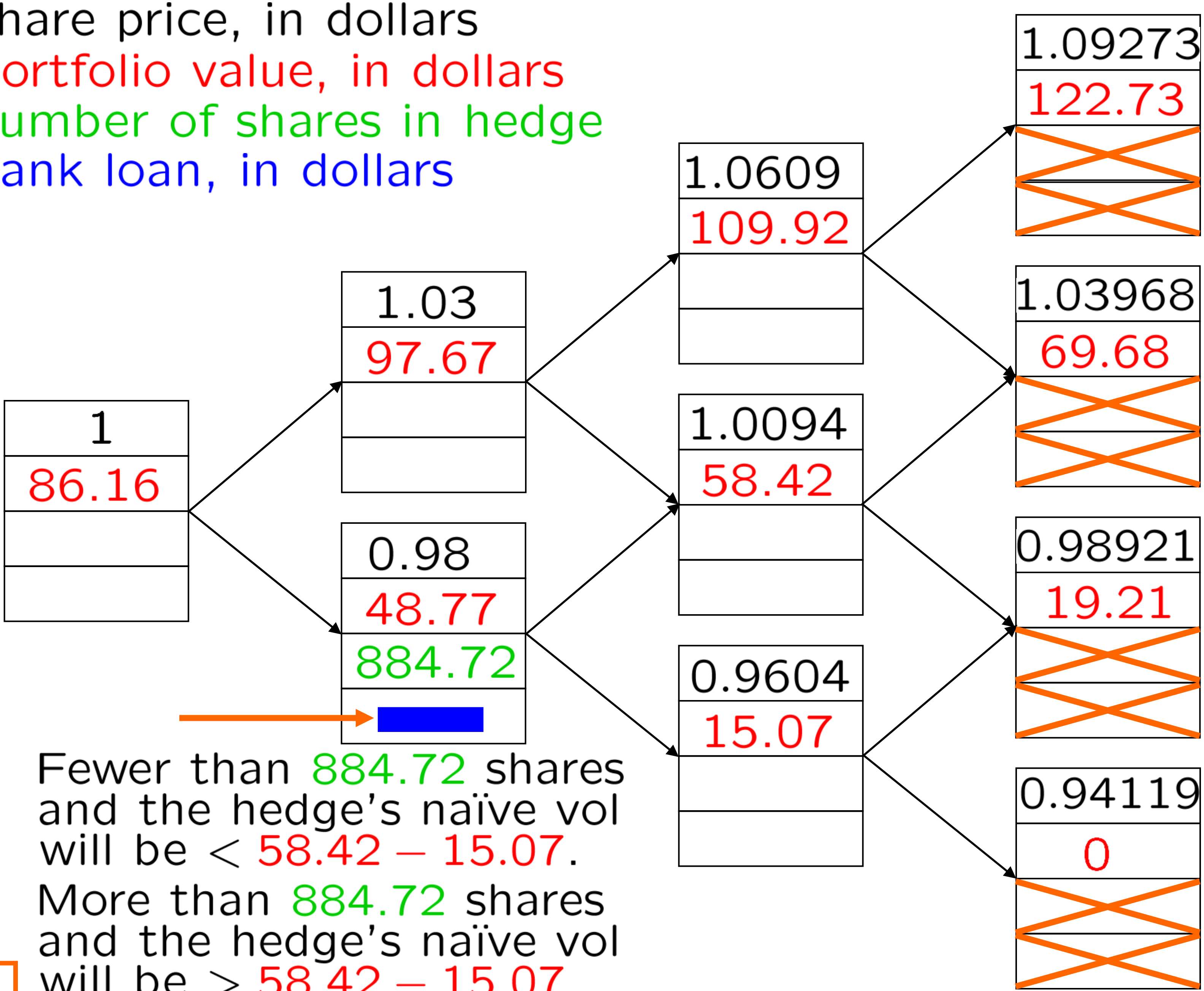
884.7218888
884.72

GAIL AND HARRY

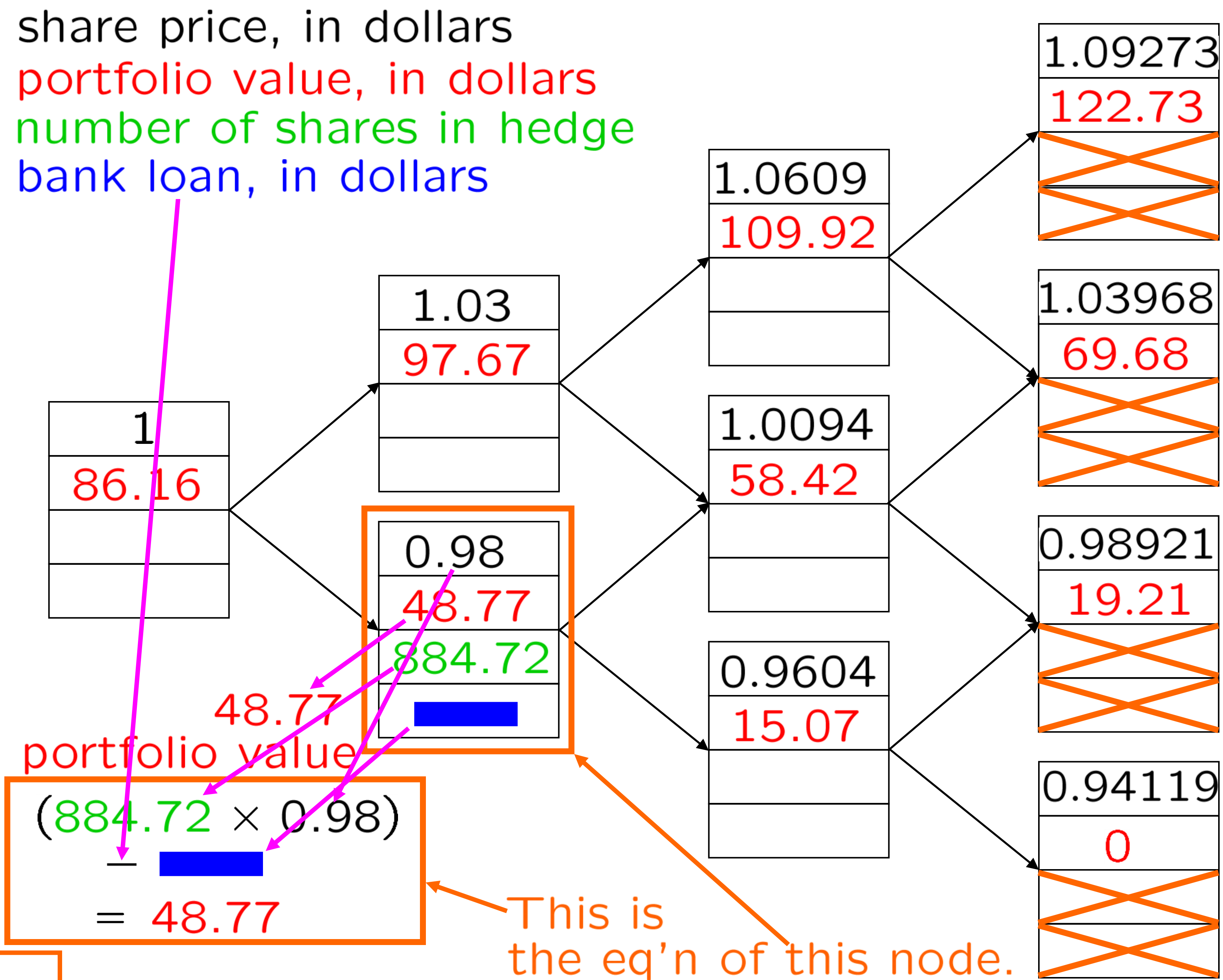
share price, in dollars
portfolio value, in dollars
number of shares in hedge
bank loan, in dollars



Fewer than 884.72 shares and the hedge's naïve vol will be < 58.42 – 15.07.

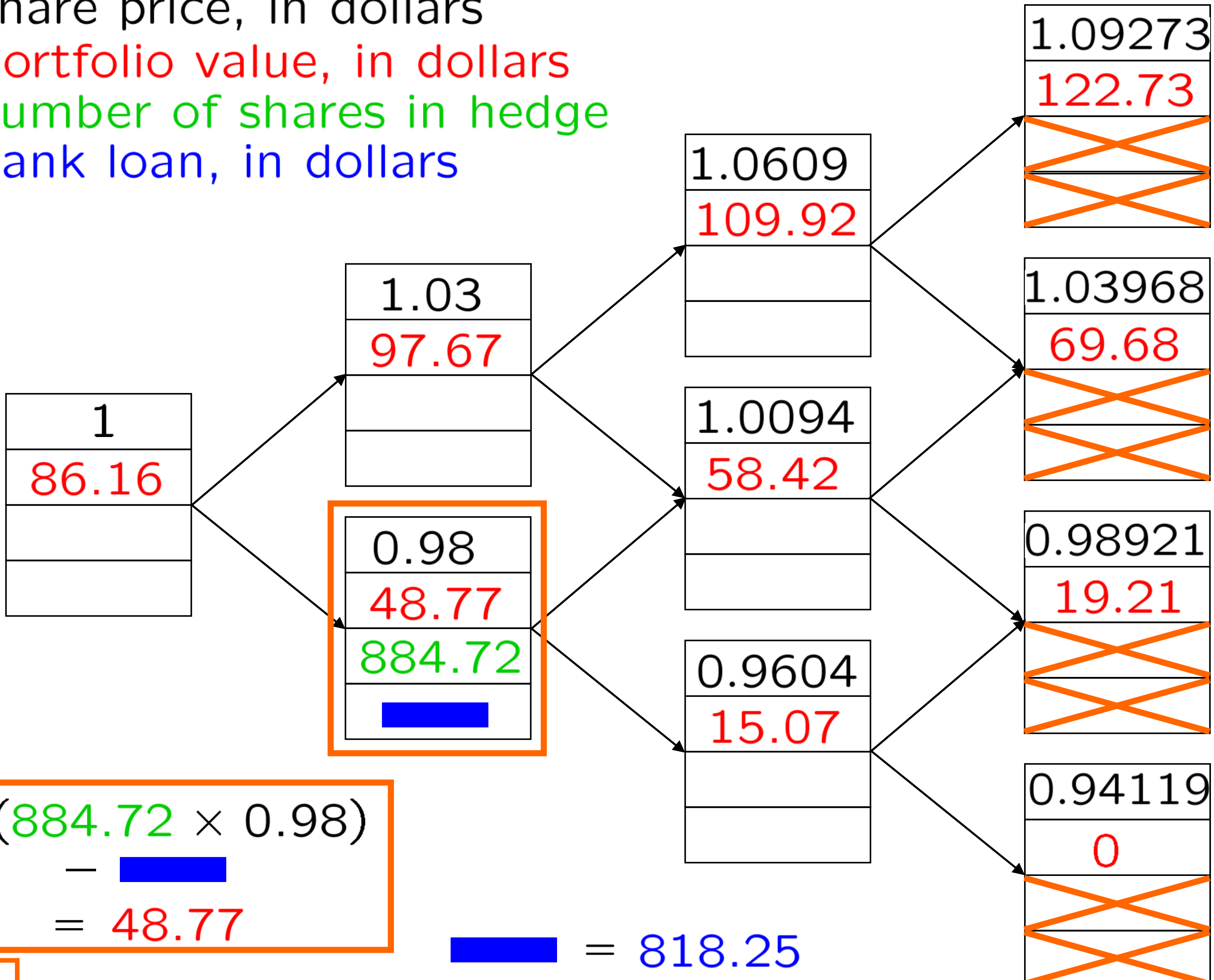
More than 884.72 shares and the hedge's naïve vol will be > 58.42 – 15.07.

GAIL AND HARRY



GAIL AND HARRY

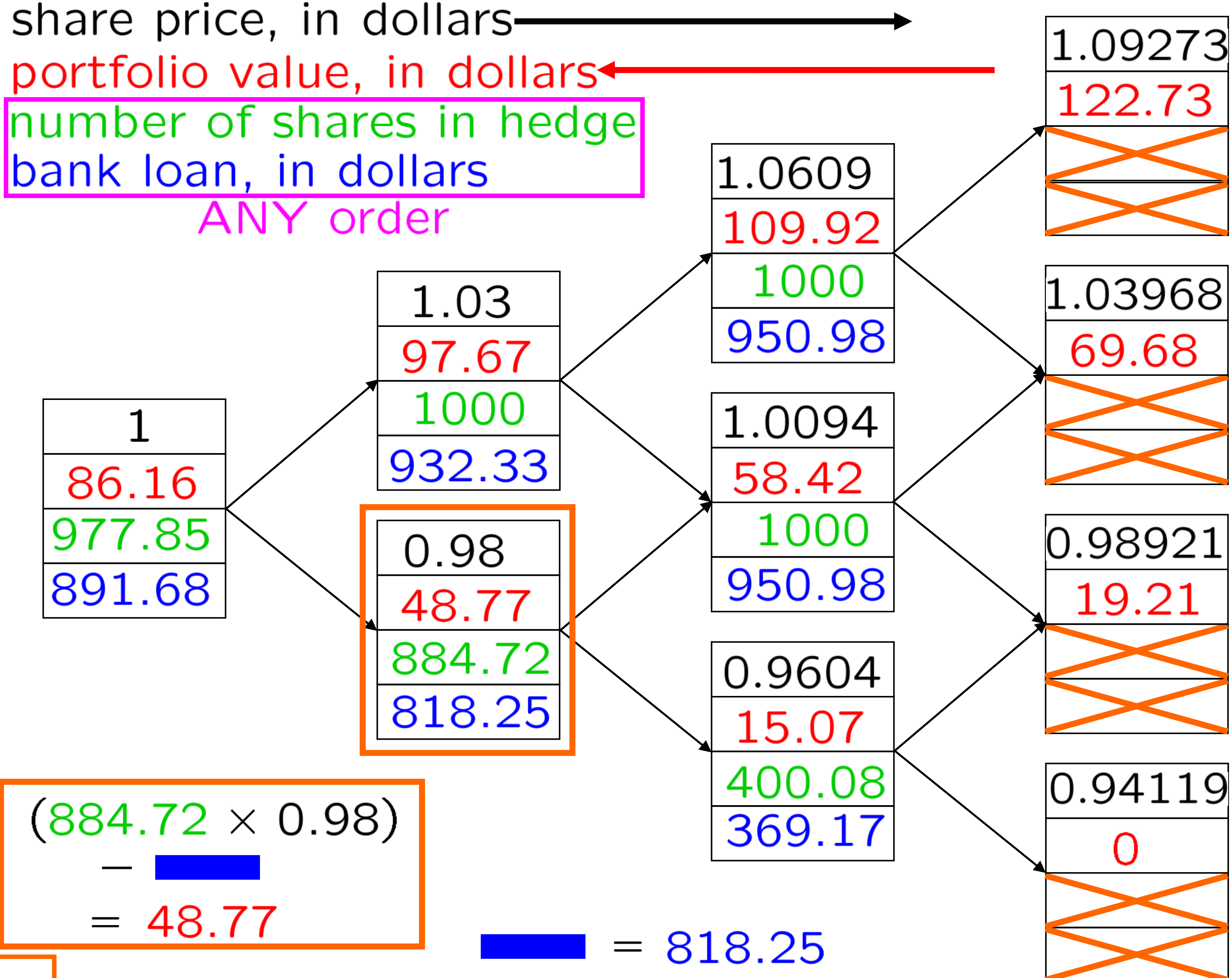
share price, in dollars
portfolio value, in dollars
number of shares in hedge
bank loan, in dollars



(884.72×0.98)
$-$ ▬
$= 48.77$

▬ $= 818.25$

GAIL AND HARRY



(884.72×0.98)

$- \blacksquare$

$= 48.77$

$\blacksquare = 818.25$

GAIL AND HARRY

share price, in dollars
portfolio value, in dollars
number of shares in hedge
bank loan, in dollars