FM 5011 Fall 2009, Midterm #1
Handout date: Thursday 22 October 2009

PRINT NAME:

Remember to read to the bottom and to SIGN YOUR NAME BELOW!

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.
Show work; a correct answer, by itself, may be insufficient for credit.

I understand the above, and I understand that cheating has **severe consequences**, from a failing grade to expulsion.

REMEMBER TO SIGN YOUR NAME:

I. Definitions: Complete the following sentences.

a. (Topic 0001, 3 pts.) Let X be a PCRV. Then the **variance** of X is defined by $\mathrm{Var}[X] = \cdots$.

b. (Topic 0001, 3 pts.) Let X and Y be non-determinstic PCRVs. The **correlation** of X and Y is defined by $\mathrm{Corr}[X,Y] = \cdots$. (You may use $\mathrm{Cov}[X,Y]$, $\mathrm{Var}[X]$ and $\mathrm{Var}[Y]$ in your answer, without defining them.)

c. (Topic 0011, 3 pts.) A **probability space** is a measure space $(\Omega, \mathcal{A}, \mu)$ such that ...

d. (Topic 0011, 3 pts.) An algebra on a set M is a collection $\mathcal{B}$ of subsets of M such that $M \in \mathcal{B}$ and such that, for all $B, C \in \mathcal{B}$, ...

e. (Topic 0011, 3 pts.) Let $(M, \mathcal{A}, \mu)$ be a measure space and let $(N, \mathcal{B})$ be a Borel space. Let $f : M \to N$ be measurable. Then $f_*\mu$ is the measure on N defined by $(f_*\mu)(B) = \cdots$.

II. True or False. (No partial credit.)

a. (Topic 0011, 3 pts.) The composition of any two measurable maps is again measurable.

b. (Topic 0011, 3 pts.) Any σ-algebra is an algebra.

c. (Topic 0008, 3 pts.) Let $\mathcal{S}$ be an identically distributed set of PCRVs, and let $n \geq 1$ be an integer. Then $\sum^n \mathcal{S}$ is also identically distributed. (Recall that $\sum^n \mathcal{S}$ is the set of all iid sums of n PCRVs taken from $\mathcal{S}$.)

d. (Topic 0002, 3 pts.) In the risk-neutral world, risky assets have the same expected returns as do risk-free assets.

e. (Topic 0010, 3 pts.) If $a_1, a_2, a_3, \ldots \geq 0$ and $a_1, a_2, a_3, \ldots \to 0$, then $a_1 + a_2 + a_3 + \cdots$ has a finite sum.

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PLEASE DO NOT WRITE BELOW THE LINE

I.

II.

III(1).

III(2).

III(3).

III(4).

III(5).

III(6).

III(7).

III. Computations. Some of your answers may involve Φ, the cumulative distribution function of the standard normal distribution. (Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.)

1. a. (calculus, 8 pts.) Find the minimum value of $f(x) = \frac{1}{2}\left(\frac{4}{x} + 25x\right)$ over all $x > 0$. (Remember to find the minimum *value*, $f(x)$.)

b. (Topic 0001, 2 pts.) Find the geometric mean of 4 and 25.

2. (Topic 0008, 10 pts.) Let $\mathcal{A}$ be an identically distributed set of PCRVs. Assume that $\mathcal{F}\delta[\mathcal{A}] = \cos t$. Compute $\mathcal{F}\delta[(\sum^5 \mathcal{A})/\sqrt{5}]$. (Recall that "$\mathcal{F}\delta$" means "the Fourier transform of the distribution of".)

3. (Topic 0008, 10 pts.) Let $f(x) = 1 - \frac{x^2}{32} + (1 - \cos x)^2$. Compute $\lim_{n \to \infty} [f(8/\sqrt{n})]^n$.

Hint: Recall that, if $\delta_n \to 0$, then $\left(1 + \frac{a}{n} + \frac{\delta_n}{n}\right)^n \to e^a$.

4. (Topic 0011, 10 pts.) Define the PCRV $X : [0,1] \to \mathbb{R}$ by

$$X(\omega) = \begin{cases} -4, & \text{if } 0 \leq \omega < 0.3; \\ 7, & \text{if } 0.3 \leq \omega < 0.8; \\ 9, & \text{if } 0.8 \leq \omega \leq 1.; \end{cases}$$

Let λ_1 be the restriction of Lebesgue measure to $[0,1]$. Write the distribution $X_*(\lambda_1)$ of X as a linear combination of point masses.

5. Let $d, u \in \mathbb{R}$, assume $d < u$. Let Y be a binary PCRV with $\Pr[Y = u] = 0.5$ and with $\Pr[Y = d] = 0.5$. Suppose that $\mathrm{E}[Y] = 0.15$ and that $\mathrm{SD}[Y] = 0.25$.

a. (Topic 0001, 5 pts.) Find u and d.

b. (Topic 0008, 5 pts.) Find the Fourier transform of the distribution of Y.

6. Let $X_1, \ldots, X_{100}$ be an iid sequence of 100 PCRVs. Let $Y := X_1 + \cdots + X_{100}$. Suppose that $\mathrm{E}[Y] = 0.05$ and that $\mathrm{SD}[Y] = 0.2$.

a. (Topic 0001, 5 pts.) Find $\mathrm{E}[X_1]$.

b. (Topic 0001, 5 pts.) Find $\mathrm{SD}[X_1]$.

7. (Topic 0009, 10 pts.) Compute $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [(e^{2x} - 5)_{+}][e^{-x^2/2}]\, dx.$