

Heads I win, tails you lose!

A personal story of arbitrage.

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The parable of the bookmaker

- 'Financial Calculus' Baxter, Rennie 1996 CUP

- Odds of 3-1 *against*
 - a reward of 3, for a stake of 1
- Odds of 3-1 *on*
 - for a stake of 3, a reward of 1
- Starting Price (SP) book making on a two-horse (A & B) race, win or lose (in a draw, all bets are off).

Quoted odds and implied probabilities

	Bookie payoff	Prob.
Horse A wins	-3	p_A
Horse A loses	1	$1 - p_A$

- If quoted price 3-1 against is fair, the implied probability of winning is $1/(1+3)$.
- $-3p_A + 1(1-p_A) = 0$ implies $p_A = 1/(1+3)$
- p_A and $1-p_B$ may not be equal for quoted prices!

Actual and subjective probabilities

- Suppose from past history horse A has a 25% chance of winning and so horse B a 75% chance i.e. 3-1 against and 3-1 on respectively, in terms of frequency.
- But there is a degree of popular sentiment reflected in the bets made at the track, adding up to \$5,000 for A and \$10,000 for B i.e. 2-1 against and 2-1 on respectively, in terms of money wagered.

Setting odds based on actual probs.

	Horse A	Horse B	
Actual prob.*	25%	75%	
Bets placed	\$5,000	\$10,000	
1. Quoted odds	3-1 against	3-1 on	
Implied prob.	25%	75%	
Profit if horse wins	-\$15,000 +\$10,000 = -\$5,000	\$1,667	Expected* profit = \$0
2. Quoted odds	13-5 against	15-4 on	
Implied prob.	28%	79%	
Profit if horse wins	-\$3,000	\$2,333	Expected* profit = \$1,000

Setting odds based on subjective probs.

	Horse A	Horse B	
Actual prob.*	25%	75%	
Bets placed	\$5,000	\$10,000	
1. Quoted odds	2-1 against	2-1 on	
Implied prob.	33%	67%	
Profit if horse wins	\$0	\$0	Break even for sure.
2. Quoted odds	9-5 against	10-4 on	
Implied prob.	36%	71%	
Profit if horse wins	\$1,000	\$1,000	For sure profit = \$1,000!

The moral...

- Slightly reducing rewards from break even odds based on money wagered allows an SP bookie to make a steady riskless profit!
- Why can't the bookie be greedy?
- Where does this lead to?
 - Arbitrage
 - Martingale measures
 - Bayesian statistics
 - Nash equilibrium
 - Aikido
 - ...

Good luck MFM 2007 students!

May you find a passion and make a living doing it!