MATH 1271 Spring 2012, Midterm #2
Handout date: Thursday 29 March 2012

PRINT YOUR NAME:

Solutions

Version D

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

$-(x^2-4x+3)=-(x-1)(x-3)$

A. (5 pts) (no partial credit) Suppose $f''(x) = -x^2 + 4x - 3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) f is concave up on $(-\infty, -3]$, down on $[-3, -1]$ and up on $[-1, \infty)$.

(b) f is concave up on $(-\infty, 1]$, down on $[1, 3]$ and up on $[3, \infty)$.

(c) f is concave down on $(-\infty, 1]$, up on $[1, 3]$ and down on $[3, \infty)$.

(d) f is concave down on $(-\infty, -3]$, up on $[-3, -1]$ and down on $[-1, \infty)$.

(e) NONE OF THE ABOVE

f'' neg 0 pos 0 neg

1 3

B. (5 pts) (no partial credit) Find the logarithmic derivative of $(2 + x^4)^{\cos x}$ w.r.t. x.

(a) $(-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))$

(b) $(\cos x)(\ln(2 + x^4))$

(c) $(-\sin x)(4x^3/(2 + x^4))$

(d) $(\cos x)(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4))$

(e) NONE OF THE ABOVE

$\frac{d}{dx}\left[(\cos x)(\ln(2+x^4))\right]$

C. (5 pts) (no partial credit) Find the derivative of $(2 + x^4)^{\cos x}$ w.r.t. x.

(a) $[(2 + x^4)^{\cos x}][(-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))]$

(b) $[(2 + x^4)^{\cos x}][(\cos x)(\ln(2 + x^4))]$

(c) $[(2 + x^4)^{\cos x}][(-\sin x)(4x^3/(2 + x^4))]$

(d) $[(2 + x^4)^{\cos x}][(\cos x)(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4))]$

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Find the logarithmic derivative of $x^2 + 7x - 8$ w.r.t. x.

(a) $\dfrac{x^2 + 7x - 8}{2x + 7}$

(b) $\dfrac{2x + 7}{x^2 + 7x - 8}$

(c) $\ln(2x + 7)$

(d) $(\ln(x^2)) + 7(\ln x) - (\ln 8)$

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Find an equation of the tangent line to $4x^2y - 2y^3 = 2$ at the point $(1, 1)$.

(a) $y - 1 = 4(x - 1)$

(b) $y - 1 = 3(x - 1)$

(c) $y - 1 = 2(x - 1)$

(d) $y - 1 = x - 1$

(e) NONE OF THE ABOVE

$8xy + 4x^2y' - 6y^2y' = 0$

$y' = \dfrac{-8xy}{4x^2 - 6y^2}$

slope $= \dfrac{-8}{4-6} = \dfrac{-8}{-2} = 4$

F. (5 pts) (no partial credit) Compute $[d/dx][\sin(\cos(e^x + 3))]$.

(a) $\cos(\cos(e^x + 3))$

(b) 0

(c) $[\cos(\cos(e^x + 3))][\cos(e^x + 3)][e^x + 3]$

(d) $[\cos(\cos(e^x + 3))][-\sin(e^x + 3)][e^x + 3]$

(e) NONE OF THE ABOVE

$=$

$[\cos(\cos(e^x+3))][-\sin(e^x+3)][e^x]$

II. True or false (no partial credit):

a. (5 pts) If f is increasing on an interval I, then $f' > 0$ on I.

F

b. (5 pts) Assume that $\lim_{x \to a} [f(x)] = 0 = \lim_{x \to a} [g(x)]$. Assume also that $\lim_{x \to a} \frac{f'(x)}{g'(x)} = -\infty$. Then $\lim_{x \to a} \frac{f(x)}{g(x)} = -\infty$.

T

c. (5 pts) Every global minimum of a function $f : \mathbb{R} \to \mathbb{R}$ occurs at a critical number for f.

T

d. (5 pts) If $f'(7) = 0$ and $f''(7) > 0$, then f has a local maximum at 7.

U F

e. (5 pts) If two functions have the same derivative, then they are equal.

F

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION D

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1ab.

III. 2.

III. 3,4.

III. 5.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. a. (5 pts) Compute $\frac{d}{dx}\left[\frac{2x^3-8}{7+(\arctan(2x))}\right]$.

$$\parallel$$

$$\frac{[7+(\arctan(2x))][6x^2]-[2x^3-8]\left[\frac{1}{1+(2x)^2}\right][2]}{[7+(\arctan(2x))]^2}$$

b. (5 pts) Compute $\frac{d}{dx}\left[(4-\sin x)^x\right]$.

$$\parallel$$

$$\left[(4-\sin x)^x\right]\left[\frac{d}{dx}\left[x\left(\ln(4-\sin x)\right)\right]\right]$$

$$\parallel$$

$$\left[(4-\sin x)^x\right]\left[\left(\ln(4-\sin x)\right)+x\left(\frac{-\cos x}{4-\sin x}\right)\right]$$

2. (10 pts) Using implicit differentiation, find $y' = dy/dx$, assuming that $(x - y^2)^5 = x$.

$$\left[5(x-y^2)^4\right]\left[1-2yy'\right]=1$$

$$\left[5(x-y^2)^4\right]-\left[10y(x-y^2)^4\right]y'=1$$

$$y'=\frac{1-5(x-y^2)^4}{-10y(x-y^2)^4}$$

3. (5 pts) Let $f(x) = 7x + x^5$. Then f is a one-to-one function. Let $g := f^{-1}$. Then $f(1) = 8$, so $g(8) = 1$. Compute $g'(8)$.

$$g'(8) = \frac{1}{f'(1)} = \frac{1}{\left[7 + 5x^4\right]_{x:\to 1}} = \frac{1}{12}$$

4. (10 pts) Find the maximal intervals of concavity for $f(x) = -3x^5 + 20x^4 + 12x - 7$. For each interval, state clearly whether f is concave up or concave down on that interval.

$$f'(x) = -15x^4 + 80x^3 + 12$$

$$f''(x) = -60x^3 + 240x^2$$
$$= -60x^2(x-4)$$

f''	pos	0^2	pos	0	neg
		0		4	

f is concave up on $(-\infty, 4]$

f is concave down on $[4, \infty)$

5. (10 pts) Compute $\lim_{x\to 1}\left[\frac{\ln x}{\cos(\pi x/2)}\right]$.

$$\frac{0}{0}$$

$$\overset{\text{l'H}}{=}$$

$$\lim_{x\to 1}\frac{1/x}{\left[-\sin(\pi x/2)\right]\left[\pi/2\right]}$$

$$=$$

$$\frac{1/1}{\left[-\sin(\pi/2)\right]\left[\pi/2\right]}$$

$$=$$

$$\frac{1}{\left[-1\right]\left[\pi/2\right]}$$

$$=$$

$$-\frac{2}{\pi}$$