MATH 1271 Spring 2013, Midterm #1
Handout date: Thursday 21 February 2013

PRINT YOUR NAME:

SOLUTIONS
VERSION B

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) (no partial credit) A line passes through $(1, 40)$ and $(5, 80)$. Find its slope. Circle one of the following answers:

(a) 0

(b) 10

(c) -10

(d) 40

(e) NONE OF THE ABOVE

$$\frac{80-40}{5-1} = \frac{40}{4} = 10$$

B. (5 pts) (no partial credit) What is the smallest number x such that $|x-3| \leq 0.005$?

(a) -2.995

(b) 3

(c) 2.995

(d) 3.005

(e) NONE OF THE ABOVE

$$3 - 0.005 \leq x \leq 3 + 0.005$$

C. (5 pts) (no partial credit) Compute $\lim_{x \to -\infty} \left[\frac{x^3 + 2x^2 - 4x}{2x^3 - 7x^2} \right]$ Circle one of the following answers:

(a) 4/7

(b) $-4/7$

(c) 1/2

(d) $-1/2$

(e) NONE OF THE ABOVE

$$\left. \frac{x^3}{2x^3} \right|_{x \neq 0} = \frac{1}{2}$$

$x \to -\infty$

$$\frac{1}{2}$$

D. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x \to 4^-} (h(x)) = 7$? Circle one of the following answers:

(a) If x is close to 4, but not equal to 4, then $h(x)$ is close to 7, but not equal to 7.

(b) If x is close to 4, but greater than 4, then $h(x)$ is close to 7.

(c) If $h(x)$ is close to 7, but not equal to 7, then x is close to 4, but less than 4.

(d) If $h(x)$ is close to 4, then x is close to 7.

(e) NONE OF THE ABOVE $\quad x \approx 4,\ x < 4 \Rightarrow h(x) \approx 7$

E. (5 pts) (no partial credit) Compute $\lim_{t \to 3} \left[\underbrace{\frac{t^2 + t - 12}{t - 3}}_{\| \, t \neq 3} \right]$. Circle one of the following answers:

$t + 4 \xrightarrow[t \to 3]{} 7$

(a) 8

(b) 7

(c) 6

(d) 5

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Compute $\lim_{x \to 0} \left[\underbrace{\frac{(8x^5 + 3x^4)(\cos x)}{4x^3 (\sin x)}}_{\| \, x \to 0} \right]$. Circle one of the following answers:

$\frac{(3x^4)(1)}{4x^3(x)} \underset{x \neq 0}{=} \frac{3}{4} \xrightarrow{x \to 0} \frac{3}{4}$

(a) 3/4

(b) -2

(c) 0

(d) This limit does not exist.

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) Let f be any function. If f is continuous at 3, then 3 is in the domain of f.

True

b. (5 pts) Let f be any algebraic function. If $\lim_{x\to\infty} f(x) = 1/3$, then $\lim_{x\to-\infty} f(x) = 1/3$.

False

c. (5 pts) Let $f(x) = |x|$. Then $f(x)$ is differentiable at $x = -1$.

True

d. (5 pts) $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$.

True

e. (5 pts) Let f be the restriction of sin to $[\pi/3, \pi/2]$. Then f is a one-to-one function.

True

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION B

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find all horizontal asymptotes to

$$y = \frac{\sqrt{4x^2 + 2x + 5}}{7x - 3} =: f(x)$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines, **NOT** numbers.)

$$f(x) \underset{x \to \pm\infty}{\sim} \frac{\sqrt{4x^2}}{7x} = \frac{2|x|}{7x} = \frac{2(\pm x)}{7x} \overset{x \neq 0}{=} \pm\frac{2}{7}$$

$x \downarrow \pm\infty$

$$\pm\frac{2}{7}$$

$y = -\frac{2}{7}$ and $y = \frac{2}{7}$ are the horizontal asymptotes

2. (15 pts) Compute $\lim_{n\to\infty}\left(1+\frac{97}{n}\right)^n$.

$x=\frac{n}{97}$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{97x}$$

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{97}$$

$$\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{97}$$

$$e^{97}$$

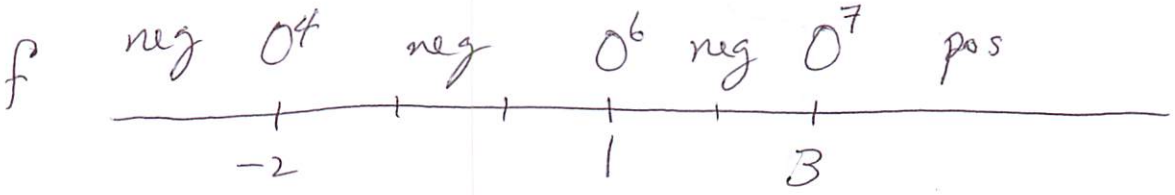
3. (10 pts) Compute $\lim_{x\to\infty}\left[\underbrace{\frac{3x^2+\sin^2 x}{4x^2+2}}_{\text{!!}\;\;f(x)}\right]$.

$$\begin{cases} 1 \\ \downarrow \\ \sin^2 x \\ \downarrow \\ 0 \end{cases} \Rightarrow \begin{cases} \dfrac{3x^2+1}{4x^2+2} \underset{x\to\infty}{\sim} \dfrac{3x^2}{4x^2} \underset{x\neq 0}{=} \dfrac{3}{4} \xrightarrow[x\to\infty]{} \dfrac{3}{4} \\ \downarrow \\ f(x) \\ \downarrow \\ \dfrac{3x^2+0}{4x^2+2} \underset{x\to\infty}{\sim} \dfrac{3x^2}{4x^2} \underset{x\neq 0}{=} \dfrac{3}{4} \xrightarrow[x\to\infty]{} \dfrac{3}{4} \end{cases}$$

By the Squeeze Thm,

$$\lim_{x\to\infty} f(x) = \frac{3}{4}$$

4. (10 pts) Let $f(x) = (x+2)^4(x-1)^6(x-3)^7$. Find all of the maximum intervals of positivity and negativity for f.



f is negative on $(-\infty, -2)$,

negative on $(-2, 1)$,

negative on $(1, 3)$

and positive on $(3, \infty)$.