MATH 1271 Spring 2013, Midterm #1
Handout date: Thursday 21 February 2013

PRINT YOUR NAME:

SOLUTIONS
VERSION D

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) What is the smallest number x such that $|x-3| \leq 0.005$?

(a) 2.995

(b) -3.005

(c) 3.005

(d) -2.995

(e) NONE OF THE ABOVE

$3-0.005 \leq x \leq 3+0.005$

B. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x \to 4^-} (h(x)) = 7$? Circle one of the following answers:

(a) If x is close to 4, but not equal to 4, then $h(x)$ is close to 7, but not equal to 7.

(b) If x is close to 4, but less than 4, then $h(x)$ is close to 7.

(c) If $h(x)$ is close to 7, but not equal to 7, then x is close to 4, but less than 4.

(d) If $h(x)$ is close to 4, then x is close to 7.

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Compute $\lim_{x \to -\infty} \left[\frac{x^3 + 2x^2 - 4x}{2x^4 - 7x^2} \right]$ Circle one of the following answers:

(a) 4/7

(b) $-4/7$

(c) 1/2

(d) $-1/2$

(e) NONE OF THE ABOVE

$\left[\frac{x^3}{2x^4} = \frac{1}{2x} \right]_{x \to -\infty}$

$\downarrow x \to -\infty$

0

D. (5 pts) (no partial credit) Compute $\lim_{t\to 3}\left[\frac{t^2+t-12}{t-3}\right]$. Circle one of the following answers:

$\| t \neq 3$

$t+4 \xrightarrow[t\to 3]{} 7$

(a) 5

(b) 6

(c) 7

(d) 8

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Compute $\lim_{x\to 0}\left[\frac{(3x^5-8x^4)(\cos x)}{4x^3(\sin x)}\right]$. Circle one of the following answers:

$? x\to 0$

$\frac{(-8x^4)(1)}{4x^3(x)} \overset{x\neq 0}{=} -\frac{8}{4} = -2 \xrightarrow{x\to 0} -2$

(a) 3/4

(b) −2

(c) 0

(d) This limit does not exist.

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) (no partial credit) A line passes through $(3, 40)$ and $(5, 80)$. Find its slope. Circle one of the following answers:

$\frac{80-40}{5-3} = \frac{40}{2} = 20$

(a) 20

(b) 10

(c) 0

(d) −10

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) Let f be any rational function. If $\lim_{x\to\infty} f(x) = 2$, then $\lim_{x\to-\infty} f(x) = 2$.

True

b. (5 pts) Let $f(x) = |x|$. Then $f(x)$ is differentiable at $x = 1$.

True

c. (5 pts) Let f be any function. If $\lim_{x\to 3} f(x)$ exists, then 3 is in the domain of f.

False

d. (5 pts) $\lim_{x\to 0} \dfrac{1-\cos x}{x} = 1$.

False

e. (5 pts) Let f be the restriction of sin to $[0, \pi]$. Then f is a one-to-one function.

False

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION D

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find all horizontal asymptotes to

$$y = \frac{\sqrt[3]{8x^3 + 2x + 5}}{5x - 3} =: f(x)$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines, **NOT** numbers.)

$$f(x) \underset{x \to \pm\infty}{\sim} \frac{\sqrt[3]{8x^3}}{5x} = \frac{2x}{5x} \underset{x \neq 0}{=} \frac{2}{5}$$

$$\downarrow \; x \to \pm\infty$$

$$\frac{2}{5}$$

$y = \frac{2}{5}$ is the only horizontal asymptote.

2. (15 pts) Compute $\lim_{n\to\infty}\left(1+\frac{27}{n}\right)^n$.

$x = \frac{n}{27}$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{27x}$$

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{27}$$

$$\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{27}$$

$$e^{27}$$

3. (10 pts) Compute $\lim_{x\to\infty}\left[\underbrace{\frac{2x^2+\cos^2 x}{4x^2+2}}_{\text{!!}\;\;f(x)}\right]$.

$$1 \qquad \frac{2x^2+1}{4x^2+2} \underset{x\to\infty}{\sim} \frac{2x^2}{4x^2} \underset{x\neq 0}{=} \frac{2}{4} \xrightarrow[x\to\infty]{} \frac{2}{4}$$

$$\text{VI} \qquad \text{VI}$$

$$\cos^2 x \qquad f(x)$$

$$\text{VI} \qquad \text{VI}$$

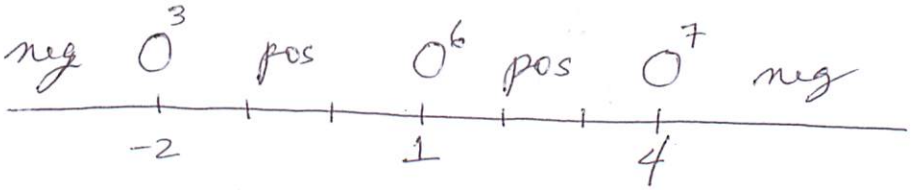
$$0 \qquad \frac{2x^2+0}{4x^2+2} \underset{x\to\infty}{\sim} \frac{2x^2}{4x^2} \underset{x\neq 0}{=} \frac{2}{4} \xrightarrow[x\to\infty]{} \frac{2}{4}$$

By the Squeeze Thm,

$$\lim_{x\to\infty} f(x) = \frac{2}{4} = \frac{1}{2}$$

4. (10 pts) Let $f(x) = -(x+2)^3(x-1)^6(x-4)^7$. Find all of the maximum intervals of positivity and negativity for f.

f



f is negative on $(-\infty, -2)$,

positive on $(-2, 1)$,

positive on $(1, 4)$

and negative on $(4, \infty)$.