MATH 1271 Spring 2013, Midterm #2
Handout date: Thursday 4 April 2013

PRINT YOUR NAME:

SOLUTIONS
Version A

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Let f be a function such that $f'(x) = 4e^{4x}$. Suppose, also, that $f(0) = 1$. Which of the following is an equation of the tangent line to the graph of f at $(0, 1)$. Circle one of the following answers:

(a) $y = 4(x - 1)$

(b) $y = 1 + 4x$

(c) $y - 1 = 4e^{4x}x$

(d) $y = 4e^{4x}(x - 1)$

(e) NONE OF THE ABOVE

slope $= f'(0) = 4e^{4\cdot 0} = 4$

$y - 1 = 4(x - 0)$

$y = 1 + 4x$

B. (5 pts) (no partial credit) Suppose $f'(x) = (x - 1)^2(x - 2)(x - 3)^2$. Which of the following is a maximal interval of increase for f? Circle one of the following answers:

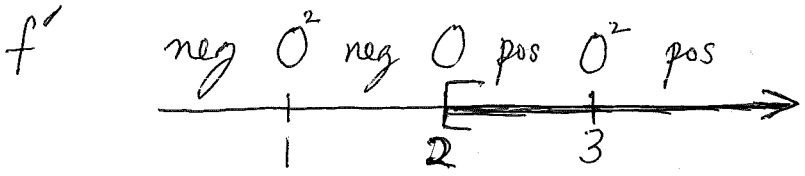
(a) $[2, \infty)$

(b) $(-2, \infty)$

(c) $[1, \infty)$

(d) $(-\infty, 1]$

(e) NONE OF THE ABOVE



C. (5 pts) (no partial credit) The Quotient Rule says that $(f/g)'$ is equal to what? Circle one of the following answers:

(a) f'/g'

(b) g'/f'

(c) $(gf' - fg')/g^2$

(d) $(fg' - gf')/g^2$

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Compute $\frac{d}{dx}[\ln|(2x+1)(3x-4)|]$. Circle one of the following answers:

(a) $\frac{2}{2x+1}+\frac{3}{3x-4}$

(b) $\left|\frac{2}{2x+1}+\frac{3}{3x-4}\right|$

(c) $\frac{6}{(2x+1)(3x+4)}$

(d) $\left|\frac{6}{(2x+1)(3x+4)}\right|$

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Compute $[d/dx][\sin^2(xy)]$. Circle one of the following answers:

(a) $2[\sin(xy)][\cos(xy)]$

(b) $[\cos^2(xy)][y+xy']$

(c) $2[\sin(xy)][y+xy']$

(d) $2[\sin(xy)][\cos(y+xy')]$

(e) NONE OF THE ABOVE

$= 2[\sin(xy)][\cos(xy)][y+xy']$

F. (5 pts) (no partial credit) Compute $\lim_{x\to\infty}(2x^2+4x-3)e^{-x}$. Circle one of the following answers:

(a) 2

(b) -3

(c) ∞

(d) 0

(e) NONE OF THE ABOVE

$= \lim_{x\to\infty}\frac{2x^2+4x-3}{e^x}$

$=$ l'H $\frac{\infty}{\infty}$ twice

$= \lim_{x\to\infty}\frac{4}{e^x} = 0$

"$e^{\infty}=\infty$" and "$\frac{4}{\infty}=0$"

II. True or false (no partial credit): $e^u(du/dx)$

a. (5 pts) Let u be any expression of x. Then $(d/dx)(e^u) = e^u$.

False

b. (5 pts) If f is increasing on an interval I, then $f' > 0$ on I.

False

c. (5 pts) Let f and g be any two functions such that $\lim_{x\to a}[f(x)] = \infty$ and $\lim_{x\to a}[g(x)] = \infty$. Then $\lim_{x\to a}[(f(x)) - (g(x))] = 0$.

False "$\infty - \infty$" is indeterminate

d. (5 pts) Let g be any function such that $\lim_{x\to\infty}[g(x)] = \infty$. Then $\lim_{x\to\infty}[(1/x)^{g(x)}] = 0$.

True "$1/\infty = 0^+$" and "$(0^+)^\infty = 0$"

e. (5 pts) Let f and g be any two functions such that $\lim_{x\to 5} f(x) = 1$ and $\lim_{x\to 5} g(x) = 0$. Then $\lim_{x\to 5} \dfrac{f(x)}{g(x)} = \infty$.

False "$\frac{1}{0}$" is (slightly) indeterminate

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5. a,b,c

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute $\frac{d}{dx}\left[\frac{e^{-x^4}}{4+\tan(x^2)}\right]$. (Here e^{-x^4} means $e^{(-x^4)}$.)

$$=\frac{[4+\tan(x^2)][e^{-x^4}][-4x^3]-[e^{-x^4}][\sec^2(x^2)][2x]}{[4+\tan(x^2)]^2}$$

2. (5 pts) Compute $\frac{d}{dx}\left[(2-\cos x)^{4+x}\right]$.

$$=[(2-\cos x)^{4+x}]\left[\frac{d}{dx}\left[(4+x)(\ln(2-\cos x))\right]\right]$$

$$=[(2-\cos x)^{4+x}]\left[(1)(\ln(2-\cos x))+(4+x)\left(\frac{\sin x}{2-\cos x}\right)\right]$$

3. (10 pts) Find an equation for the tangent line to $7x^3 - 5xy + y^2 = 4x - y$ at $(1, 3)$.

$m :=$ slope of this tangent line

$$21x^2 - 5y - 5xy' + 2yy' = 4 - y'$$

1 3 1 m 3 m m

$$21 - 15 - 5m + 6m = 4 - m$$

$$6 + m = 4 - m$$

$$2m = -2$$

$$m = -1$$

$$\boxed{y - 3 = (-1)(x - 1)}$$

4. (10 pts) Compute $\lim_{x\to 0} (e^x + \sin x)^{5/x}$.

$$\|$$

$$e^{\lim_{x\to 0} (5/x)(\ln(e^x+\sin x))}$$

$$\|$$

$$e^{\lim_{x\to 0} \frac{5(\ln(e^x+\sin x))}{x}}$$

$$\| \text{l'H} \tfrac{0}{0}$$

$$e^{\lim_{x\to 0} \frac{5\left(\frac{e^x+\cos x}{e^x+\sin x}\right)}{1}}$$

$$\|$$

$$e^{\frac{5\left(\frac{1+1}{1+0}\right)}{1}} = e^{10}$$

5. Let $y = x^3$. Then $\triangle y = ax^2(\triangle x) + bx(\triangle x)^2 + c(\triangle x)^3$, for some real numbers a, b, c.

a. (5 pts) Compute a, b and c.

$$\triangle y = (x + \triangle x)^3 - x^3$$

$$= \cancel{x^3} + 3x^2(\triangle x) + 3x(\triangle x)^2 + (\triangle x)^3 - \cancel{x^3}$$

$a = 3$ $\quad$ $b = 3$ $\quad$ $c = 1$

b. (5 pts) Assuming $\triangle x \neq 0$, compute $\dfrac{\triangle y}{\triangle x}$.

$\| \triangle x \neq 0$

$$3x^2 + 3x(\triangle x) + (\triangle x)^2$$

c. (5 pts) Compute $\lim\limits_{\triangle x \to 0} \dfrac{\triangle y}{\triangle x}$.

$\|$

$$3x^2$$