MATH 1271 Spring 2013, Midterm #2
Handout date: Thursday 4 April 2013

PRINT YOUR NAME:

SOLUTIONS
Version B

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) The Quotient Rule says that $(f/g)'$ is equal to what? Circle one of the following answers:

(a) f'/g'

(b) g'/f'

(c) $(fg' - gf')/g^2$

(d) $(gf' - fg')/g^2$

(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Compute $[d/dx][\sin^2(xy)]$. Circle one of the following answers:

(a) $2[\sin(xy)][\cos(xy)]$

(b) $[\cos^2(xy)][y + xy']$

(c) $2[\sin(xy)][\cos(xy)][y + xy']$

(d) $2[\sin(xy)][\cos(y + xy')]$

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Compute $\lim_{x\to\infty} (2x^2 + 4x - 3)e^{-x}$. Circle one of the following answers:

(a) 0

(b) -3

(c) ∞

(d) 2

(e) NONE OF THE ABOVE

$$\| \quad \lim_{x\to\infty} \frac{2x^2+4x-3}{e^x}$$

$$\| \; l'H \tfrac{\infty}{\infty} \text{ twice}$$

$$\lim_{x\to\infty} \frac{4}{e^x} = 0$$

$$\boxed{\text{“}e^\infty = \infty\text{” and “}\frac{4}{\infty} = 0\text{”}}$$

D. (5 pts) (no partial credit) Let f be a function such that $f'(x) = 3e^{4x-4}$. Suppose, also, that $f(1) = 5$. Which of the following is an equation of the tangent line to the graph of f at $(1, 5)$. Circle one of the following answers:

(a) $y = 1 + 3(x - 5)$

(b) $y = 5 + 3(x - 1)$

(c) $y = 1 + 3e^{4x-4}(x - 5)$

(d) $y = 5 + 3e^{4x-4}(x - 1)$

(e) NONE OF THE ABOVE

slope $= f'(1) = 3e^{4\cdot 1-4} = 3$

$y - 5 = 3(x-1)$

$y = 5 + 3(x-1)$

E. (5 pts) (no partial credit) Suppose $f'(x) = (x-1)^8(x-2)^6(x-3)^5$. Which of the following is a maximal interval of increase for f? Circle one of the following answers:

(a) $[2, \infty)$

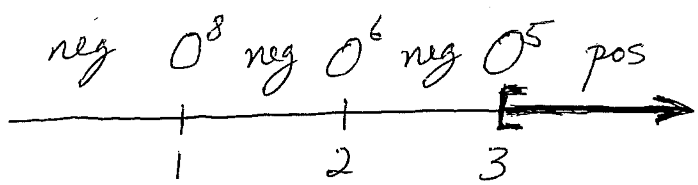
(b) $(-2, \infty)$

(c) $[1, \infty)$

(d) $(-\infty, 1]$

(e) NONE OF THE ABOVE

f'



$[3, \infty)$

F. (5 pts) (no partial credit) Compute $\dfrac{d}{dx}\left[\ln|(2x+1)(3x-4)|\right]$. Circle one of the following answers:

(a) $\dfrac{2}{2x+1} + \dfrac{3}{3x-4}$

(b) $\left|\dfrac{2}{2x+1} + \dfrac{3}{3x-4}\right|$

(c) $\dfrac{6}{(2x+1)(3x+4)}$

(d) $\left|\dfrac{6}{(2x+1)(3x+4)}\right|$

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) Let f and g be any two functions such that $\lim_{x\to a}[f(x)] = \infty$ and $\lim_{x\to a}[g(x)] = \infty$. Then $\lim_{x\to a}[(f(x)) - (g(x))] = 0$.

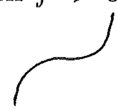
False "$\infty - \infty$" is indeterminate

b. (5 pts) Let g be any function such that $\lim_{x\to\infty}[g(x)] = \infty$. Then $\lim_{x\to\infty}[(1/x)^{g(x)}] = 0$.

True "$1/\infty = 0^+$" and "$(0^+)^\infty = 0$"

c. (5 pts) If f is increasing on an interval I, then $f' > 0$ on I.

False



d. (5 pts) Let f and g be any two functions such that $\lim_{x\to 5} f(x) = 0$ and $\lim_{x\to 5} g(x) = \infty$. Then $\lim_{x\to 5} \dfrac{f(x)}{g(x)} = 0$.

True "$\frac{0}{\infty} = 0$"

e. (5 pts) Let u be any expression of x. Then $(d/dx)(e^u) = e^u(du/dx)$.

True chain rule

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION B

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5. a,b,c

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute $\frac{d}{dx}\left[\frac{e^{-x^3}}{4+\tan(x^2)}\right]$. (Here e^{-x^3} means $e^{(-x^3)}$.)

$$\| $$

$$\frac{[4+\tan(x^2)][e^{-x^3}][-3x^2]-[e^{-x^3}][\sec^2(x^2)][2x]}{[4+\tan(x^2)]^2}$$

2. (5 pts) Compute $\frac{d}{dx}\left[(2+\sin x)^{3-x}\right]$.

$$\| $$

$$\left[(2+\sin x)^{3-x}\right]\left[\frac{d}{dx}\left[(3-x)(\ln(2+\sin x))\right]\right]$$

$$\| $$

$$\left[(2+\sin x)^{3-x}\right]\left[(-1)(\ln(2+\sin x))+(3-x)\left(\frac{\cos x}{2+\sin x}\right)\right]$$

3. (10 pts) Find an equation for the tangent line to $5x^3 - 2xy + y^2 = 5x + y$ at $(1, 3)$.

$m :=$ slope of this tangent line

$$15\underset{1}{x^2} - 2\underset{3}{y} - 2\underset{1}{x}\,\underset{\cdot m}{y'} + 2\underset{3}{y}\,\underset{m}{y'} = 5 + \underset{m}{y'}$$

$$15 - 6 - 2m + 6m = 5 + m$$

$$9 + 4m = 5 + m$$

$$3m = -4$$

$$m = -\frac{4}{3}$$

$$\boxed{y - 3 = -\frac{4}{3}(x - 1)}$$

4. (10 pts) Compute $\lim_{x \to 0} (e^x - 4\sin x)^{5/x}$.

$$\|$$

$$e^{\lim_{x \to 0} (5/x)(\ln(e^x - 4\sin x))}$$

$$\|$$

$$e^{\lim_{x \to 0} \frac{5(\ln(e^x - 4\sin x))}{x}}$$

$$\| \; l'H \tfrac{0}{0}$$

$$e^{\lim_{x \to 0} \frac{5\left(\frac{e^x - 4\cos x}{e^x - 4\sin x}\right)}{1}}$$

$$\|$$

$$e^{\frac{5\left(\frac{1-4}{1-0}\right)}{1}} = e^{-15}$$

5. Let $y = x^4$. Then $\triangle y = px^3(\triangle x) + qx^2(\triangle x)^2 + rx(\triangle x)^3 + s(\triangle x)^4$, for some real numbers p, q, r, s.

a. (5 pts) Compute p, q, r and s.

$$\Delta y = (x+\Delta x)^4 - x^4$$

$$= \cancel{x^4} + 4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 - \cancel{x^4}$$

$$p = 4 \qquad q = 6 \qquad r = 4 \qquad s = 1$$

b. (5 pts) Assuming $\triangle x \neq 0$, compute $\frac{\triangle y}{\triangle x}$.

$$= \quad (\Delta x \neq 0)$$

$$4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3$$

c. (5 pts) Compute $\lim\limits_{\triangle x \to 0} \frac{\triangle y}{\triangle x}$.

$$= 4x^3$$