CALCULUS

Definite integration and Riemann sum problems

NEW

0590-1. Let $f(x) = 2 + 2x^2$.
NEW

a. Compute $L_4 S_{-2}^{2} f$.
Sketch f over $[-2, 2]$ and add,
into your sketch, the four
rectangles represented by $L_4 S_{-2}^{2} f$.

b. Compute $M_4 S_{-2}^{2} f$.
Sketch f over $[-2, 2]$ and add,
into your sketch, the four
rectangles represented by $M_4 S_{-2}^{2} f$.

c. Compute $R_4 S_{-2}^{2} f$.
Sketch f over $[-2, 2]$ and add,
into your sketch, the four
rectangles represented by $R_4 S_{-2}^{2} f$.

NEW 0590-2. Let $f(x) = e^x + 6$.

a. Compute $L_2 S_0^8 f$ to three decimal places.

b. Compute $M_2 S_0^8 f$ to three decimal places.

c. Compute $R_2 S_0^8 f$ to three decimal places.

NEW 0590-3. Let $f(x) = \sin^2 x$.

a. Compute $L_3 S_0^{2\pi} f$ to three decimal places.

b. Compute $M_3 S_0^{2\pi} f$ to three decimal places.

c. Compute $R_3 S_0^{2\pi} f$ to three decimal places.

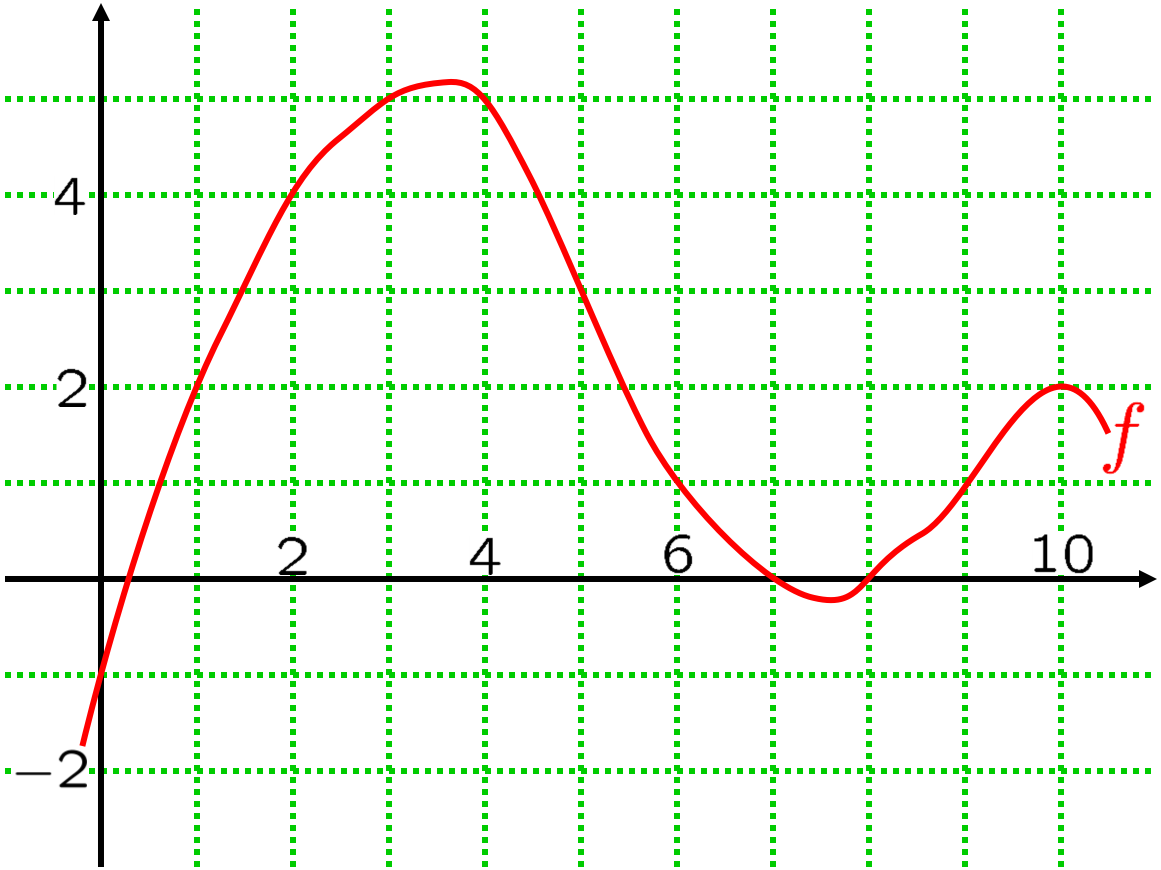
0590-4. A car's acceleration is positive from time 0 to time 24 seconds, and its velocity at various times is given in the table below.

NEW

time (secs)	0	4	8	12	16	20	24
velocity (ft/sec)	0	40	56	68	77	81	83

Find upper and lower estimates for the distance traveled by the car over these 24 seconds.

0590-5. The gph of a function f appears below.
NEW



Estimate $\int_0^{10} f(x)\,dx$ by computing

(a) $L_5 S_0^{10} f$, (b) $M_5 S_0^{10} f$

and (c) $R_5 S_0^{10} f$.

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0590-6. Express the area under $y = e^{-x^2/5}$
NEW
from $x = -2$ to $x = 0$ as a limit of midpoint Riemann sums. (Don't evaluate the limit.)

0590-7. Express the area under $y = \sqrt{x^3 + x + 5}$
NEW
from $x = 1$ to $x = 4$ as a limit of left endpoint Riemann sums. (Don't evaluate the limit.)

0590-8. Express the area under $y = \cos\left(x^4 - x\right)$
NEW
from $x = 0$ to $x = 5$ as a limit of right endpt Riemann sums. (Don't evaluate the limit.)

NEW 0590-9. Express $\displaystyle\int_2^4 \frac{e^{-x^2}}{\sqrt{\pi}}\,dx$ as a limit of midpoint Riemann sums.
(Don't evaluate the limit.)

NEW 0590-10. Let $f(x) = 2x^3$.

a. Write $R_n S_0^2 f$ as a rational expression in n (*i.e.*, as one polynomial in n divided by another).

b. Compute $\displaystyle\lim_{n\to\infty} R_n S_0^2 f$.

0590-11. The limit
NEW

$$\lim_{n\to\infty}\left[\frac{3}{n}\sum_{j=1}^{n}\left(\cos^2\left(-4+j(3/n)\right)\right)\right]$$

represents the area under $y = f(x)$ from $x = a$ to $x = b$, for some choice of $f(x)$, a and b.

a. Find $f(x)$, a and b.

b. Express the limit as a definite integral.

0590-12. The limit
NEW

$$\lim_{n\to\infty}\left[\frac{5}{n}\sum_{j=0}^{n-1}\left(\cos\left(\frac{1}{3+j(5/n)}\right)\right)\right]$$

represents the area under $y = f(x)$
from $x = a$ to $x = b$,
for some choice of $f(x)$, a and b.

a. Find $f(x)$, a and b.

b. Express the limit
as a definite integral.

0590-13. Let $f(x) = 2 - \sqrt{4 - x^2}$.
NEW

a. Sketch the graph of $y = f(x)$.

b. Compute $\int_{-2}^{2} f(x)\,dx$, by interpreting this integral as an area.