

CALCULUS

Problems involving horizontal asymptotes

NEW

WARNING: In this homework, do NOT use l'Hôpital's rule. It has not been covered yet.

0250-1. Using the graph below of f ,

NEW

find these limits:

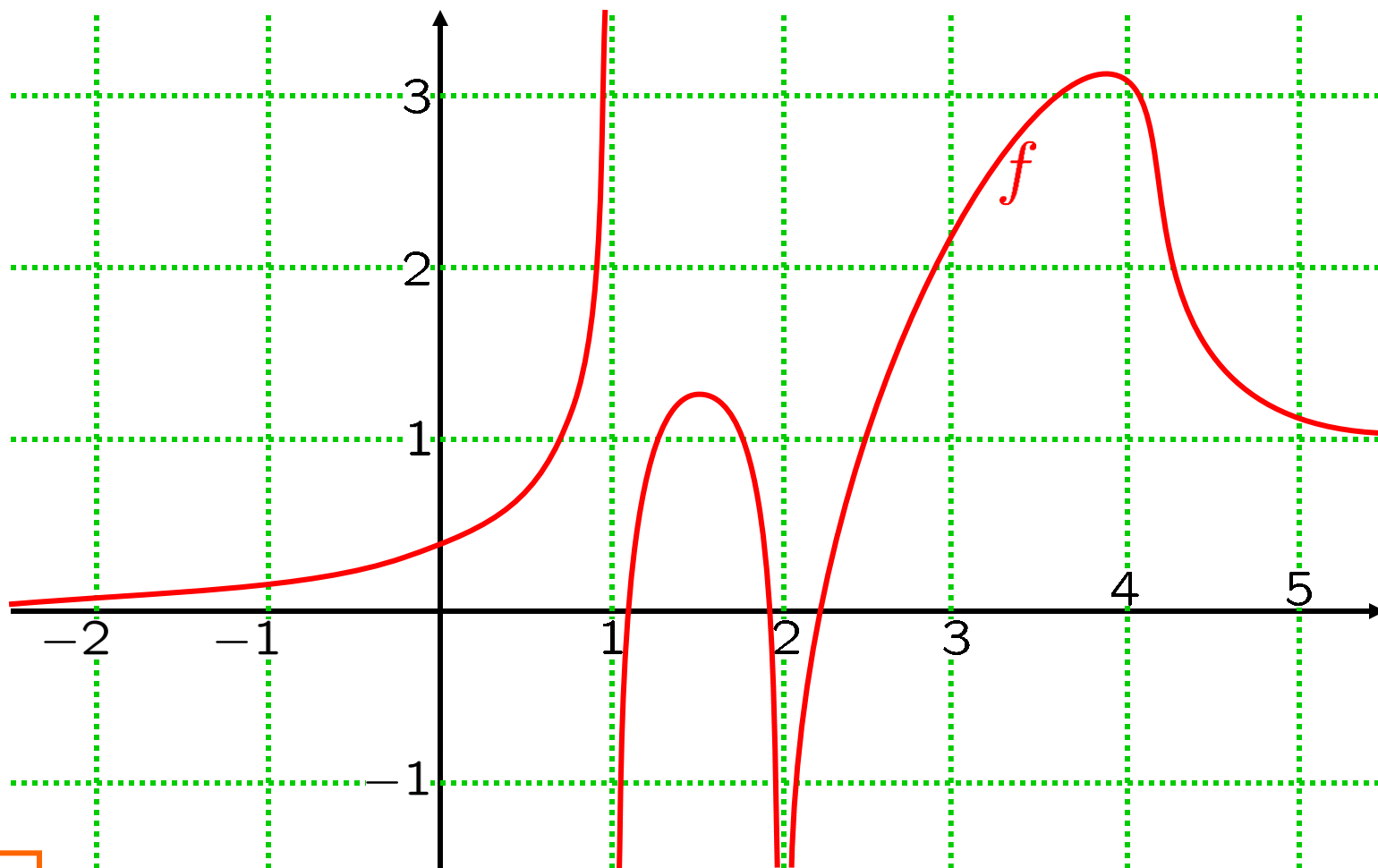
a. $\lim_{x \rightarrow -\infty} f(x)$

b. $\lim_{x \rightarrow 1^-} f(x)$

c. $\lim_{x \rightarrow 1^+} f(x)$

d. $\lim_{x \rightarrow 2} f(x)$

e. $\lim_{x \rightarrow \infty} f(x)$



Sketch a graph of a function $h : \mathbb{R} \rightarrow \mathbb{R}$ s.t.

$$h \text{ is even, i.e., } h(-x) = h(x),$$

$$\lim_{x \rightarrow -\infty} h(x) = 0,$$

$$\lim_{x \rightarrow -3} h(x) = \infty,$$

$$\lim_{x \rightarrow -2^-} h(x) = -\infty, \quad h(-2) = \lim_{x \rightarrow -2^+} h(x) = 1,$$

$$\lim_{x \rightarrow 0} h(x) = 2 \quad \text{and} \quad h(0) = 2.$$



0250-3.
NEW

Sketch a graph of a function $h : \mathbb{R} \rightarrow \mathbb{R}$ s.t.

$$h \text{ is odd, i.e., } h(-x) = -(h(x)),$$

$$\lim_{x \rightarrow -\infty} h(x) = 2,$$

$$\lim_{x \rightarrow -3} h(x) = \infty,$$

$$\lim_{x \rightarrow -2^-} h(x) = -\infty, \quad h(-2) = \lim_{x \rightarrow -2^+} h(x) = 1,$$

$$\lim_{x \rightarrow -1} h(x) = -\infty \quad \text{and} \quad h(1) = -1.$$

0250-4. Compute
NEW

a. $\lim_{x \rightarrow \infty} \frac{x^3 + 2x - 1}{5x^3 - 7}$

Do NOT use
l'Hôpital's rule.

b. $\lim_{x \rightarrow \infty} \frac{x^4 + 1000000}{-x^4 + 1}$

Do NOT use
l'Hôpital's rule.

c. $\lim_{x \rightarrow \infty} \frac{2x + 1000000}{x^3 - 3}$

Do NOT use
l'Hôpital's rule.

d. $\lim_{x \rightarrow \infty} \frac{8x^4 + 1}{4x^3 + 1000000}$

Do NOT use
l'Hôpital's rule.

0250-5. Compute
NEW

a. $\lim_{x \rightarrow -\infty} \frac{x^3 + 2x - 1}{5x^3 - 7}$

Do NOT use l'Hôpital's rule.

b. $\lim_{x \rightarrow -\infty} \frac{x^4 + 1000000}{-x^4 + 1}$

Do NOT use l'Hôpital's rule.

c. $\lim_{x \rightarrow -\infty} \frac{2x + 1000000}{x^3 - 3}$

Do NOT use l'Hôpital's rule.

d. $\lim_{x \rightarrow -\infty} \frac{8x^4 + 1}{4x^3 + 1000000}$

Do NOT use l'Hôpital's rule.

NOTE: These are limits at $-\infty$; in the last problem, they were at ∞ .

0250-6. Compute

NEW

a. $\lim_{x \rightarrow -\infty} \sqrt[3]{\frac{54x^3 + 2}{-2x^3 + 3x - 4}}$

Do NOT use l'Hôpital's rule.

b. $\lim_{x \rightarrow \infty} \sqrt[5]{\frac{-96x^4 - 2x^3 + 1}{2 + 5x - 3x^4}}$

Do NOT use l'Hôpital's rule.

c. $\lim_{x \rightarrow -\infty} \frac{\sqrt{36x^6 + 8x^2 - x}}{x^3 - 4x^2 + 5}$

Do NOT use l'Hôpital's rule.

d. $\lim_{x \rightarrow -\infty} \left(\sqrt{x^4 + 8x + 4} - \sqrt{x^4 + 6x - 3} \right)$

Do NOT use l'Hôpital's rule.

0250-7. Find the (maximal) intervals

NEW

where $x^2(x - 1)^3(x - 2)^2$
is positive and negative,

then compute $\lim_{x \rightarrow -\infty} x^2(x - 1)^3(x - 2)^2,$

then compute $\lim_{x \rightarrow \infty} x^2(x - 1)^3(x - 2)^2.$

0250-8. Suppose, $\forall x < 100$, that

$$\frac{\sqrt{3x^3 - 2x^2 - 8}}{-2x^3 + 5x + 7} < f(x) < \frac{\sqrt{3x^4 - 16x^3 - 48x}}{-2x^4 + 10x + 30}.$$

Compute $\lim_{x \rightarrow -\infty} f(x)$.