CALCULUS
Optimization
NEW

0510-1. Among all pairs of numbers $x, y \in \mathbb{R}$
NEW
such that $2x + y = 16$, we seek
the pair whose product xy is maximized.
We examine the following table.

x	$y = 16 - 2x$	$xy = x(16 - 2x)$
1	14	14
2	12	24
3	10	30
4	8	32
5	6	30
6	4	24

a. Based on this table, guess a solution.

b. Find the exact solution by finding the maximum of all the values of $f(x) = x(16 - 2x)$ at critical numbers.

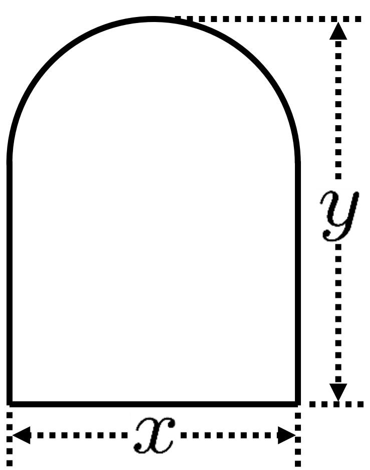
0510-2. Among all pairs of numbers $x, y \in \mathbb{R}$ such that $2x - y = 14$, find the pair whose product xy is minimized.
NEW

0510-3. Maximize $u^3 v$ subject to $3u + 4v = 8$.
NEW

0510-4. Minimize $r + s$ subject to $r^4 + s^4 = 81$.
NEW

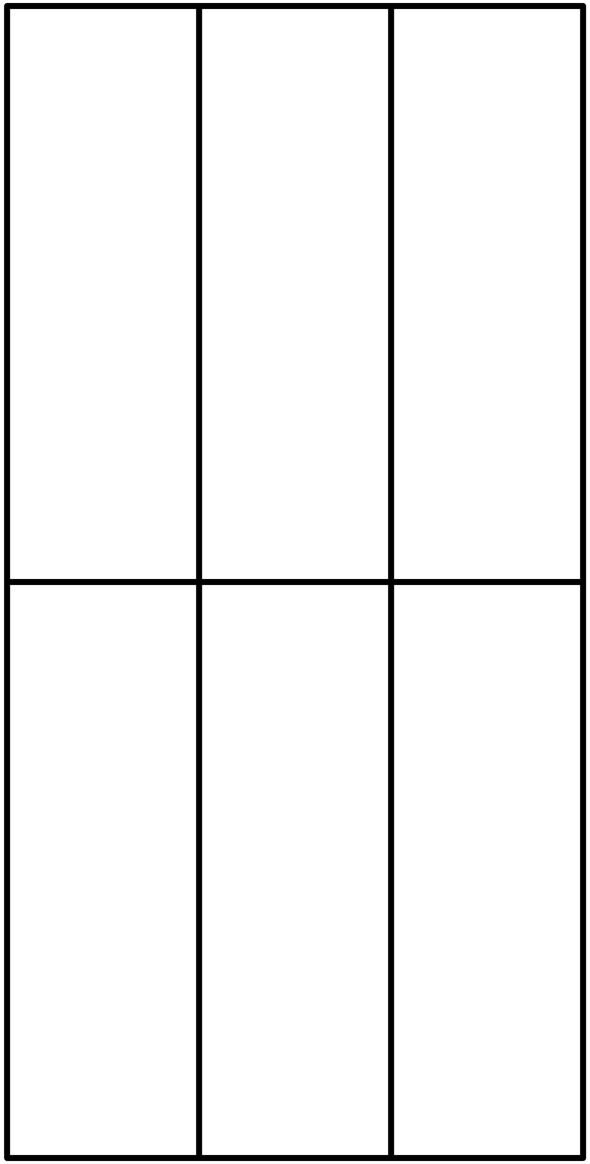
WARNING: Be sure to minimize, not maximize.

0510-5. The top of the object in the diagram is a semicircle. The total perimeter of the object is 32 meters. Find the dimensions x and y that maximize the area enclosed.
NEW



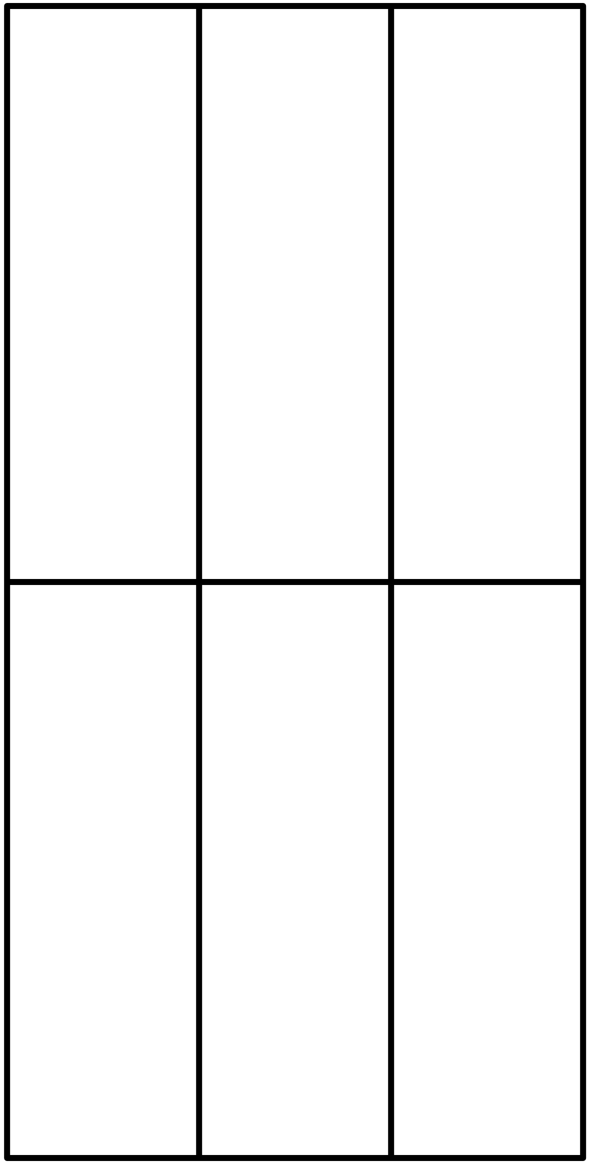
0510-6. We plan to build a fenced area with six pens for livestock, as shown. We wish for the total length of the fencing (including both the perimeter and the interior fencing) to be 200 yards. Find the maximum possible enclosed area.

NEW



0510-7. NEW

We plan to build a fenced area with six pens for livestock, as shown. We wish for the total area enclosed (including all six pens) to be 100 feet^2. Each foot of fencing costs one dollar. Find the minimum possible cost of the fencing.



0510-8. We have 650 meters^2 of material from which to build an open-topped cylindrical container. Find the radius and height that maximizes the volume enclosed.
NEW

0510-9. Let L be the line $2x + y = -4$.
NEW

a. Find the point P on L closest to $(9, 8)$, by minimizing $(x - 9)^2 + (y - 8)^2$,
subject to $2x + y = -4$.

b. Find an equation of the line N that is perpendicular to L and passes through the point $(9, 8)$.

c. Find the point Q that is on the intersection of L and N.

d. Sketch the graph of L, and then add in $(9, 8)$, P, Q and N.

0510-10. NEW Let E be the ellipse

$$4x^2 + y^2 = 72.$$

Find the dimensions of a rectangle inscribed in E (with sides parallel to the axes of E) whose area is maximal.

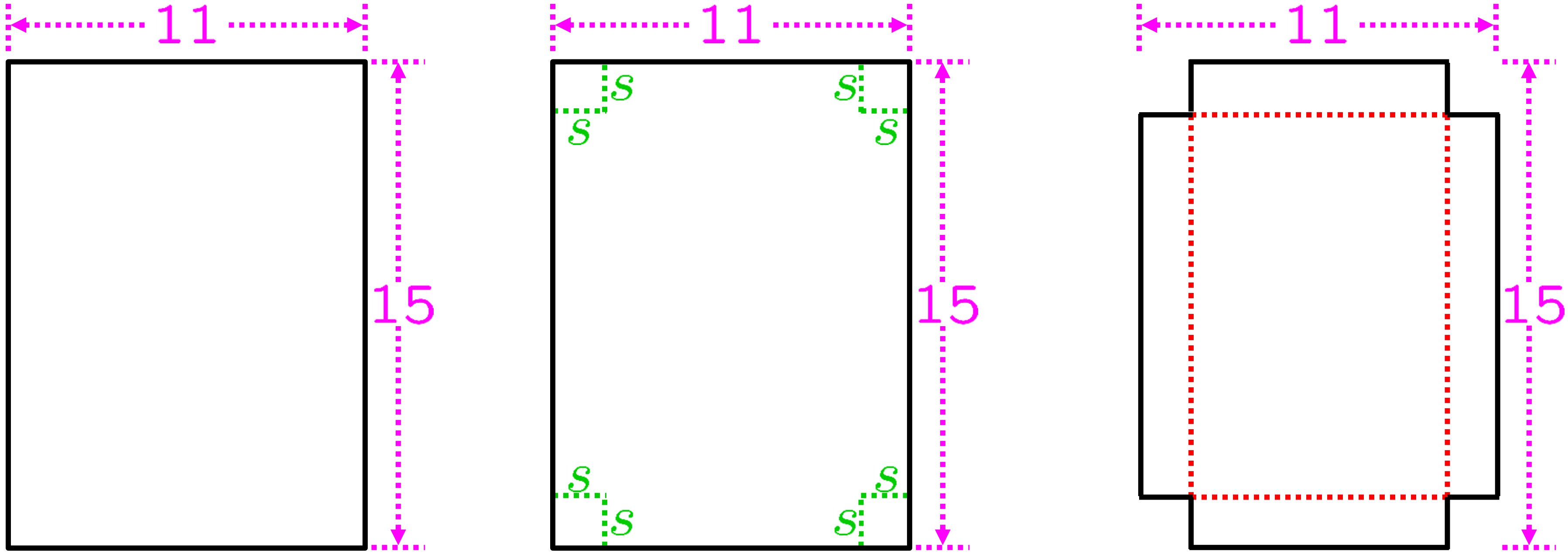
0510-11. NEW Let S be a sphere of radius 15. Find the radius and height of a right circular cone inscribed in S whose volume is maximal.

0510-12. NEW We build two holding pens with 90 feet of fencing. One is in the shape of a square. The shape of the other is an isosceles right triangle. Find the maximum total area that can be enclosed in the two pens.

OPTIONAL PROBLEM,
NOT ASSIGNED

0510-13. On an (11 cm) $\times$ (15 cm) sheet of paper, we mark a green-dashed square in each corner, each one of side length s. After cutting out these squares, we fold along the red-dashed lines as shown, creating an open-topped box. What is the maximum volume of such a box?

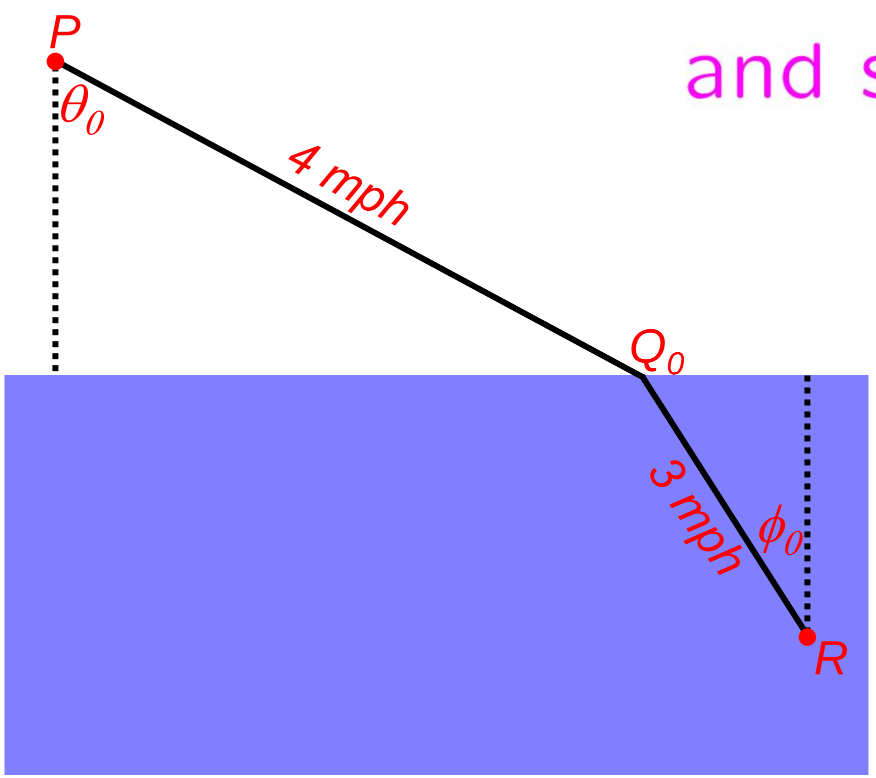
NEW



OPTIONAL PROBLEM,
NOT ASSIGNED

0510-14. I must travel from point P on land to point R in the water. My land speed is 4 mph. My water speed is 3 mph. I choose the point Q_0 so as to minimize travel time. Define θ_0 and ϕ_0 as shown in the picture

NEW



and show that $\dfrac{\sin\theta_0}{\sin\phi_0} = \dfrac{4}{3}$.

Note: This is called Snell's Law.

Note: You may assume that $\exists$a unique Q_0 that minimizes the travel time from P to R.