CALCULUS
Linear approximation
NEW

0540-1. Find the linearization of

NEW

$$f(x) = -x^3 + x^2$$

at $x = 5$.

That is, find m and a s.t. the linear function

$$L(x) = a + m(x - 5)$$

has the same 1-jet at $x = 5$ as does $f(x)$.

That is, find m and a s.t. the linear function

$$L(x) = a + m(x - 5)$$

satisfies: $L(5) = f(5)$ and $L'(5) = f'(5)$.

0540-2. Find the linearization of

NEW

$$f(x) = \csc x$$

at $x = \pi/3$.

That is, find m and a s.t. the linear function

$$L(x) = a + m(x - (\pi/3))$$

has the same 1-jet at $x = \pi/3$ as does $f(x)$.

That is, find m and a s.t. the linear function

$$L(x) = a + m(x - (\pi/3))$$

satisfies: $L(\pi/3) = f(\pi/3)$

and $L'(\pi/3) = f'(\pi/3)$.

0540-3. Let $y = \dfrac{(3x^3 + x)(\tan^2 x)}{e^{4x}}$.
NEW

Compute $\triangle y$ and dy.

0540-4. Let $x = \dfrac{2t - 3}{\sec(2t + \sqrt{2})}$.
NEW

Compute $\triangle x$ and dx.

0540-5. Let $z = \dfrac{e^{-2q^2+4}}{\sin(q + 5)}$.
NEW

a. Compute $[\triangle z]_{q:\to 0, \triangle q:\to 0.0002}$.

b. Compute $[dz]_{q:\to 0, dq:\to 0.0002}$.

0540-6. a. Compute $(2.9998)^7$.
NEW

b. Approx. $(2.9998)^7$ by differentials.

c. Let $L(x)$ be the linearization of $f(x) = x^7$ at $x = 3$.

Compute $L(2.9998)$.

0540-7. Let θ be the number of radians in 60.1°.
NEW

Approximate $\sec\theta$ by differentials.

0540-8. Approx. $\tan(0.0002)$ by differentials.
NEW

0540-9. We need to paint a ball whose radius is 5 meters.
NEW

The coat of paint is to be 0.003 meters thick, so, after painting, the radius will be 5.003 meters.

a. Let $V = \frac{4}{3}\pi r^3$. Compute ΔV and dV.

b. Using ΔV, compute the exact volume of paint that will be needed.

c. Using dV, estimate the volume of paint that will be needed.

d. Compute 0.003 times the surface area of a ball of radius 5 meters.

0540-10. A hemisphere of radius r has volume $\frac{2}{3}\pi r^3$.
NEW

Imagine that we work at an architecture firm that specializes in building hemispherical domes. A client asks us to build a dome in the shape of a hemisphere whose radius is $25 \pm \frac{1}{12}$ feet.

Up to some error, its volume will be $\frac{2}{3}\pi(25^3)$ cubic feet.

Using differentials, estimate that error.