CALCULUS

Problems involving horizontal asymptotes

OLD

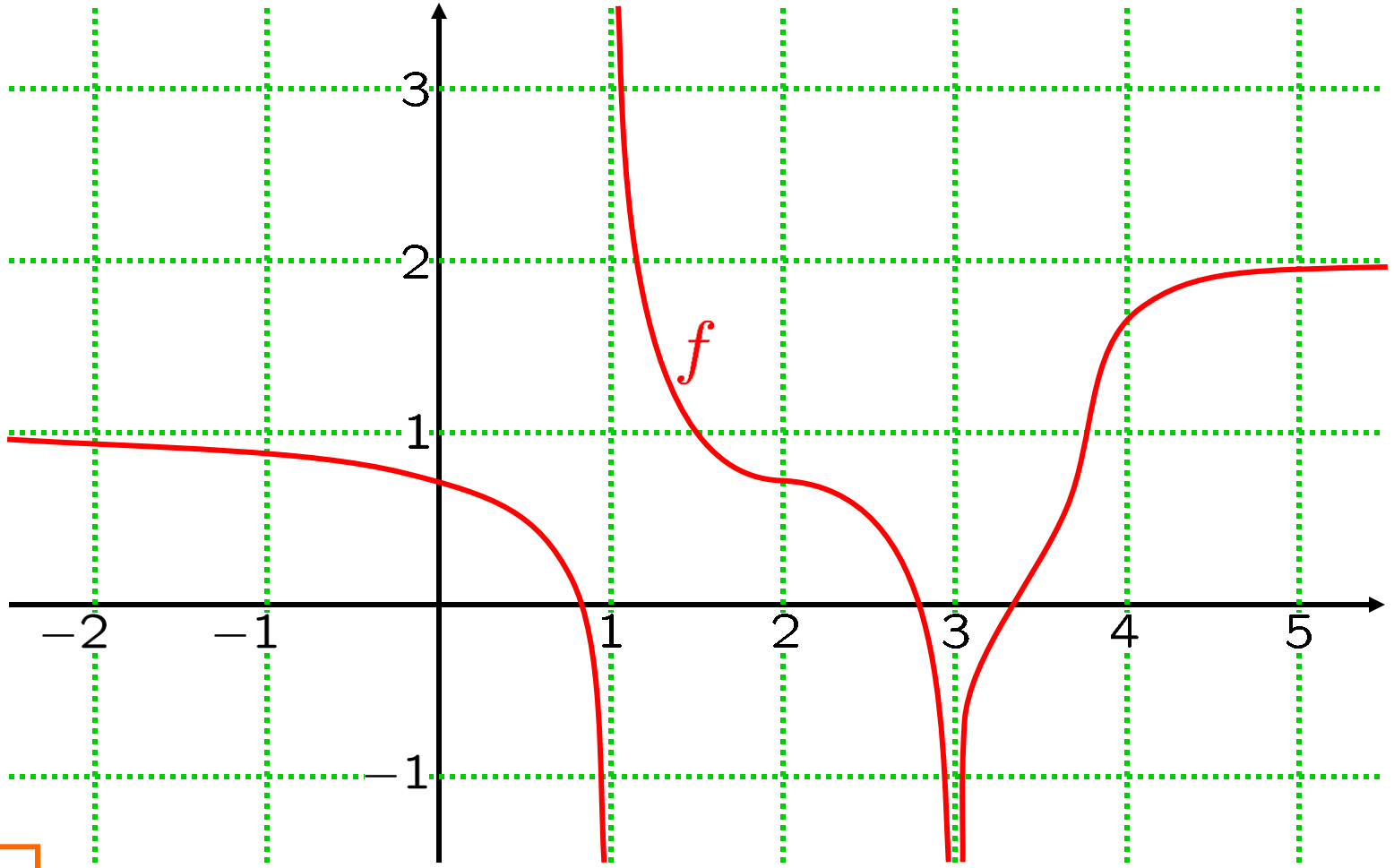
WARNING: In this homework, do NOT use l'Hôpital's rule. It has not been covered yet.

0250-1. Using the graph below of f,

OLD

find these limits: a. $\lim_{x \to -\infty} f(x)$ b. $\lim_{x \to 1^-} f(x)$

c. $\lim_{x \to 1^+} f(x)$ d. $\lim_{x \to 3} f(x)$ e. $\lim_{x \to \infty} f(x)$



0250-2.

OLD

Sketch a graph of a function $h : \mathbb{R} \to \mathbb{R}$ s.t.

h is even, *i.e.*, $h(-x) = h(x)$,

$\lim_{x \to -\infty} h(x) = 2$, $\lim_{x \to -3} h(x) = -\infty$,

$\lim_{x \to -2^-} h(x) = \infty$, $\lim_{x \to -2^+} h(x) = -\infty$,

$\lim_{x \to -1} h(x) = \infty$ and $h(0) = 1$.

0250-3.

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Sketch a graph of a function $f : \mathbb{R} \to \mathbb{R}$ s.t.

f is odd, *i.e.*, $f(-x) = -(f(x))$,

$\lim_{x \to -\infty} f(x) = 2$, $\lim_{x \to -3} f(x) = -\infty$,

$\lim_{x \to -2^-} f(x) = \infty$, $\lim_{x \to -2^+} f(x) = -\infty$

and $\lim_{x \to -1} f(x) = \infty$.

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0250-4. Compute

a. $\lim_{x\to\infty} \dfrac{x^3+2x-1}{x+5}$

Do NOT use l'Hôpital's rule.

b. $\lim_{x\to\infty} \dfrac{x^3+1000000}{x^4+1}$

Do NOT use l'Hôpital's rule.

c. $\lim_{x\to\infty} \dfrac{x^4+1}{x^3+1000000}$

Do NOT use l'Hôpital's rule.

d. $\lim_{x\to\infty} \dfrac{2x^3+1}{7x^3+1000000}$

Do NOT use l'Hôpital's rule.

0250-5. Compute

OLD

a. $\lim_{x \to -\infty} \dfrac{x^3 + 2x - 1}{x + 5}$ Do NOT use l'Hôpital's rule.

b. $\lim_{x \to -\infty} \dfrac{x^3 + 1000000}{x^4 + 1}$ Do NOT use l'Hôpital's rule.

c. $\lim_{x \to -\infty} \dfrac{x^4 + 1}{x^3 + 1000000}$ Do NOT use l'Hôpital's rule.

d. $\lim_{x \to -\infty} \dfrac{2x^3 + 1}{7x^3 + 1000000}$ Do NOT use l'Hôpital's rule.

NOTE: These are limits at $-\infty$; in the last problem, they were at ∞.

0250-6. Compute

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a. $\lim_{x \to -\infty} \sqrt[3]{\dfrac{2x^3+1}{16x^3+1000000}}$ Do NOT use l'Hôpital's rule.

b. $\lim_{x \to \infty} \sqrt[4]{\dfrac{48x^4+x^2+1}{2+x+3x^4}}$ Do NOT use l'Hôpital's rule.

c. $\lim_{x \to -\infty} \dfrac{\sqrt{9x^4+x^2+1}}{x^2-x+1000000}$ Do NOT use l'Hôpital's rule.

d. $\lim_{x \to \infty} \left(\sqrt{x^2+5x} - \sqrt{x^2+9x}\right)$

Do NOT use l'Hôpital's rule.

0250-7. Find the (maximal) intervals
OLD

where $x(x-1)^2(x-2)^4$

is positive and negative,

then compute $\lim_{x \to -\infty} x(x-1)^2(x-2)^4$,

then compute $\lim_{x \to \infty} x(x-1)^2(x-2)^4$.

OLD

0250-8. Suppose, $\forall x > 100$, that

$$\frac{x^2+2x+8}{2x^2+3x+7} < f(x) < \frac{5x^2+16x+48}{10x^2+10x+30}.$$

Compute $\lim_{x\to\infty} f(x)$.