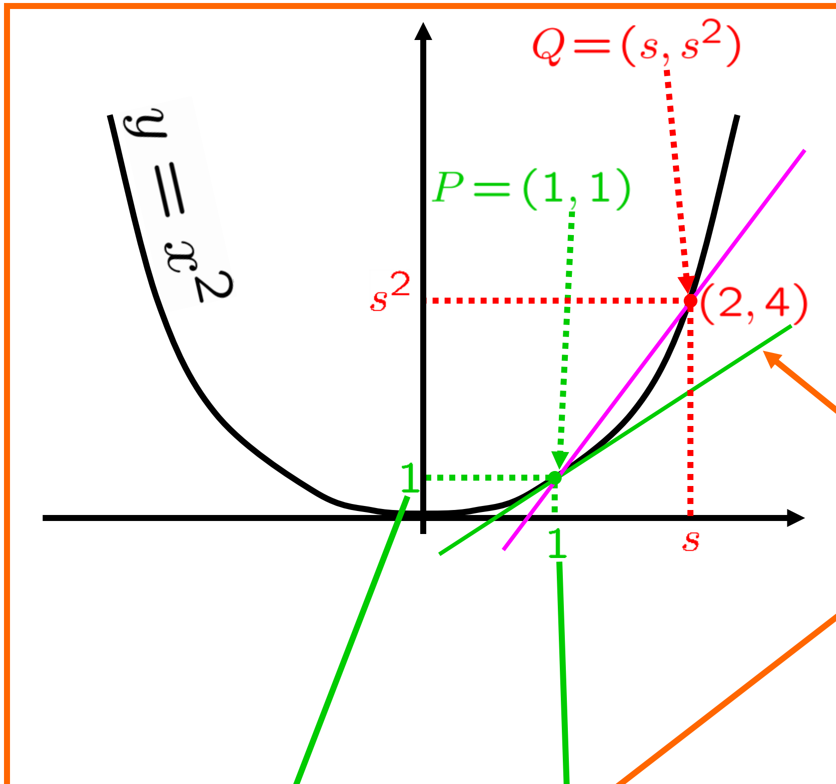
CALCULUS
Rates of change and slopes of lines

EXAMPLE: Find an equation of the tangent line to the parabola at the point $P=(1,1)$.



m := slope of the green line

Eq'n of the green line:

$$y - 1 = m(x-1)$$

Goal: Find m

$s := 2$

$Q = (s, s^2)$ is on the graph.

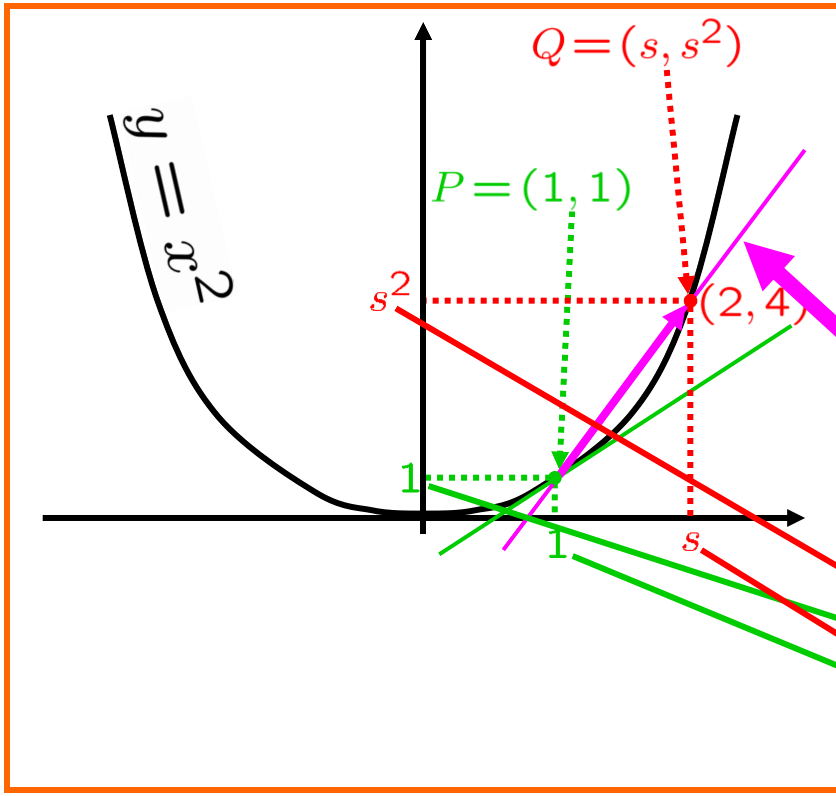
$m \approx m_{PQ}$:= slope of the purple line

Equation of the line through (x_0, y_0) of slope m:

$$y - y_0 = m(x - x_0)$$

1   1

EXAMPLE: Find an equation of the tangent line to the parabola at the point $P = (1,1)$.



$m :=$ slope of the green line

Eq'n of the green line:

$$y - 1 = m(x - 1)$$

Goal: Find m

$s := 2 \leftarrow 1.5$

$Q = (s, s^2)$ is on the graph.

$m \approx$ poor approximation, but to be improved . . .

$m_{PQ} :=$ slope of the purple line

$$= \frac{\text{rise}}{\text{run}} = \frac{s^2 - 1}{s - 1} = \frac{4 - 1}{2 - 1} = 3$$

Equation of the line through (x_0, y_0) of slope m:

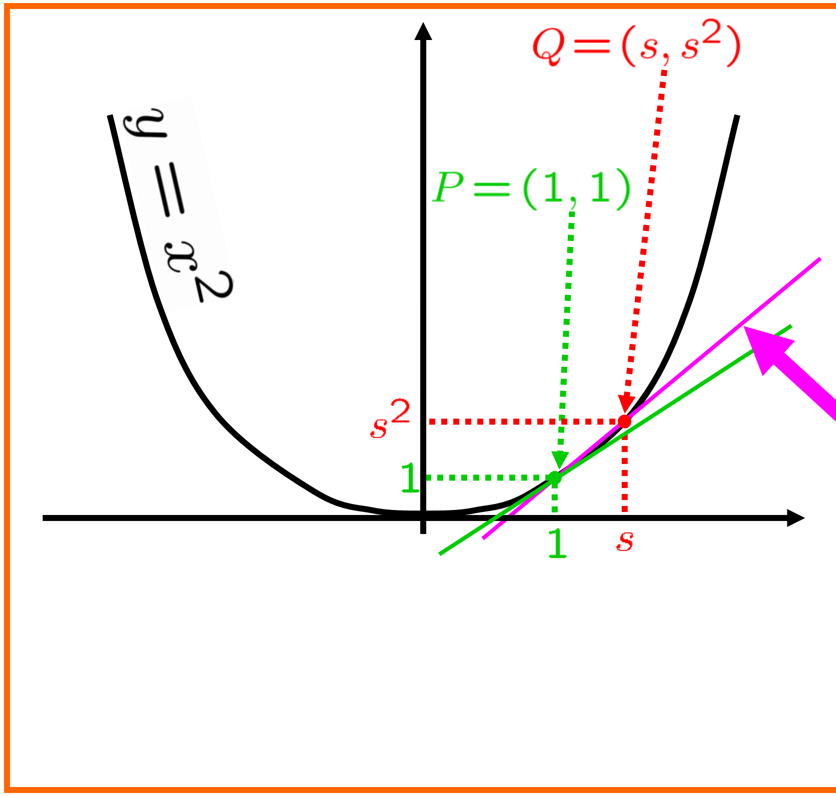
$$y - y_0 = m(x - x_0)$$

Idea: Redo this, with

$s := 1.5, 1.1, 1.01, 1.001, \ldots$

and find the limit of the m_{PQ}s.

EXAMPLE: Find an equation of the tangent line to the parabola at the point $P = (1, 1)$.



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m
$\wr\wr$ better!
m_{PQ} := slope of the purple line

$$= \frac{\text{rise}}{\text{run}} = \frac{s^2 - 1}{s - 1} = \frac{2.25 - 1}{1.5 - 1}$$

$= 2.5$

exercise

SKILL
comp slope

Equation of the line through (x_0, y_0) of slope m:

$$y - y_0 = m(x - x_0)$$

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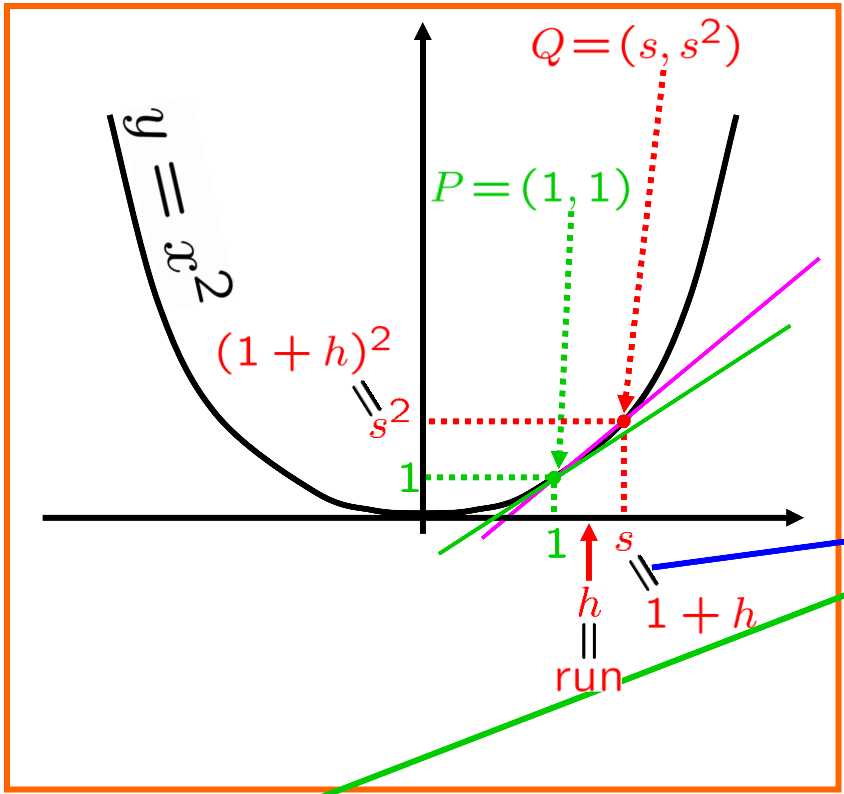

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$$y - 1 = m(x-1)$$

Goal: Find m

Traditional to use $1+h$ instead of s

$s := 1.5$

$Q = (s, s^2)$ is on the graph.

m $\approx$ m_{PQ} := slope of the purple line

$$= \frac{\text{rise}}{\text{run}} = \frac{s^2 - 1}{s - 1} = \frac{2.25 - 1}{1.5 - 1}$$

$$= 2.5$$

$$m = \lim_{s \to 1} \frac{s^2 - 1}{s - 1}$$

$$\frac{(1+h)^2 - 1}{(1+h) - 1}$$

Idea: Redo this, with $h = s - 1$

$s := 1.5, 1.1, 1.01, 1.001, \ldots$

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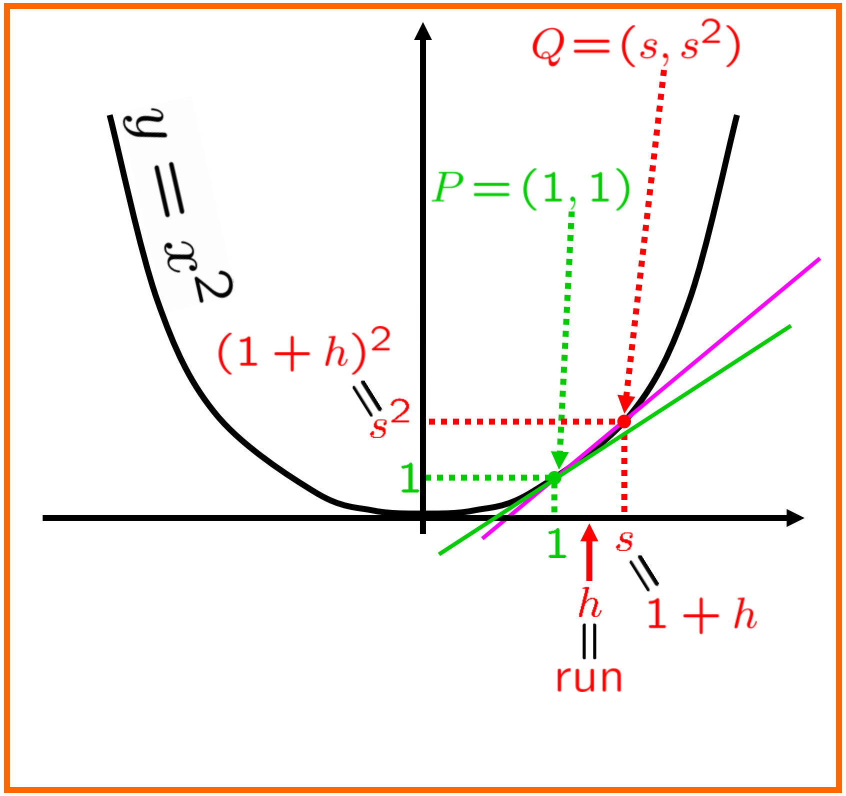
EXAMPLE: Find an equation of the tangent line to the parabola at the point $P=(1,1)$.



$m :=$ slope of the green line

Eq'n of the green line:

$$y-1=m(x-1)$$

Goal: Find m

Traditional to use $1+h$ instead of s

$s := 1.5$

$h \to 0$ instead of $s \to 1$

$Q=(s,s^2)$ is on the graph.

$m \approx m_{PQ} :=$ slope of the purple line

$$= \frac{\text{rise}}{\text{run}} = \frac{s^2-1}{s-1} = \frac{2.25-1}{1.5-1} = 2.5$$

$$m = \lim_{s\to 1} \frac{s^2-1}{s-1}$$

$$= \lim_{h\to 0} \frac{(1+h)^2-1}{(\cancel{1}+h)-\cancel{1}}$$

Idea: Redo this, with $h=s-1$

$h := 0.5, 0.1, 0.01, 0.001, \ldots$

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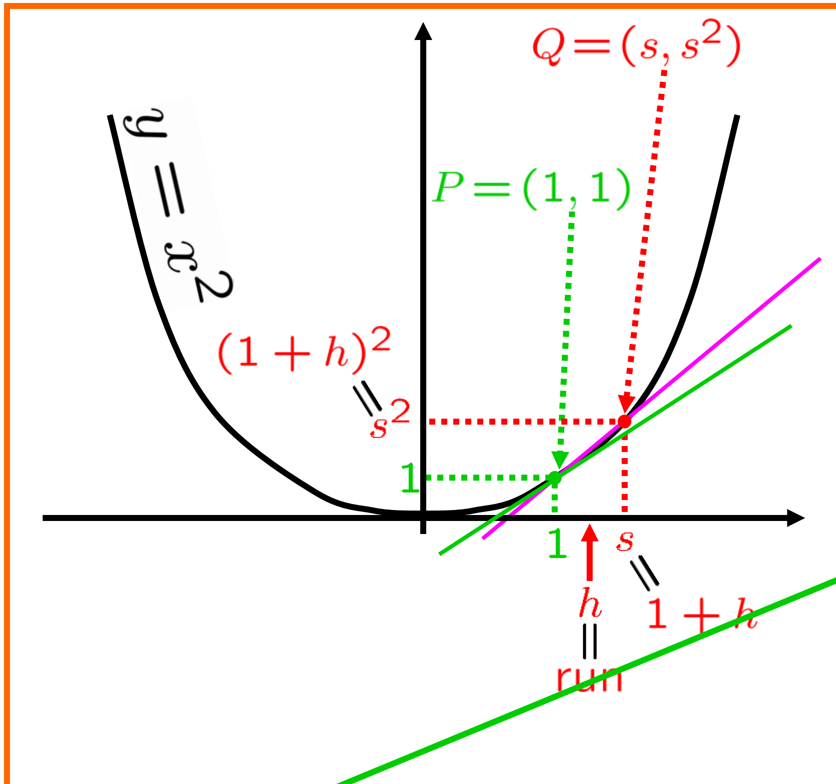

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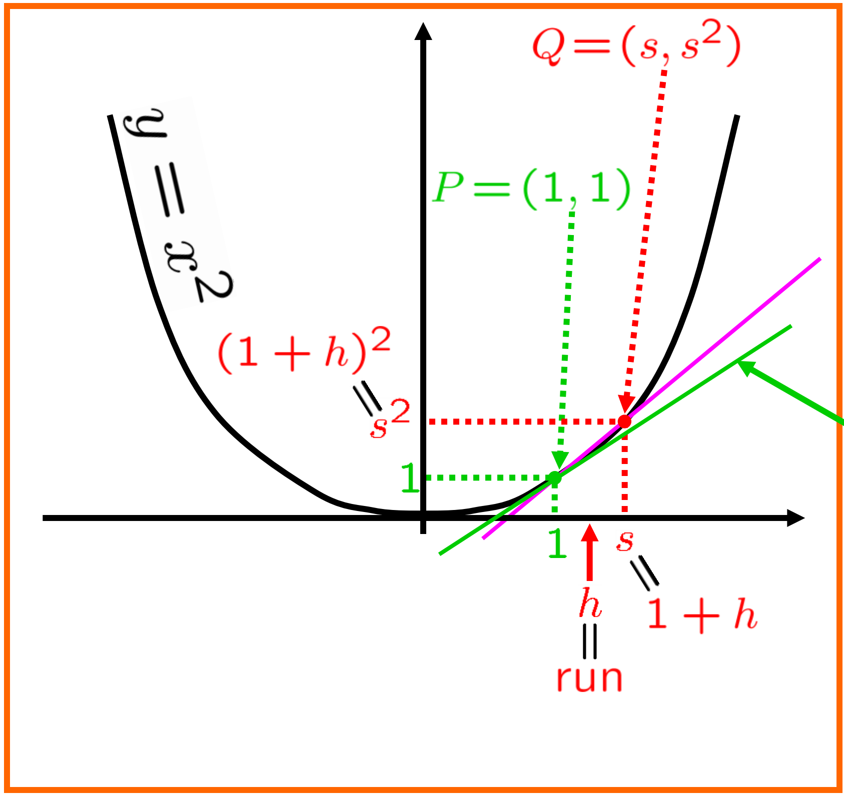
$$m = \lim_{s\to 1} \frac{s^2-1}{s-1}$$

$$= \lim_{h\to 0} \frac{(1+h)^2-1}{(1+h)-1} = \lim_{h\to 0} \frac{(1+h)^2-1}{h} = \cdots = 2$$

exercise

DONE!

EXAMPLE: Find an equation of the tangent line to the parabola at the point $P = (1, 1)$.



m := slope of the green line

Eq'n of the green line:

$$y - 1 = 2(x - 1)$$

Traditional to use $1+h$ instead of s

$s := 1.5$

$h \to 0$ instead of $s \to 1$

$Q = (s, s^2)$ is on the graph.

$m \approx m_{PQ}$:= slope of the purple line

$$= \frac{\text{rise}}{\text{run}} = \frac{s^2 - 1}{s - 1} = \frac{2.25 - 1}{1.5 - 1}$$

$$= 2.5$$

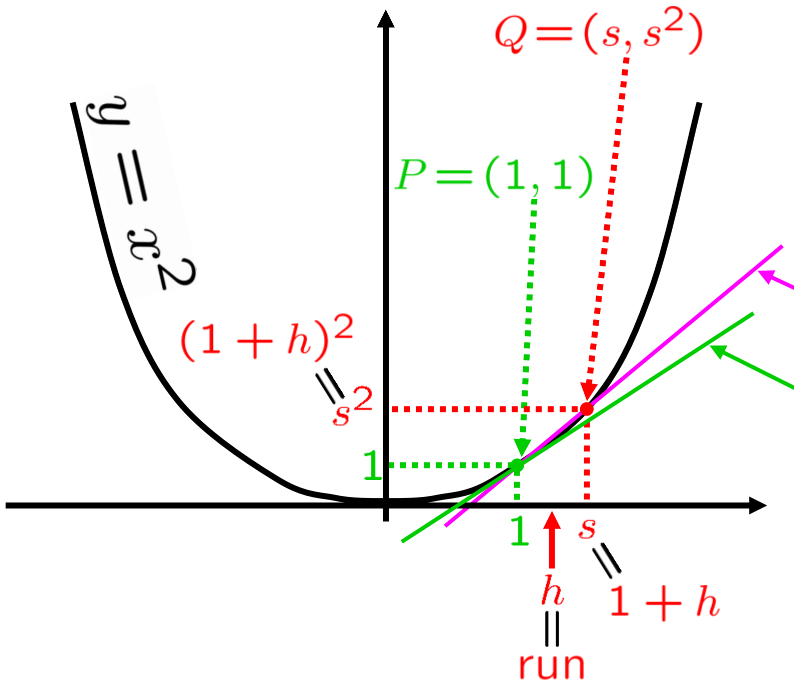
$$m = \lim_{s \to 1} \frac{s^2 - 1}{s - 1}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 - 1}{(1 + h) - 1} = \lim_{h \to 0} \frac{(1+h)^2 - 1}{h} = \cdots = 2$$

exercise

DONE!

EXAMPLE: Find an equation of the tangent line to the parabola at the point $P = (1, 1)$.



$m :=$ slope of the green line

Eq'n of the green line:

$$y - 1 = 2(x - 1)$$

secant line

tangent line

SOME TERMINOLOGY

difference quotient

$$\frac{s^2 - 1}{s - 1}$$

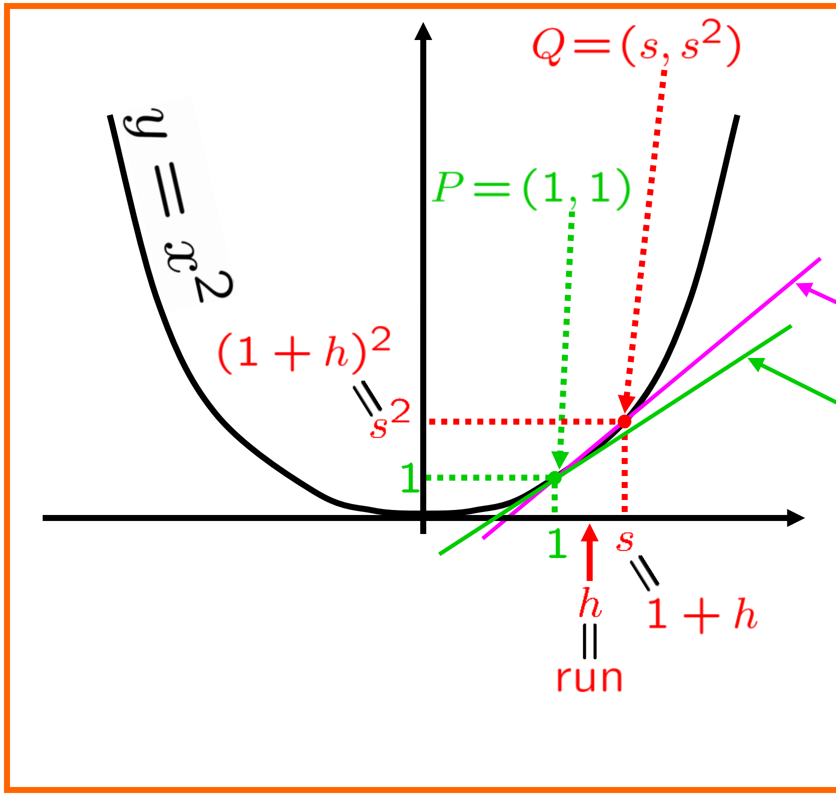
$$m = \lim_{s \to 1} \frac{s^2 - 1}{s - 1}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 - 1}{(\not{1} + h) - \not{1}} = \lim_{h \to 0} \frac{(1+h)^2 - 1}{h} = \cdots = 2$$

exercise

Given two points on a line, the slope is computed as a difference quotient.

The slope of a secant line is computed as a difference quotient.



The slope of a tangent line is computed as a limit of difference quotients.

secant line

tangent line

difference quotient

$$\frac{s^2-1}{s-1}$$

$$m = \lim_{s\to 1}\frac{s^2-1}{s-1}$$

$$= \lim_{h\to 0}\frac{(1+h)^2-1}{(1+h)-1} = \lim_{h\to 0}\frac{(1+h)^2-1}{h} \underset{\text{exercise}}{=} \cdots = 2$$

Given two points on a line, the slope is computed as a difference quotient.

The slope of a secant line is computed as a difference quotient.

$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y\text{-coordinate}}{\text{change in } x\text{-coordinate}}$$

The slope of a tangent line is computed as a limit of difference quotients.

Given two positions of a moving particle, *e.g.*, a falling ball, the average velocity is computed as a difference quotient.

$$\parallel$$

$$\frac{\text{change in position}}{\text{change in time}}$$

The instantaneous velocity is computed as a limit of difference quotients.

Given *any* two quantities related by a formula, the average rate of change of one w.r.t. the other is computed as a difference quotient.

$$= \frac{\text{change one quantity}}{\text{change the other}}$$

The instantaneous rate of change of one w.r.t. the other is computed as a limit of difference quotients.

THIS IS THE THEME OF DIFFERENTIAL CALCULUS!!

Farenheit is related to Celcius by

$$F = (9/5)C + 32$$

Rate of change is always: 9/5. (See §1.1, Example 1.1, p. 3, and §1.1, Exercise 12, p. 6.)

Tax is related to (adjusted gross) income by a formula.

Rate of change not constant, (See §1.1, Exercise 16, p. 6.) and is called the "marginal tax rate".

This "other" quantity is often time, but not always.

For example . . .

Given *any* two quantities related by a formula, the average rate of change of one w.r.t. the other is computed as a difference quotient.

$$\frac{\text{change one quantity}}{\text{change the other}}$$

The instantaneous rate of change of one w.r.t. the other is computed as a limit of difference quotients.

THIS IS THE THEME OF DIFFERENTIAL CALCULUS!!

If

you graph some function,

then

average rates of change ⟷ slopes of secant lines

and

instantaneous rates of change ⟷ slopes of tangent lines

Let's see this through an example . . .

or function

Given *any* two quantities related by a formula,

the average rate of change of one w.r.t. the other

$=$ $\dfrac{\text{change one quantity}}{\text{change the other}}$

is computed as a difference quotient.

The instantaneous rate of change of one w.r.t. the other

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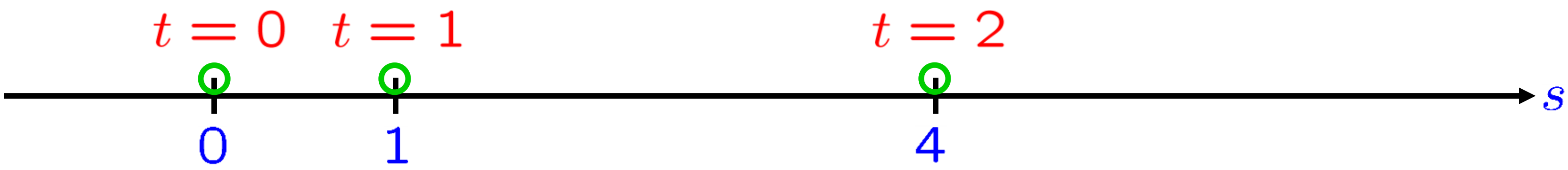
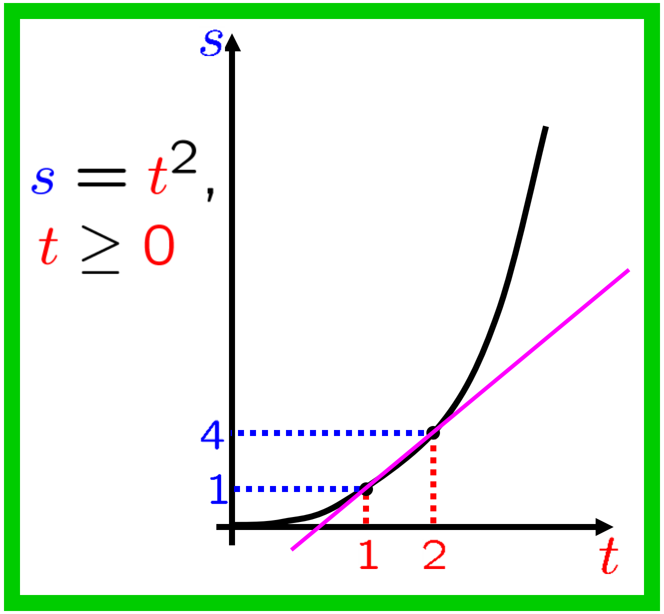
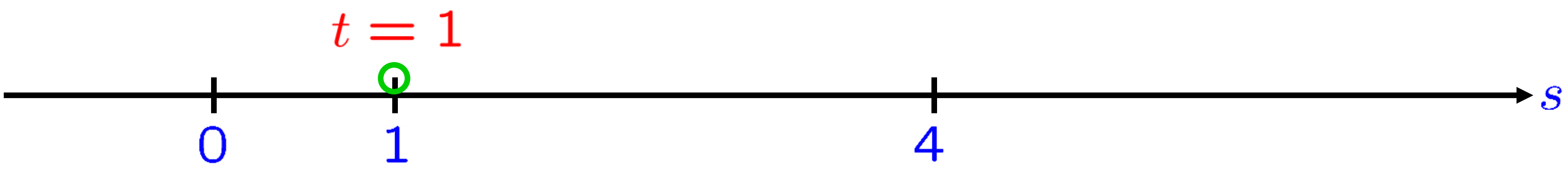


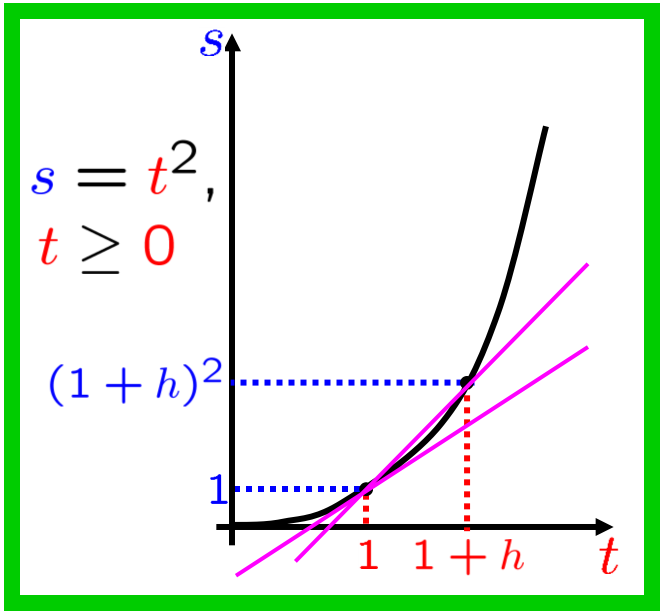


Average velocity between 1 and 2 seconds

$$= \frac{4 - 1}{2 - 1}$$

= slope of the secant line





Instantaneous velocity at the one second mark

$$= \lim_{h \to 0} \frac{(1+h)^2 - 1^2}{(1+h) - 1}$$

= slope of the tangent line

If

you graph the function,

then

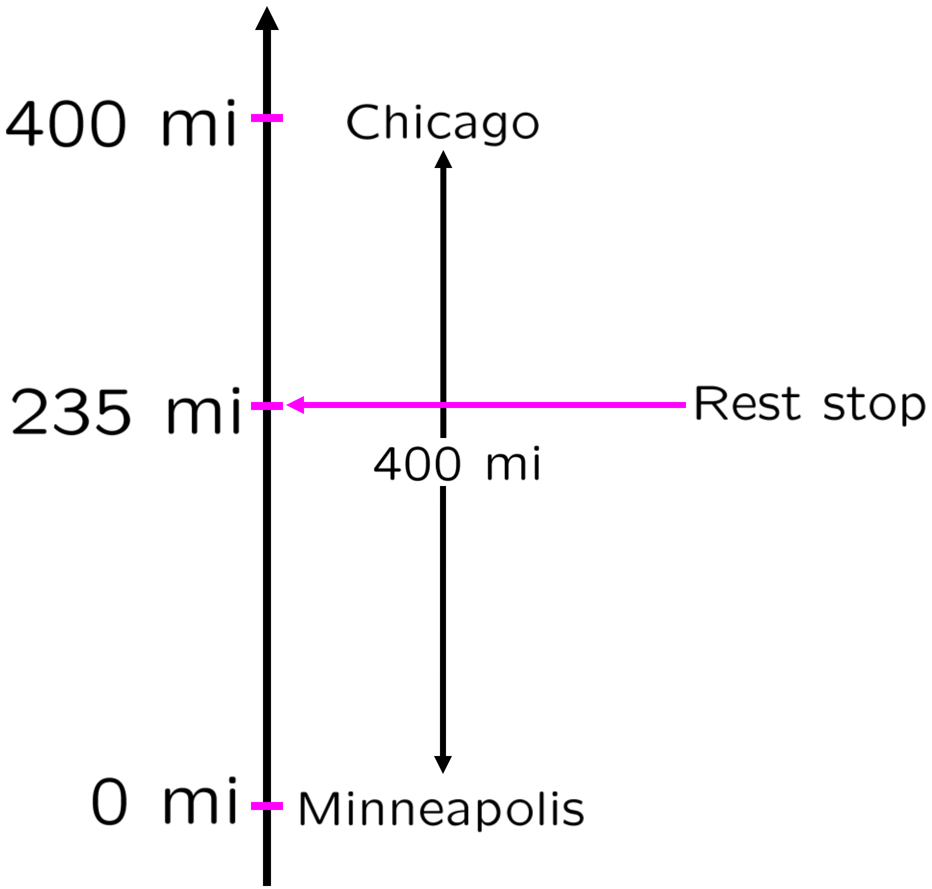
average rates of change ⟷ slopes of secant lines

and

instantaneous rates of change ⟷ slopes of tangent lines

Let's see this through

another example . . .

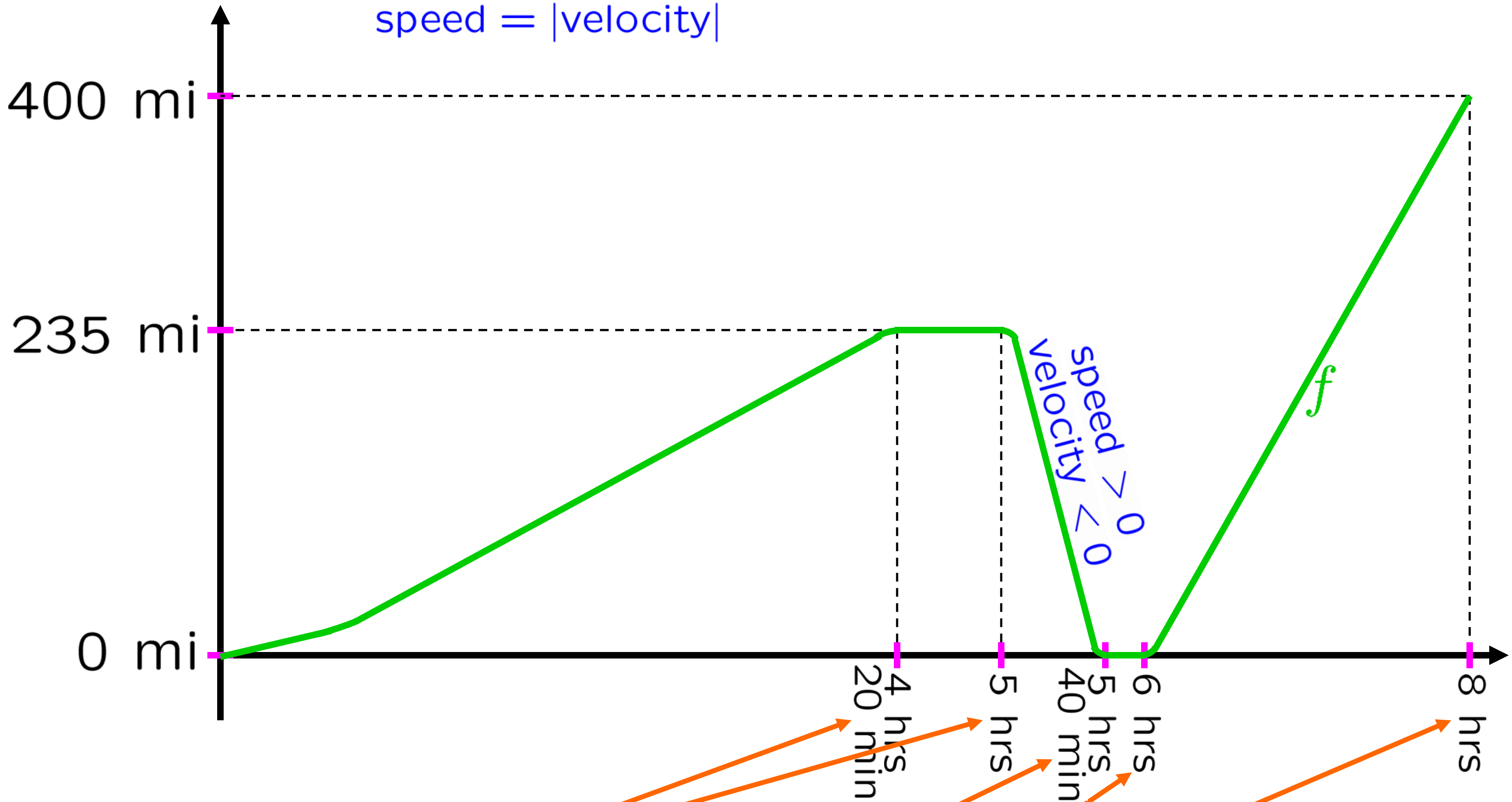


0 hrs, leave Minneapolis
4 hrs 20 mins, start break
5 hrs, notice problem,
head back
5 hrs 40 mins, arrive Minneapolis

6 hrs, leave Minneapolis
8 hrs, arrive Chicago

velocity = rate of change in position w.r.t. time

speed = |velocity|



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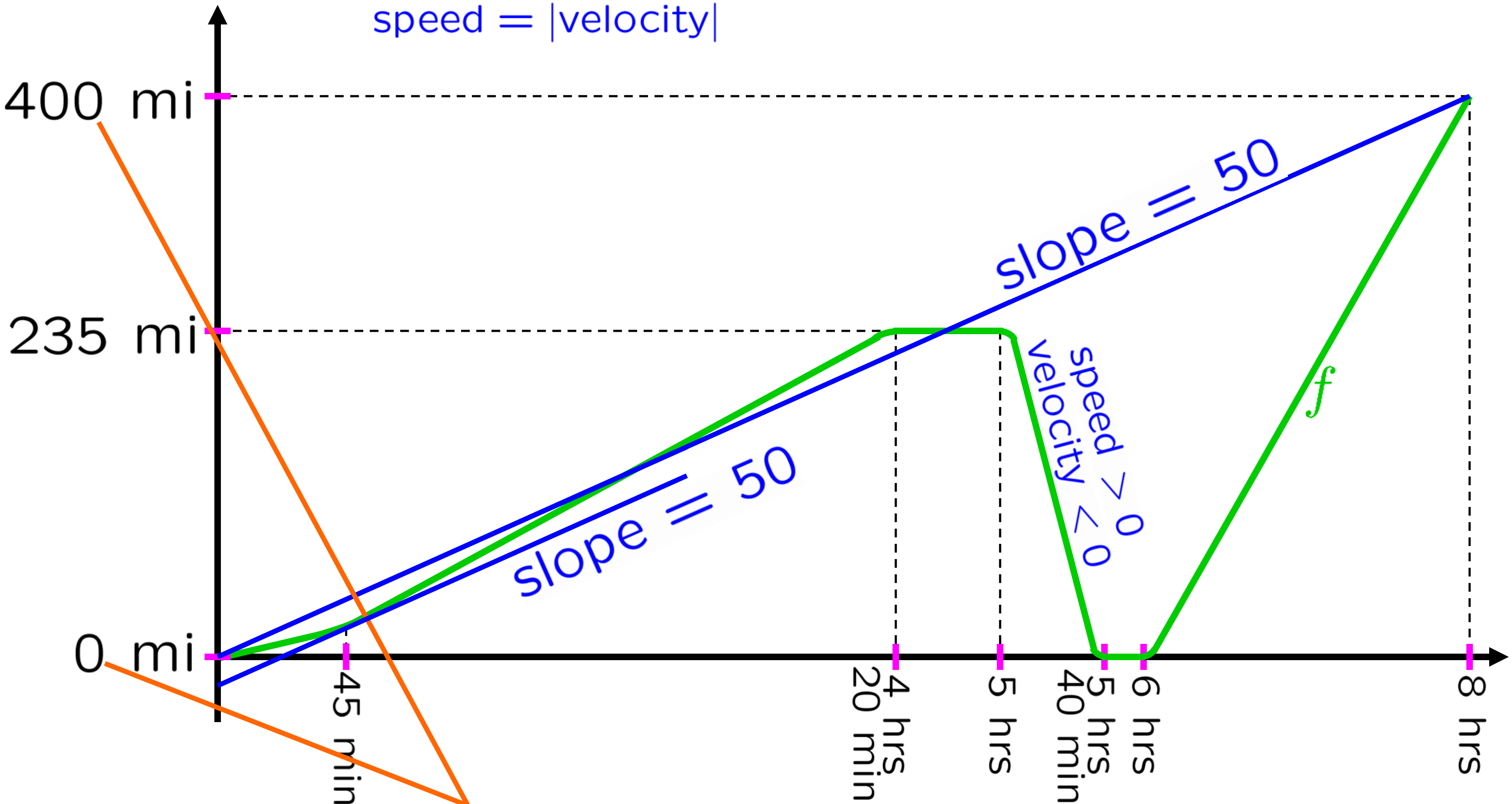

Average velocity over the eight hrs:

$$\frac{[f(8)] - [f(0)]}{8 - 0} = \frac{400 - 0}{8} = 50$$

Average velocity is 50 mph from 0 hrs to 8 hrs.

instantaneous velocity is 50 mph at some time (45 min)

EXAMPLE 1:

SKILL
comp avg vel

If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$.

(a) Find the average velocity over the given time intervals:

(i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$

(iv) $[1, 1.01]$ (v) $[1, 1.001]$

(b) Estimate the instantaneous velocity when $t = 1$.

EXAMPLE 1: $y = 10t - 1.86t^2$

SKILL
comp avg vel

(a) Find the average velocity over the given time intervals:

(i) $[1, 2]$ (ii) $[1, y = 10t - 1.86t^2 1, 1.1]$

(iv) $[1, 1.01]$ (v) $[1, 1.001]$

(a) Find the average velocity over the given

(b) time intervals:nstantaneous velocity when

(i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$

$\frac{[y]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}}$ $[1$ average velocity over $[1, 1+h]$

timate the instantaneous velocity when $t = 1$.

EXAMPLE 1: $y = 10t - 1.86t^2$

SKILL
comp avg vel

(a) Find the average velocity over the given time intervals:

(i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$

(iv) $[1, 1.01]$ (v) $[1, 1.001]$

(b) Estimate the instantaneous velocity when $t = 1$.

$$\frac{[y]_{t:\rightarrow 1}^{t:\rightarrow 1+h}}{[t]_{t:\rightarrow 1}^{t:\rightarrow 1+h}} = \text{average velocity over } [1, 1+h]$$

(a)(i) $h :\rightarrow 1$ (ii) $h :\rightarrow 0.5$ (iii) $h :\rightarrow 0.1$

(iv) $h :\rightarrow 0.01$ (v) $h :\rightarrow 0.001$

$$= \frac{[10(1+h) - 1.86(1+h)^2] - [10(1) - 1.86(1)^2]}{[1+h] - [1]}$$

$$= \frac{[10((1+h) - (1))] - [1.86((1+h)^2 - (1^2))]}{[1+h] - [1]}$$

EXAMPLE 1: $y = 10t - 1.86t^2$

SKILL
comp avg vel

(a) Find the average velocity over the given time intervals:

(i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$

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(b) Estimate the instantaneous velocity when $t = 1$.

$$\frac{[y]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}} = \text{average velocity over } [1, 1+h]$$

$$= \frac{[10((1+h)-(1))] - [1.86(\underbrace{(1+h)^2}_{1^2+2(1)(h)+h^2}-(1^2))]}{[1+h]-[1]}$$

$$\frac{[10((1+h)-(1))] - [1.86((1+h)^2-(1^2))]}{[1+h]-[1]}$$

EXAMPLE 1: $y = 10t - 1.86t^2$ SKILL comp avg vel

(a) Find the average velocity over the given time intervals:

(i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$

(iv) $[1, 1.01]$ (v) $[1, 1.001]$

(b) Estimate the instantaneous velocity when $t = 1$.

$$\frac{[y]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}} = \text{average velocity over } [1, 1+h]$$

$$= \frac{[10((\cancel{1}+h) - (\cancel{1}))] - [1.86(\overbrace{(1+h)^2}^{\cancel{1^2} + 2(1)(h) + h^2} - (\cancel{1^2}))]}{[\cancel{1}+h] - [\cancel{1}]}$$

$$= \frac{[10h - 1.86(2h + h^2)]}{h}$$

EXAMPLE 1: $y = 10t - 1.86t^2$

SKILL
comp avg vel

(a) Find the average velocity over the given time intervals:

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$$\frac{[y]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}} = \text{average velocity over } [1, 1+h]$$

$$= \frac{[10h - 1.86(2h + h^2)]}{h}$$

$$\overset{h \neq 0}{=} 10 - 1.86(2 + h)$$

$$= 10 - 3.72 - 1.86h = 6.28 - 1.86h$$

EXAMPLE 1: $y = 10t - 1.86t^2$

SKILL
comp avg vel

(a) Find the average velocity over the given time intervals:

(i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$

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(b) Estimate the instantaneous velocity when $t = 1$.

$$\frac{[y]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}} = \text{average velocity over } [1, 1+h]$$

(a)(i) $h :\to 1$ (ii) $h :\to 0.5$ (iii) $h :\to 0.1$

(iv) $h :\to 0.01$ (v) $h :\to 0.001$

$\overset{h\neq 0}{=}$ $6.28 - 1.86h$

$h \neq 0$

$6.28 - 1.86h$

EXAMPLE 1: $y = 10t - 1.86t^2$

SKILL
comp avg vel

(a) Find the average velocity over the given time intervals:

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(iv) $h :\to 0.01$ (v) $h :\to 0.001$

$\overset{h \neq 0}{=}$

(a) $6.28 - 1.86h$

(i) $6.28 - (1.86)(1) = 4.42$

(ii) $6.28 - (1.86)(0.5) = 5.35$

(iii) $6.28 - (1.86)(0.1) = 6.094$

(iv) $6.28 - (1.86)(0.01) = 6.2614$

(v) $6.28 - (1.86)(0.001) = 6.27814$

(b) 6.28

EXAMPLE 2:

The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion

$$s = 2[\sin(\pi t)] + 3[\cos(\pi t)],$$

where t is measured in seconds.

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$
(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

EXAMPLE 2: $s = 2[\sin(\pi t)] + 3[\cos(\pi t$

SKILL
comp avg vel

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$

SKILL
comp avg vel

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}}$$

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

EXAMPLE 2: $s = 2[\sin(\pi t)] + 3[\cos(\pi t)]$

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

SKILL
comp avg vel

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}}$$

$t :\to 1 + h$ $t :\to 1$ $t :\to 1 + h$ $t :\to 1$

$$= \frac{2[(\sin(\pi(1+h))) - (\sin(\pi))] + 3[(\cos(\pi(1+h))) - (\cos(\pi))]}{h}$$

-1

$$= \frac{2[\sin(\pi(1+h))] + 3[1 + (\cos(\pi(1+h)))]}{h}$$

EXAMPLE 2: $s = 2[\sin(\pi t)] + 3[\cos(\pi t)]$

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

SKILL
comp avg vel

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}} = \frac{2[\underbrace{\sin(\pi(1+h))}_{\sin(\pi+\pi h)}]+3[1+(\underbrace{\cos(\pi(1+h))}_{\cos(\pi+\pi h)})]}{h}$$

$$\frac{2[\sin(\pi(1+h))]+3[1+(\cos(\pi(1+h)))]}{h}$$

EXAMPLE 2: $s = 2[\sin(\pi t)] + 3[\cos(\pi t)]$

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

SKILL
comp avg vel

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}}$$

$\forall x \in \mathbb{R}$, $\sin(\pi + x) = -\sin x$

$\forall x \in \mathbb{R}$, $\cos(\pi + x) = -\cos x$

$\sin(\pi + \pi h)$ $\cos(\pi + \pi h)$

$$= \frac{2[\sin(\pi(1+h))] + 3[1 + (\cos(\pi(1+h)))]}{h}$$

$$= \frac{-2[\sin(\pi h)] + 3[1 - (\cos(\pi h))]}{h}$$

EXAMPLE 2: $s = 2[\sin(\pi t)] + 3[\cos(\pi t)]$

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

SKILL
comp avg vel

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}}$$

(a) (i) $h :\to 1$ (ii) $h :\to 0.1$
(iii) $h :\to 0.01$ (iv) $h :\to 0.001$

Exercise: Make these substitutions.

$$= \frac{-2[\sin(\pi h)] + 3[1 - (\cos(\pi h))]}{h}$$

To do (b),
it helps to rewrite this...

$$= -2\left[\frac{\sin(\pi h)}{h}\right] + 3\left[\frac{1 - (\cos(\pi h))}{h}\right]$$

EXAMPLE 2: $s = 2[\sin(\pi t)] + 3[\cos(\pi t)]$

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

SKILL
comp avg vel

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}}$$

To do (b),
it helps to rewrite this. . .

$$= -2\left[\frac{\sin(\pi h)}{h}\right] + 3\left[\frac{1-(\cos(\pi h))}{h}\right]$$

To do (b),
it helps to rewrite this. . .

$$= -2\pi\left[\frac{\sin(\pi h)}{\pi h}\right] + 3\pi\left[\frac{1-(\cos(\pi h))}{\pi h}\right]$$

EXAMPLE 2: $s = 2[\sin(\pi t)] + 3[\cos(\pi t)]$

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(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}}$$

$$= -2\pi\left[\frac{\sin(\pi h)}{\pi h}\right] + 3\pi\left[\frac{1-(\cos(\pi h))}{\pi h}\right]$$

$h \to 0$: $\left[\frac{\sin(\pi h)}{\pi h}\right] \to 1$ and $\left[\frac{1-(\cos(\pi h))}{\pi h}\right] \to 0$

$$-2\pi\left[\frac{\sin(\pi h)}{\pi h}\right] + 3\pi\left[\frac{1-(\cos(\pi h))}{\pi h}\right]$$

$\frac{\sin x}{x}$, $x \to 0$: $\frac{\sin x}{x} \to 1$

Topic 0230

IOU §4.3, p. 68, l.-7

$\frac{1-(\cos x)}{x}$, $x \to 0$: $\frac{1-(\cos x)}{x} \to 0$

Topic 0230

IOU §4.3, p. 69, l-10

EXAMPLE 2: $s = 2[\sin(\pi t)] + 3[\cos(\pi t)]$

(a) Find the average velocity for each time period:

(i) $[1, 2]$ (ii) $[1, 1.1]$

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

SKILL
comp avg vel

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}} = -2\pi\left[\frac{\sin(\pi h)}{\pi h}\right] + 3\pi\left[\frac{1-(\cos(\pi h))}{\pi h}\right]$$

$h \to 0$: $\frac{\sin(\pi h)}{\pi h} \to 1$, $\frac{1-(\cos(\pi h))}{\pi h} \to 0$, $3\pi \cdot 0 \to 0$

(b) -2π ~~cm/s~~

Note: When dealing with expressions of t,
one often replaces every "h" with "$\triangle t$".
When dealing with expressions of x,
one often replaces every "h" with "$\triangle x$".

More on this later (in: approx. by differentials).

SKILL	SKILL	SKILL
Slope secant & tangent	Diff quot & derivative	Slope pos/neg
Whitman problems	Whitman problems	Whitman problems
§2.1, p. 23-24, #1-3,5,7	§2.1, p. 23-24, #4,6	§2.1, p. 24, #8

$$\frac{[s]_{t:\to 1}^{t:\to 1+h}}{[t]_{t:\to 1}^{t:\to 1+h}} = -2\pi\left[\frac{\sin(\pi h)}{\pi h}\right] + 3\pi\left[\frac{1-(\cos(\pi h))}{\pi h}\right]$$

$$\underbrace{\left[\frac{\sin(\pi h)}{\pi h}\right]}_{\xrightarrow{h \to 0}\ 1} \qquad \underbrace{\left[\frac{1-(\cos(\pi h))}{\pi h}\right]}_{\xrightarrow{h \to 0}\ 0}$$

STOP

(b) -2π cm/s