CALCULUS
Intervals of increase/decrease and intervals of concavity

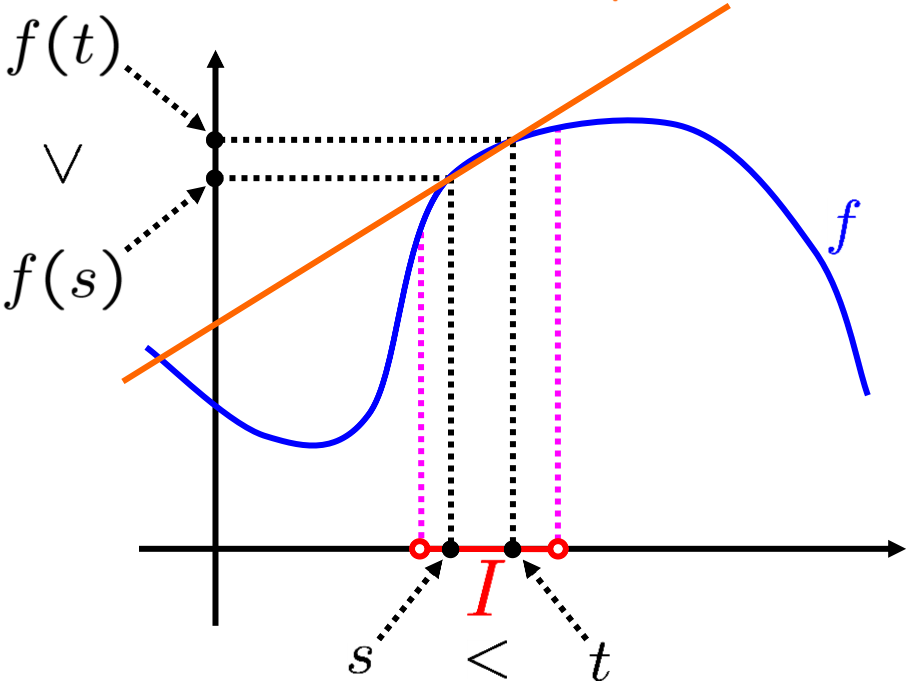
DEFINITION: Let I be an interval.

A function f is called **increasing on** I if

$f(s) < f(t)$ whenever $s, t \in I$ and $s < t$.

e.g.:

slope $= \dfrac{[f(t)] - [f(s)]}{t - s} > 0$



all

"secant lines run uphill" (slopes > 0)

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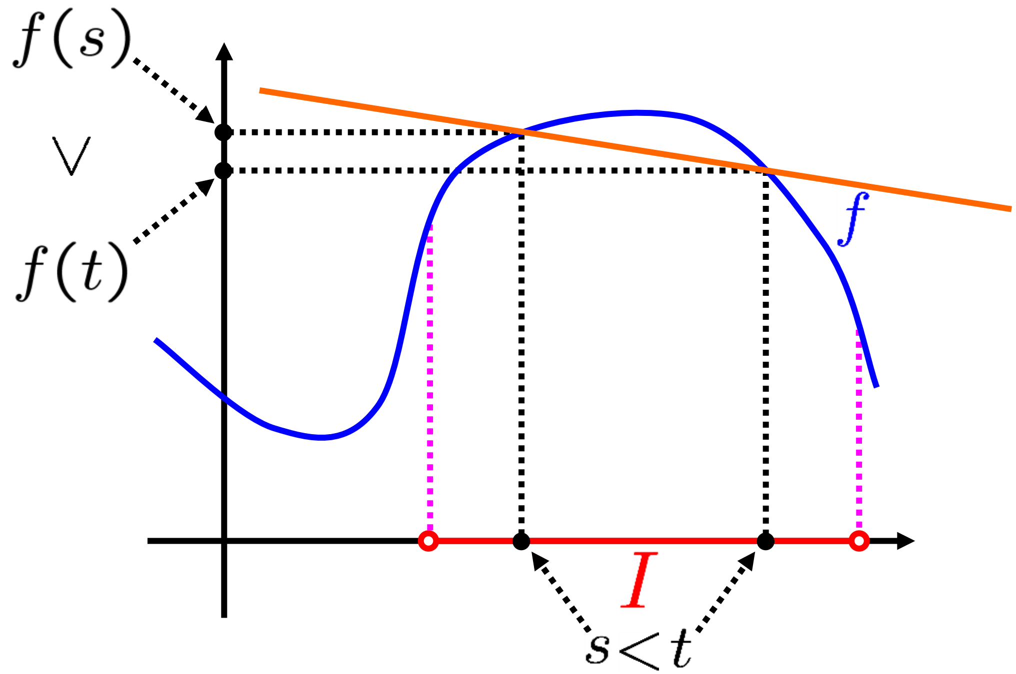
non-*e.g.*:



some

"secant line runs downhill"

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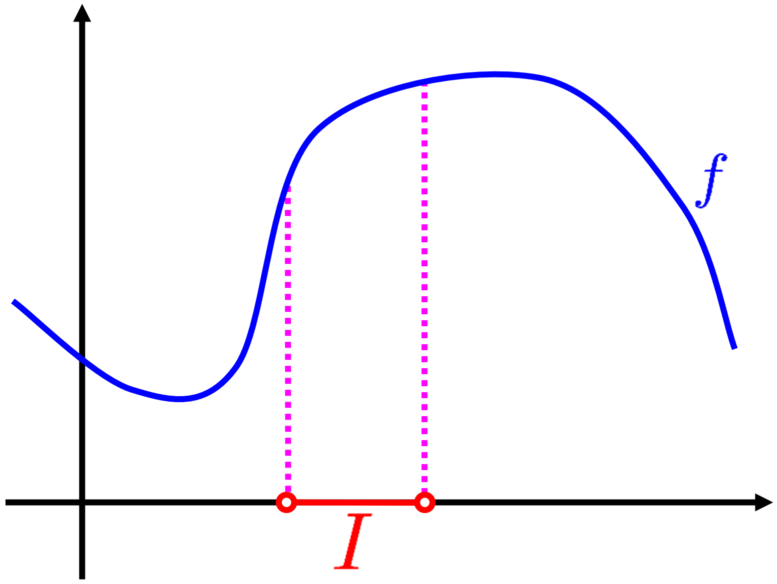
e.g.:



Typical to make the interval as large as possible. . .

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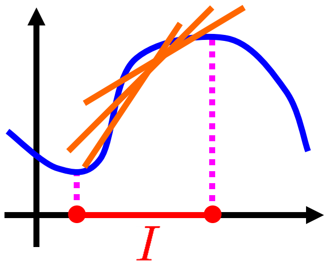
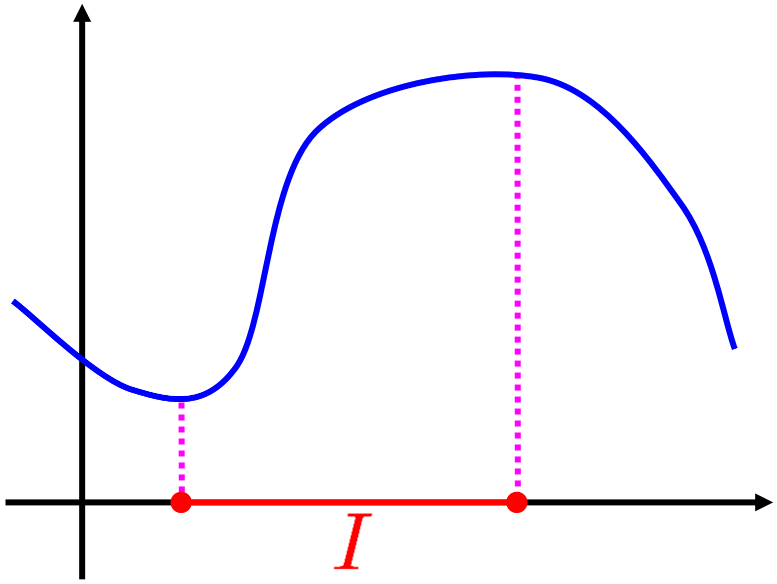


"secant lines run uphill" (slopes > 0)

e.g.:



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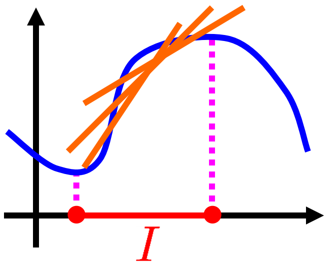
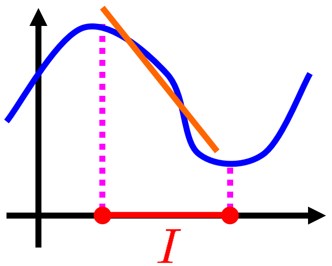
"secant lines run uphill" (slopes > 0)



A function f is called **decreasing on** I if

$$f(s) > f(t) \quad \text{whenever } s, t \in I \text{ and } s < t.$$

"secant lines run downhill" (slopes < 0)

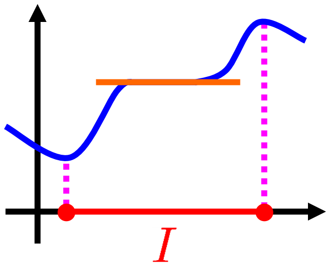


(semi-increasing)

A function f is called **nondecreasing on** I if

$$f(s) \leq f(t) \quad \text{whenever } s, t \in I \text{ and } s \leq t.$$

"secant lines don't run downhill" (slopes ≥ 0)

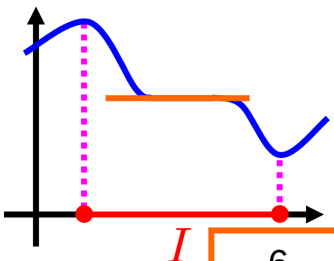


(semi-decreasing)

A function f is called **nonincreasing on** I if

$$f(s) \geq f(t) \quad \text{whenever } s, t \in I \text{ and } s \leq t.$$

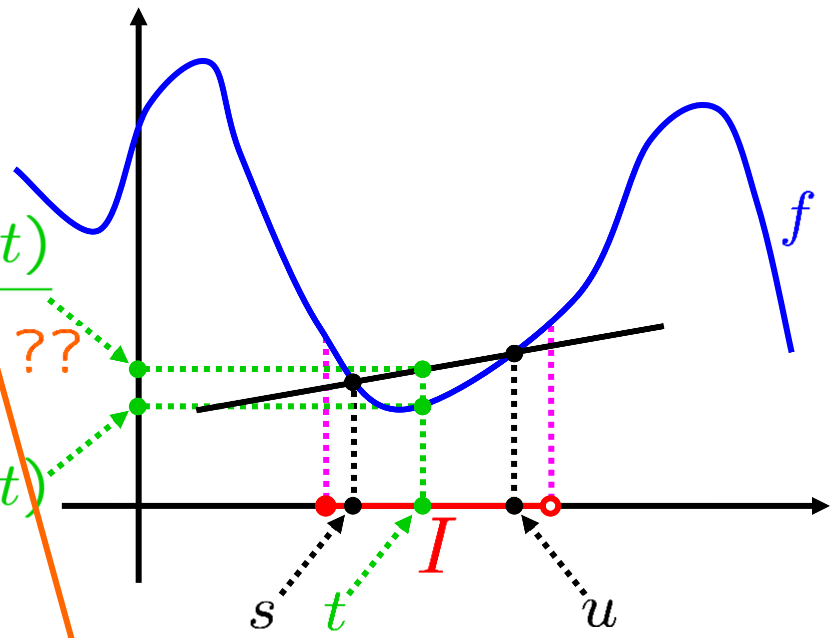
"secant lines don't run uphill" (slopes ≤ 0)



cf. §5.4, pp. 100–101, DEFINITION: Let I be an interval.

A function f is called **concave up on** I if the secant line segment from $(s, f(s))$ to $(u, f(u))$ lies above the graph of f, whenever $s, u \in I$.

e.g.:



this is linear in t

$$\frac{(f(u))(t-s) + (f(s))(u-t)}{u-s}$$

$f(s)$, for $t :\to s$

$f(u)$, for $t :\to u$

$\forall t \in (s, u)$,

$$\frac{(f(u))(t-s) + (f(s))(u-t)}{u-s} > f(t)$$

cf. §5.4, pp. 100–101, DEFINITION: Let I be an interval.

A function f is called **strictly** **concave up (convex)** on I if

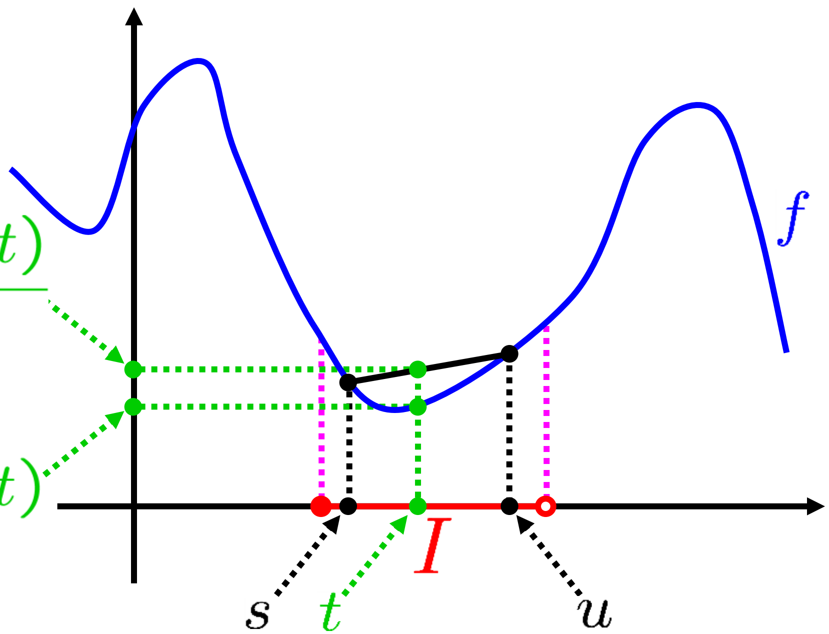
$$\forall t \in (s,u),\quad \frac{(f(u))(t-s)+(f(s))(u-t)}{u-s} > f(t)$$

whenever $s, u \in I$.

e.g.:

$$\frac{(f(u))(t-s)+(f(s))(u-t)}{u-s}$$

$f(t)$



$\forall t \in (s,u)$,

$$\frac{(f(u))(t-s)+(f(s))(u-t)}{u-s} > f(t)$$

cf. §5.4, pp. 100–101, DEFINITION: Let I be an interval.

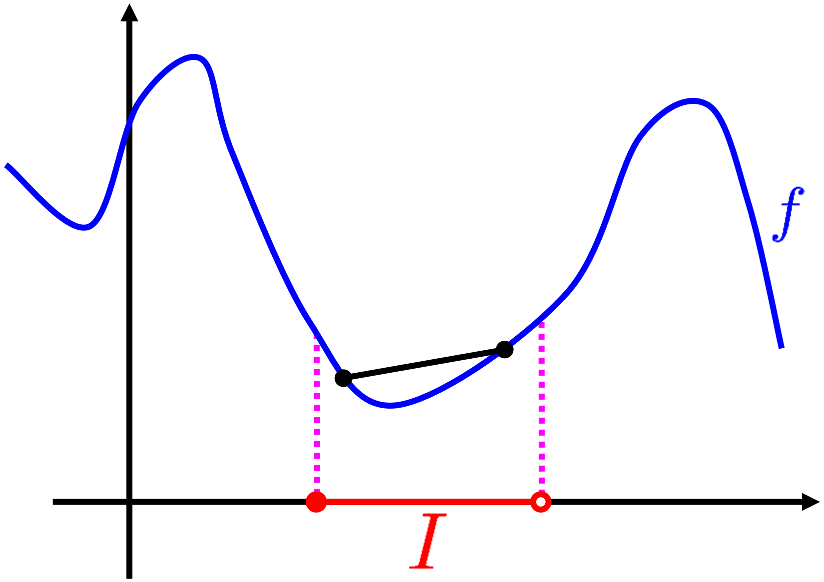
A function f is called **concave up on** I if **(convex)** **strictly**

the secant line segment from $(s, f(s))$ to $(u, f(u))$

lies above the graph of f,

whenever $s, u \in I$.

e.g.:



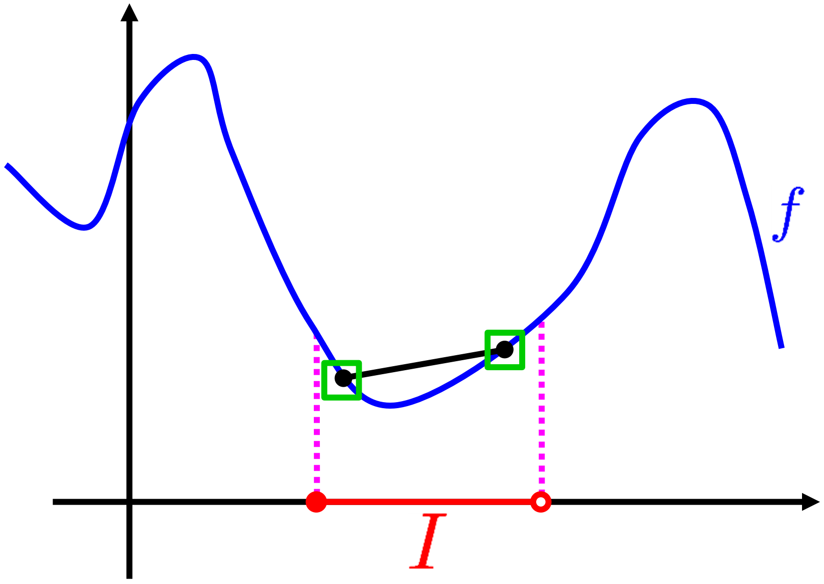
cf. §5.4, pp. 100–101, DEFINITION: Let I be an interval.

strictly

(convex)

A function f is called **concave up on** I if
the secant line segment from $(s, f(s))$ to $(u, f(u))$
lies above the graph of f,
whenever $s, u \in I$.

e.g.:



cf. §5.4, pp. 100–101, DEFINITION: Let I be an interval.

strictly (convex)

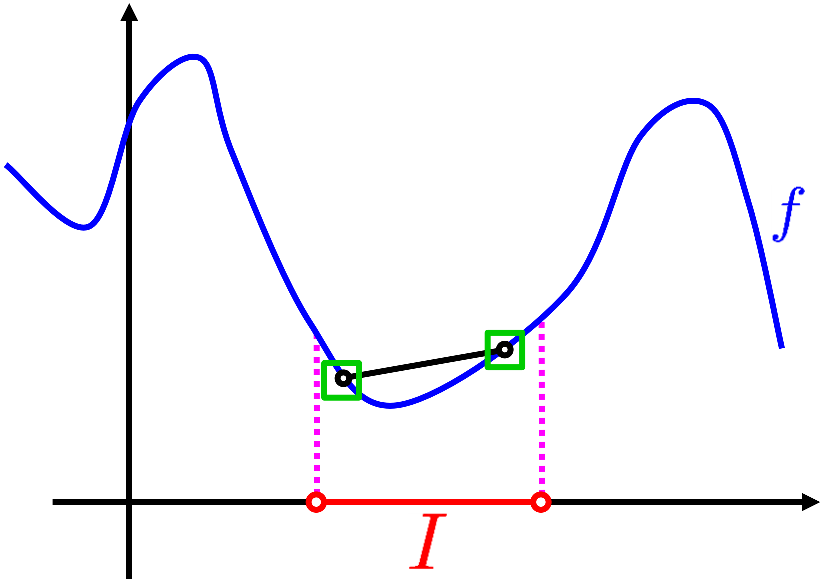
A function f is called **concave up on** I if

the [open] secant line segment from $(s, f(s))$ to $(u, f(u))$

lies above the graph of f,

whenever $s, u \in I$.

e.g.:



Typical to make the interval as large as possible. . .

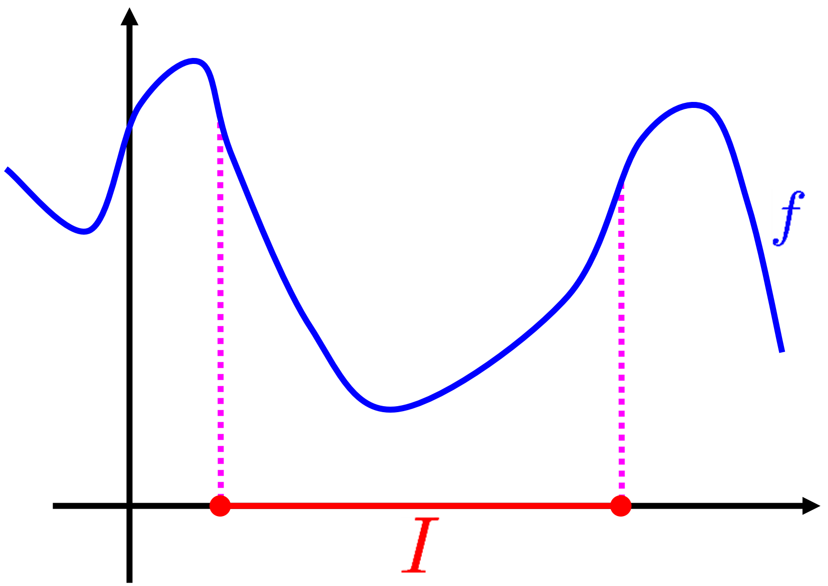
cf. §5.4, pp. 100–101, DEFINITION: Let I be an interval.

strictly (convex)

A function f is called **concave up on** I if the secant line segment from $(s, f(s))$ to $(u, f(u))$ lies above the graph of f, whenever $s, u \in I$.

open

e.g.:



Typical to make the interval as large as possible. . .

FACT:

Say f diff. at all pts of I.

Then:

f is concave up on I

iff

the graph of f lies above all of its "tangents" on I.

cf. §5.4, pp. 100–101, DEFINITION: Let I be an interval.

strictly

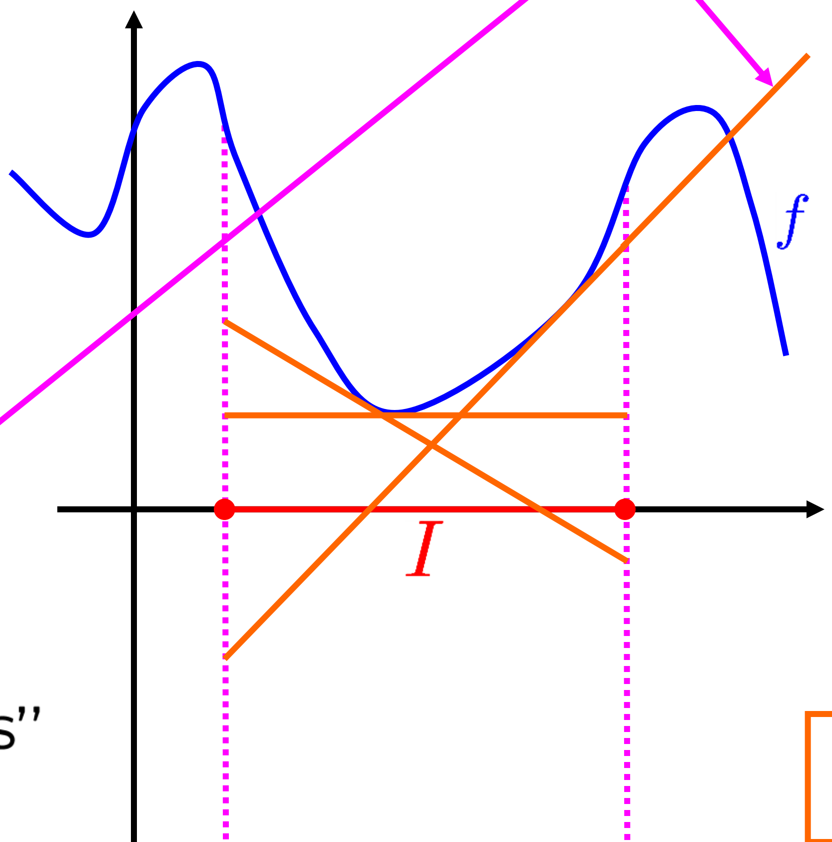
(convex)

A function f is called **concave up on** I if
the secant line segment from $(s, f(s))$ to $(u, f(u))$

open

lies above the graph of f,
whenever $s, u \in I$.

e.g.:



FACT:

Say f diff. at all pts of I.

Then:

f is concave up on I

iff

the graph of f

lies above all of its "tangents"

on I.

§5.4

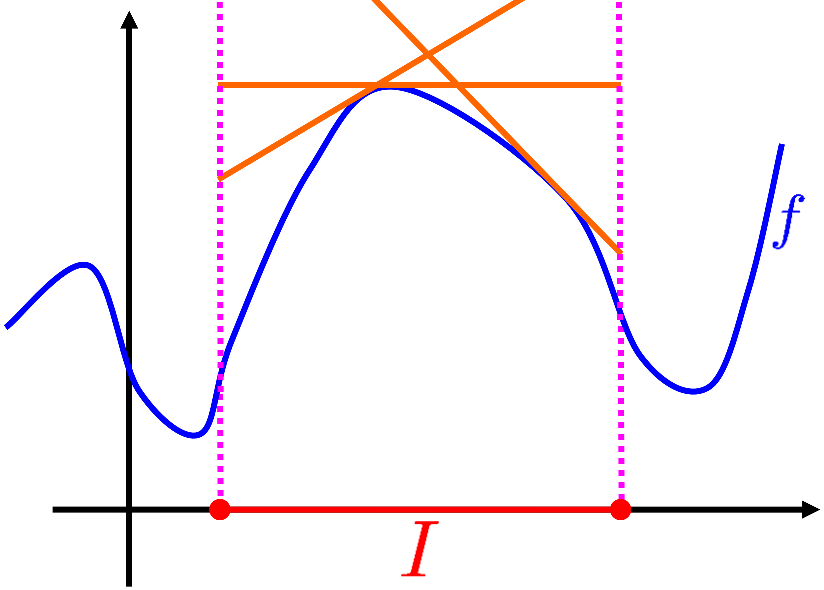
cf. §5.4, pp. 100–101, DEFINITION: Let I be an interval.

strictly

(concave)

A function f is called **concave down on** I if
the open secant line segment from $(s, f(s))$ to $(u, f(u))$
lies below the graph of f,
whenever $s, u \in I$.

e.g.:



FACT:

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Then:

f is concave down on I

iff

the graph of f

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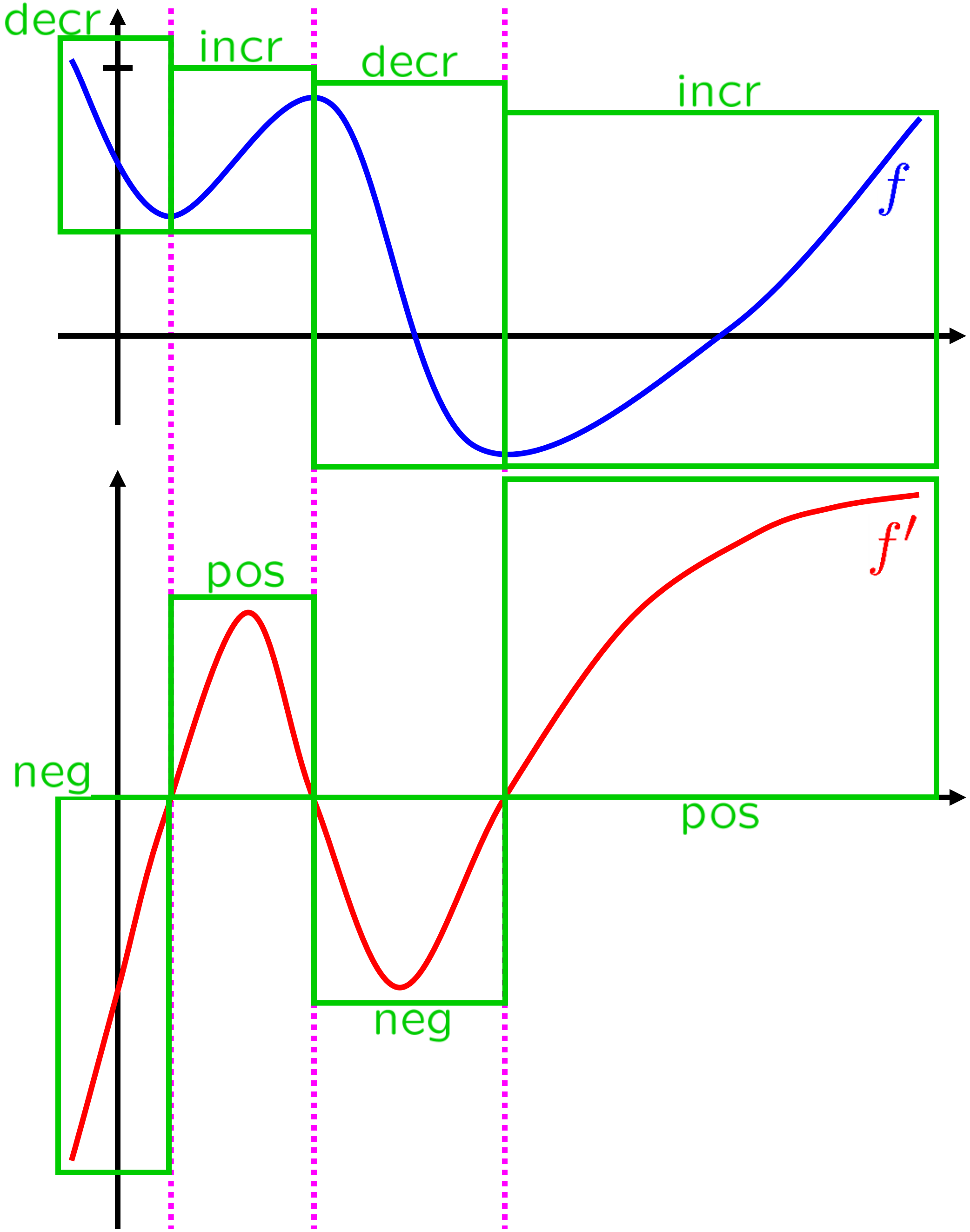
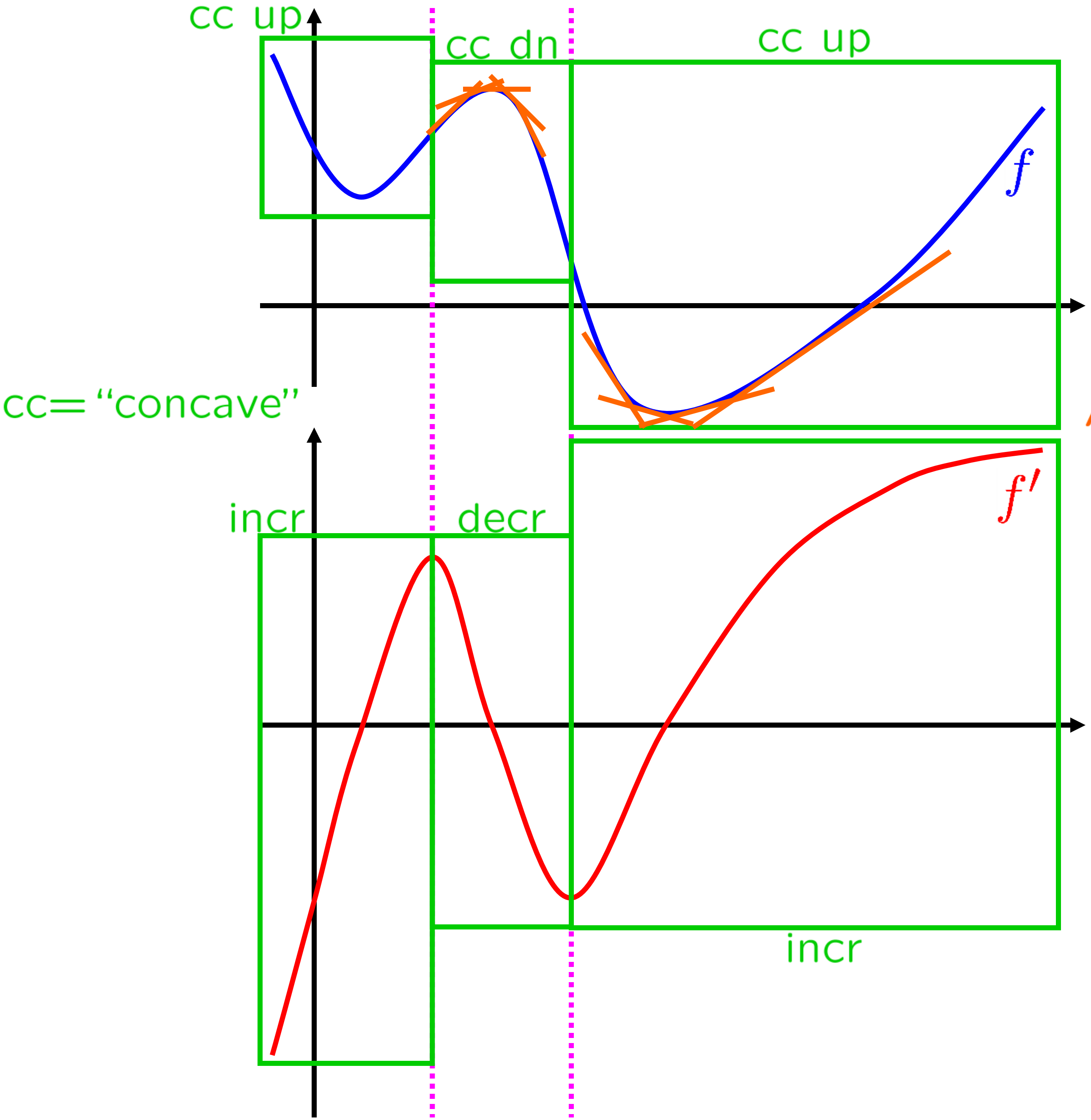
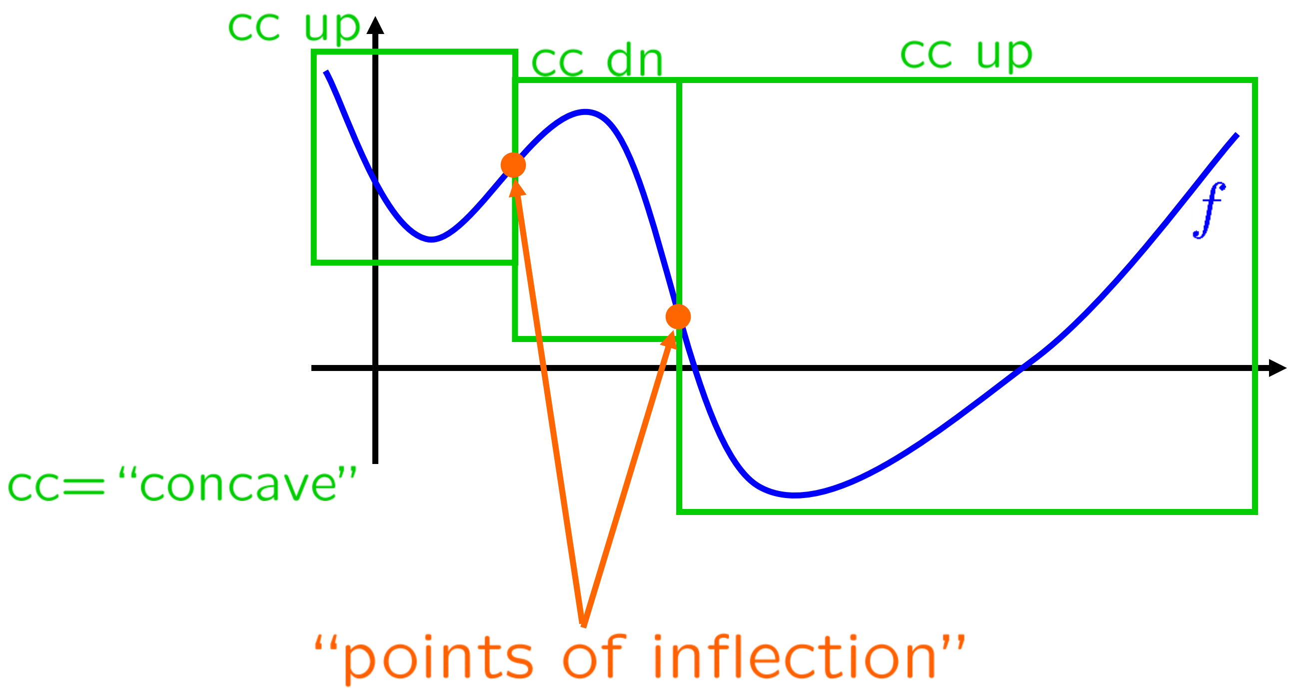

decr
incr
decr
incr
f
pos
f'
neg
pos
neg



TANGENT LINE SLOPES
negative
less negative
positive
more positive

TANGENT LINE SLOPES ARE INCREASING


cc up
cc dn
cc up
f
cc= "concave"
"points of inflection"


STOP