CALCULUS
The chain rule

Let $f(x) = x^3$.

Let $y := f(x) = x^3$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^3)$.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^3)]$

$$\frac{\triangle z}{\triangle x} \underset{\substack{\triangle x \neq 0 \\ \triangle y \neq 0}}{=} \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

UNNEEDED

f is 1-1

$\triangle y = [f(x + \triangle x)] - [f(x)]$

$$\triangle x \neq 0 \Rightarrow x \neq x + \triangle x \Rightarrow f(x) \neq f(x + \triangle x) \Rightarrow \triangle y \neq 0$$

Note: $\frac{d}{dx}[(\sin x)(x^3)]$ may look similar,
but is actually *very* different.

$$\|$$

$$(\sin x)(3x^2) + (\cos x)(x^3)$$

§3.5

Let $f(x) = x^3$.

Let $y := f(x) = x^3$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^3)$.

Goal: $\underbrace{\frac{d}{dx}[\sin(x^3)]}_{\frac{dz}{dx}}$

$$\frac{\triangle z}{\triangle x} \quad \underset{\triangle x \neq 0}{=} \quad \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$$\triangle x \to 0$$

$$\triangle x \neq 0 \Rightarrow x \neq x + \triangle x \Rightarrow f(x) \neq f(x + \triangle x) \Rightarrow \triangle y \neq 0$$

Note: $\frac{d}{dx}[(\sin x)(x^3)]$ may look similar, but is actually *very* different.

$$\frac{d}{dx}[(\sin x)(x^3)] = (\sin x)(3x^2) + (\cos x)(x^3)$$

Let $f(x) = x^3$.

Let $y := f(x) = x^3$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^3)$.

Is z an expr. of y?

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^3)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$\frac{dz}{dx}$ $\qquad \triangle x \to 0 \qquad$??? $\qquad \frac{dy}{dx}$

$\triangle x \neq 0 \Rightarrow x \neq x + \triangle x \Rightarrow f(x) \neq f(x + \triangle x) \Rightarrow \triangle y \neq 0$

y is a DEPENDENT variable, so

there are no "expressions of y".

Note: $\frac{d}{dx}[(\sin x)(x^3)]$ may look similar,

but is actually *very* different.

$$\| $$

$(\sin x)(3x^2) + (\cos x)(x^3)$

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z an expr. of x.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^3)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$$\frac{dz}{dx} \qquad \triangle x \to 0 \qquad ??? \qquad \frac{dy}{dx}$$

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$(\sin x)(3x^2) + (\cos x)(x^3)$

Let $f(x) = x^3$.

Let $y := f(x) = x^3$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^3)$.

z an expr. of x.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^3)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$$\frac{dz}{dx} \qquad ??? \qquad \frac{dy}{dx}$$

$\triangle x \to 0$

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y is a DEPENDENT variable, so there are no "expressions of y".

$\triangle z = [g(f(x + \triangle x))] - [g(f(x))]$

ADD

$\triangle y = [f(x + \triangle x)] - [f(x)]$

$y = f(x)$

$y + \triangle y = f(x + \triangle x)$

§3.5

Let $f(x) = x^3$.

Let $y := f(x) = x^3$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^3)$.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^3)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

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there are no "expressions of y".

$\triangle z = [g(f(x + \triangle x))] - [g(f(x))]$

$y + \triangle y = f(x + \triangle x) \qquad y = f(x)$

$y = f(x)$

$y + \triangle y = f(x + \triangle x)$

§3.5

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Let $y := f(x) = x^3$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) =$ sin(x^3).

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^3)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

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there are no "expressions of y".

$$\triangle z = [g(f(x + \triangle x))] - [g(f(x))] = [g(y + \triangle y)] - [g(y)]$$

$$y + \triangle y = f(x + \triangle x) \qquad y = f(x)$$

Let $f(x) = x^3$.

Let $y := f(x) = x^3$ and let $g :=$ sin.

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$\frac{dz}{dx}$??? $\frac{dy}{dx}$

$\triangle x \to 0$

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y is a DEPENDENT variable, so

there are no "expressions of y".

$$\triangle z = [g(f(x + \triangle x))] - [g(f(x))] = [g(y + \triangle y)] - [g(y)]$$

$$\frac{\triangle z}{\triangle y} = \frac{[g(y + \triangle y)] - [g(y)]}{\triangle y}$$

§3.5

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y is a DEPENDENT variable, so

there are no "expressions of y".

$$\triangle y \underset{\triangle x \neq 0}{=} \left[\frac{\triangle y}{\triangle x}\right][\triangle x] \underset{\triangle x \to 0}{\to} \left[\frac{dy}{dx}\right][0] = 0$$

$$\frac{\triangle z}{\triangle y} = \frac{[g(y + \triangle y)] - [g(y)]}{\triangle y} \underset{\triangle x \to 0}{\to} g'(y)$$

§3.5

Let $f(x) = x^3$.

Let $y := f(x) = x^3$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^3)$.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^3)]$

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$$\frac{dz}{dx} \qquad \triangle x \to 0 \qquad g'(y) \qquad \frac{dy}{dx}$$

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$$\frac{\triangle z}{\triangle y} = \frac{[g(y + \triangle y)] - [g(y)]}{\triangle y} \underset{\triangle x \to 0}{\to} g'(y)$$

§3.5

Let $f(x) = x^3$.

Let $y := f(x) = x^3$ and let $g :=$ sin.

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$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^3)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$$\frac{dz}{dx} \qquad g'(y) \qquad \frac{dy}{dx}$$

$\triangle x \to 0$

$\triangle x \neq 0 \Rightarrow x \neq x + \triangle x \Rightarrow f(x) \neq f(x + \triangle x) \Rightarrow \triangle y \neq 0$

$$\frac{dz}{dx} = [g'(y)]\left[\frac{dy}{dx}\right] = [\sin'(x^3)]\left[\frac{d}{dx}(x^3)\right]$$

$$= [\cos(x^3)][3x^2] \blacksquare$$

§3.5

Let $f(x) = x^3$. ← x^4

Let $y := f(x) = x^3$ and let $g := \sin$.

Let $z := g(y) = g(f(x)) = \sin(x^3)$.

$\frac{dz}{dx}$

Goal: $\frac{d}{dx}[\sin(x^3)]$ ← x^4

$$\frac{\triangle z}{\triangle x} \underset{\substack{\triangle x \neq 0 \\ \triangle y \neq 0}}{=} \left[\frac{\triangle z}{\triangle y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

NEEDED ($\triangle y \neq 0$)

$\triangle x \to 0$: $\frac{\triangle z}{\triangle x} \to \frac{dz}{dx}$, $\frac{\triangle z}{\triangle y} \to g'(y)$, $\frac{\triangle y}{\triangle x} \to \frac{dy}{dx}$

$$\triangle x \neq 0 \Rightarrow x \neq x + \triangle x \Rightarrow f(x) \neq f(x + \triangle x) \Rightarrow \triangle y \neq 0$$

NOT TRUE FOR $f(x) = x^4$

$$\frac{dz}{dx} = [g'(y)]\left[\frac{dy}{dx}\right] = [\sin'(x^3)]\left[\frac{d}{dx}(x^3)\right]$$

$$= [\cos(x^3)][3x^2] \blacksquare$$

§3.5

Let $f(x) = x^4$.

Let $y := f(x) = x^4$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^4)$.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^4)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\square z}{\square y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$\triangle y \neq 0$

UNNEEDED

$$\frac{\square z}{\square y} := \begin{cases} \frac{\triangle z}{\triangle y}, & \text{if } \triangle y \neq 0 \\ g'(y), & \text{if } \triangle y = 0 \end{cases}$$

$$\triangle z = [g(f(x + \triangle x))] - [g(f(x))] = [g(y + \triangle y)] - [g(y)]$$

$$\triangle y = 0 \quad \Rightarrow \quad \triangle z = 0$$

§3.5

Let $f(x) = x^4$.

Let $y := f(x) = x^4$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^4)$.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^4)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\square z}{\square y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$$\left[\frac{\square z}{\square y}\right] \approx g'(y)$$

$\triangle x \to 0$

Say $\triangle x \approx 0$, but $\neq 0$. $\quad \triangle y \approx 0$

$$\frac{\triangle z}{\triangle y} = \frac{[g(y+\triangle y)] - [g(y)]}{\triangle y} \underset{\triangle y \neq 0}{\approx} g'(y)$$

$$\frac{\square z}{\square y} := \begin{cases} \dfrac{\triangle z}{\triangle y}, & \text{if } \triangle y \neq 0 \\ g'(y), & \text{if } \triangle y = 0 \end{cases}$$

$$\triangle y \underset{\triangle x \neq 0}{=} \left[\frac{\triangle y}{\triangle x}\right][\triangle x] \underset{\triangle x \to 0}{\to} \left[\frac{dy}{dx}\right][0] = 0$$

§3.5

Let $f(x) = x^4$.

Let $y := f(x) = x^4$ and let $g := \sin$.

Let $z := g(y) = g(f(x)) = \sin(x^4)$.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^4)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\square z}{\square y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$$\frac{dz}{dx} \qquad \triangle x \to 0 \qquad g'(y) \qquad \frac{dy}{dx}$$

Say $\triangle x \approx 0$, but $\neq 0$. $\triangle y \approx 0$

$$\frac{\triangle z}{\triangle y} = \frac{[g(y+\triangle y)] - [g(y)]}{\triangle y} \underset{\triangle y \neq 0}{\approx} g'(y)$$

$$\frac{\square z}{\square y} := \begin{cases} \frac{\triangle z}{\triangle y}, & \text{if } \triangle y \neq 0 \\ g'(y), & \text{if } \triangle y = 0 \end{cases}$$

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Let $f(x) = x^4$.

Let $y := f(x) = x^4$ and let $g := \sin$.

Let $z := g(y) = g(f(x)) = \sin(x^4)$.

$$\frac{dz}{dx}$$

Goal: $\frac{d}{dx}[\sin(x^4)]$

$$\frac{\triangle z}{\triangle x} \underset{\triangle x \neq 0}{=} \left[\frac{\square z}{\square y}\right]\left[\frac{\triangle y}{\triangle x}\right]$$

$$\frac{dz}{dx} \qquad \triangle x \to 0 \qquad g'(y) \qquad \frac{dy}{dx}$$

$$\frac{\square z}{\square y} := \begin{cases} \dfrac{\triangle z}{\triangle y}, & \text{if } \triangle y \neq 0 \\ g'(y), & \text{if } \triangle y = 0 \end{cases}$$

$$\frac{dz}{dx} = [g'(y)]\left[\frac{dy}{dx}\right] = [\sin'(x^4)]\left[\frac{d}{dx}(x^4)\right]$$

$$= [\cos(x^4)]\,[4x^3] \blacksquare$$

Let $f(x) = x^4$.

Let $y := f(x) = x^4$ and let $g :=$ sin.

Let $z := g(y) = g(f(x)) = \sin(x^4)$.

$$\overbrace{\frac{d}{dx}[\sin(x^4)]}^{\frac{dz}{dx}}$$

Goal: $\frac{d}{dx}[\sin(x^4)]$

$[y = f(x)$ and $z = g(y)] \quad \Rightarrow$

$$\frac{dz}{dx} = \left[g'(y)\right]\left[\frac{dy}{dx}\right]$$

$$\frac{dz}{dx} = \left[g'(y)\right]\left[\frac{dy}{dx}\right]$$

y is a DEPENDENT variable, so
there is no d/dy, technically, but. . .

Sloppy, but common:

$$\frac{dz}{dy} = g'(y)$$

$[y = f(x)$ and $z = g(y)]$ $\Rightarrow$

$$\frac{dz}{dx} = \left[g'(y)\right]\left[\frac{dy}{dx}\right]$$

Chain Rule: $$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$[y = f(x)$ and $z = g(y)] \quad \Rightarrow$

$$\frac{dz}{dx} = \left[g'(y)\right]\left[\frac{dy}{dx}\right]$$

$$\frac{d}{dx}[g(y)] = \left[g'(y)\right]\left[\frac{dy}{dx}\right]$$

Chain Rule: $$\frac{d}{dx}[g(f(x))] = \left[g'(f(x))\right]\left[\frac{d}{dx}(f(x))\right]$$

to differentiate the result. . .

plugged into a function

an expression of x

take the derivative of the function

plug in the expression

and multiply by

the derivative of the expression

Chain Rule: $$\frac{d}{dx}[g(f(x))] = [g'(f(x))]\left[\frac{d}{dx}(f(x))\right]$$

e.g.: $$\frac{d}{dx}[\sin(\cot x)] = [\cos(\cot x)][-\csc^2 x] \blacksquare$$

$(d/dx)(\cot x) = -\csc^2 x$

$\sin' = \cos$

$(d/dx)(\sin(x)) = \cos(x)$

§3.5

- to differentiate the result. . .
- plugged into a function
- an expression of x
- take the derivative of the function
- plug in the expression
- and multiply by
- the derivative of the expression

Chain Rule: $$\frac{d}{dx}[g(f(x))] = \left[g'(f(x))\right]\left[\frac{d}{dx}(f(x))\right]$$

$$(d/dx)(\tan x) = \sec^2 x$$

e.g.: $$\frac{d}{dx}[e^{\tan x}] = [\ e^{\tan x}\ ][\sec^2 x] \blacksquare$$

§3.5 $(d/dx)(e^x) = e^x$ $\quad (e^\bullet)' = e^\bullet$

to differentiate the result. . .

plugged into a function

an expression of x

take the derivative of the function

plug in the expression

and multiply by

the derivative of the expression

Chain Rule: $\frac{d}{dx}[g(f(x))] = [g'(f(x))]\left[\frac{d}{dx}(f(x))\right]$

e.g.: $\frac{d}{dx}[\cos^3 x] = [3\cos^2 x][-\sin x]$ ■

$(d/dx)(\cos x) = -\sin x$

$(\bullet^3)' = 3\bullet^2$

Chain Rule: $\frac{d}{dt}[g(f(t))] = \left[g'(f(t))\right]\left[\frac{d}{dt}(f(t))\right]$

Chain Rule: $\frac{d}{ds}[g(f(s))] = \left[g'(f(s))\right]\left[\frac{d}{ds}(f(s))\right]$

"f then g"

the composite of g and f

$(g \circ f)(x)$

Chain Rule: $\frac{d}{dx}[g(f(x))] = \left[g'(f(x))\right]\left[\frac{d}{dx}(f(x))\right]$

$(g \circ f)'(x) =$

$= [(g' \circ f)(x)]\,[f'(x)]$

Chain Rule: $(g \circ f)' = [g' \circ f] \cdot f'$

§3.5

$$\frac{d}{dx}\left[\sin\left(\tan\left(x^7\right)\right)\right] = \left[\cos\left(\tan\left(x^7\right)\right)\right]\left[\sec^2\left(x^7\right)\right]\left[7x^6\right]$$

Take the derivative of the function.

Plug in the expression.

Multiply by the derivative of the expression.

$$\frac{d}{dx}\left(\frac{e^x \sin x}{\cos^7(x^3)}\right) =$$

$$[\cos(x^3)]^7$$

$$7[\cos(x^3)]^6$$

$$\frac{[\cos^7(x^3)][e^x \sin x + e^x \cos x] - [e^x \sin x][7\cos^6(x^3)][-\sin(x^3)][3x^2]}{\cos^{14}(x^3)}$$

$$\left[\frac{e^x \sin x}{\cos^7(x^3)}\right]_{x:\to 3}$$

§3.5

Goal: the complementary formula

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\sin' = \cos$$

$$\frac{d}{dx}[\cos x] = \frac{d}{dx}\left[\sin\left(\frac{\pi}{2} - x\right)\right]$$

$$= \left[\sin'\left(\frac{\pi}{2} - x\right)\right]\left[\frac{d}{dx}\left(\frac{\pi}{2} - x\right)\right]$$

$$= \left[\cos\left(\frac{\pi}{2} - x\right)\right][-1]$$

$$= [\sin x][-1]$$

$$= -\sin x$$

§3.5

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\tan' = \sec^2$$

Goal: the complementary formula

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\tan\left(\frac{\pi}{2} - x\right)\right]$$

$$= \left[\tan'\left(\frac{\pi}{2} - x\right)\right]\left[\frac{d}{dx}\left(\frac{\pi}{2} - x\right)\right]$$

$$= \left[\sec^2\left(\frac{\pi}{2} - x\right)\right][-1]$$

$$= [\csc^2 x][-1]$$

$$= -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = (\sec x)(\tan x)$$

$$\sec' = (\sec)(\tan)$$

Goal: the complementary formula

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\sec\left(\frac{\pi}{2} - x\right)\right]$$

$$= \left[\sec'\left(\frac{\pi}{2} - x\right)\right]\left[\frac{d}{dx}\left(\frac{\pi}{2} - x\right)\right]$$

$$= \left[((\sec)(\tan))\left(\frac{\pi}{2} - x\right)\right][-1]$$

$$= [(\csc x)(\cot x)][-1]$$

$$= -(\csc x)(\cot x)$$

STOP