CALCULUS
Logarithmic differentiation

$$\text{la} := \ln|\bullet|$$

$$\text{la}' = \frac{1}{\bullet}$$

Logarithmic Derivative

MULTIPLY BY $f(x)$ . . .

$$[f(x)]\left[\frac{d}{dx}\left[\underset{\text{function}}{\underline{\text{la}}}\Big(\underset{\text{expression}}{\boxed{f(x)}}\Big)\right]\right] \underset{f(x)\neq 0}{=} \left[\cancel{\frac{f(x)}{f(x)}}\right]\left[\frac{d}{dx}[f(x)]\right]$$

Principle of Logarithmic Differentiation

$$\frac{d}{dx}[f(x)] \underset{f(x)\neq 0}{=} [f(x)]\underbrace{\left[\frac{d}{dx}[\text{la}(f(x))]\right]}_{\text{logarithmic derivative of } f(x)}$$

Principle of Logarithmic Differentiation:

To compute the derivative of an expression, multiply the expression by its logarithmic derivative.

Works well when la simplifies the expression.

e.g.:

UNNEEDED

$$\forall x > 0,\quad \frac{d}{dx}[x^x] \overset{x^x \neq 0}{=} [\ x^x\ ]\left[\frac{d}{dx}[\boxed{\mathrm{la}}(\ x^x\ )]\right]$$

x^x is not def'd in a nbd of 0, or of any negative number.

Principle of Logarithmic Differentiation

$$\underline{\frac{d}{dx}[f(x)]} \underset{f(x) \neq 0}{=} \underline{[f(x)]}\ \underbrace{\left[\underline{\frac{d}{dx}[\mathrm{la}(f(x))]}\right]}_{\text{logarithmic derivative of } f(x)}$$

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$$\forall x > 0,\ \frac{d}{dx}[x^x] = [\ x^x\ ]\left[\frac{d}{dx}[\boxed{\ln}(\ x^x\ )]\right]$$

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To compute the derivative of an expression,
multiply the expression
by its logarithmic derivative.

Works well when
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e.g.:

$$\forall x > 0,\ \frac{d}{dx}[x^x] = [\ x^x\ ]\left[\underbrace{\frac{d}{dx}[\underbrace{\ln(\,x^{\boxed{x}}\,)}_{x\cdot\ln(x)}]}_{\cancel{1}(\ln(x)) + \underbrace{x(1/x)}_{1}}\right]$$

$$= [\ x^x\ ][\,1 + (\ln\ x)\,] \blacksquare$$

Next: Properties of la. . .

Principle of Logarithmic Differentiation:

To compute the derivative of an expression, multiply the expression by its logarithmic derivative.

Works well when la simplifies the expression.

Facts:

$$\mathrm{la}\,(PQ) = (\mathrm{la}\,P) + (\mathrm{la}\,Q)$$

$$\mathrm{la}\,(P/Q) = (\mathrm{la}\,P) - (\mathrm{la}\,Q)$$

$$\mathrm{la}\left(P^{\boxed{k}}\right) \underset{P^k \text{ exists}}{=} k\,(\mathrm{la}\,P)$$

non-*e.g.*: $\mathrm{la}\left((-1)^{1/2}\right)$ DNE, but $(1/2)\,(\mathrm{la}(-1)) = 0$

e.g.: $\mathrm{la}\left((-1)^2\right) = 2\,(\mathrm{la}(-1))$

Note: $\ln\left((-1)^2\right) = 0,$ but $2\,(\ln(-1))$ DNE

cf. §3.1, pp. 45–48 THE POWER RULE

If n is any real number
and if $f(x) = x^n$,
then $f'(x) \underset{x \neq 0}{=} nx^{n-1}$.

Next: $x = 0$

Proof: $f'(x) \underset{x \neq 0}{=} [x^n]\left[\frac{d}{dx}[\mathrm{la}(x^{\boxed{n}})]\right] = [x^n]\left[\frac{d}{dx}[n\,(\mathrm{la}\,x)]\right]$

$= [x^n][n(1/x)] \underset{x \neq 0}{=} nx^{n-1}$

x^n exists
UNNEEDED

QED

$x \neq 0 \Rightarrow x^n \neq 0$

Note: $\forall n < 1$, both $f'(0)$ and $n \cdot 0^{n-1}$ DNE.
If $n = 1$, then $f'(0) = 1$, but $n \cdot 0^{n-1}$ DNE.

$$\frac{d}{dx}[f(x)] \underset{f(x) \neq 0}{=} [f(x)]\left[\frac{d}{dx}[\mathrm{la}(f(x))]\right]$$

cf. §3.1, pp. 45–48 THE POWER RULE

If n is any real number
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Proof:

$$f'(x) \underset{x \neq 0}{=} [x^n]\left[\frac{d}{dx}[\text{la}(x^n)]\right] = [x^n]\left[\frac{d}{dx}[n\,(\text{la}\,x)]\right]$$

$$= [x^n][n(1/x)] \underset{x \neq 0}{=} nx^{n-1}$$

QED

Next: exponential functions

Note: $\forall n < 1$, both $f'(0)$ and $n \cdot 0^{n-1}$ DNE.
If $n = 1$, then $f'(0) = 1$, but $n \cdot 0^{n-1}$ DNE.
If $n > 1$ and if n is irrational,
then $f'(0)$ DNE, but $n \cdot 0^{n-1} = 0$.
If $n > 1$ and if n is odd/even,
then $f'(0)$ DNE, but $n \cdot 0^{n-1} = 0$.
If $n > 1$ and if n is even/odd,
then $f'(0) = 0 = n \cdot 0^{n-1}$. ~~even/even~~
If $n > 1$ and if n is odd/odd,
then $f'(0) = 0 = n \cdot 0^{n-1}$.

§4.7

$10^x > 0$, so $\mathrm{la}(10^x) = \ln(10^x)$

$$\frac{d}{dx}[10^x] = [10^x]\left[\frac{d}{dx}(\ln[10^{\boxed{x}}])\right]$$

$$= [10^x]\left[\frac{d}{dx}(x \cdot \boxed{\ln[10]})\right]$$

$$= [10^x][\ln 10]\xcancel{\left[\frac{d}{dx}[x]\right]}$$

$$= [10^x][\ln 10]$$

$10 :\to b$

$$\forall b > 0,\ \frac{d}{dx}[b^x] = [b^x][\ln b]$$

Next: logarithmic functions

Goal: $\frac{d}{dx}[\log_2 x]$

$2 :\rightarrow b$

$\forall b \in (0,\infty)\backslash\{1\}$,

$$\frac{d}{dx}[\log_b x] \overset{x>0}{=} \frac{1}{x[\ln b]}$$

ALTERNATE APPROACH. . .

$$2^{\log_2 x} = x$$

TAKE d/dx

$$\underbrace{2^{\log_2 x}}_{x}[\ln 2]\left[\frac{d}{dx}[\log_2 x]\right] \overset{x>0}{=} 1$$

$$\frac{d}{dx}[\log_2 x] \overset{x>0}{=} \frac{1}{x[\ln 2]}$$

DIVIDE BY $x \ln 2$

$$\forall b > 0,\ \frac{d}{dx}[b^x] = [b^x][\ln b]$$

$b :\rightarrow 2$

$$\frac{d}{dx}[2^x] = [2^x][\ln 2]$$

$$[2^\bullet]' = [2^\bullet][\ln 2]$$

Assuming $x > 0$, $b > 0$ and $b \neq 1$.

$$b^{\log_b x} = x$$

TAKE ln

$$\ln\left(b^{\boxed{\log_b x}}\right) = \ln x$$

$$[\log_b x]\,[\ln b] = \ln x$$

DIVIDE BY $\ln b$

$$\log_b x = \frac{\ln x}{\ln b}$$

"If the $\log_b x$ button
on your calculator breaks,
but not the ln button,
you can still calculate $\log_b x$."

TAKE d/dx

$$\frac{d}{dx}[\log_b x] = \frac{d}{dx}\left[\frac{\ln x}{\boxed{\ln b}}\right] = \left[\frac{1}{\ln b}\right]\left[\boxed{\frac{d}{dx}[\ln x]}\right] = \left[\frac{1}{\ln b}\right]\left[\boxed{\frac{1}{x}}\right] = \frac{1}{x[\ln b]}$$

$$\boxed{\forall b \in (0,\infty)\backslash\{1\},\quad \frac{d}{dx}[\log_b x] \overset{x>0}{=} \frac{1}{x[\ln b]}}$$

ALTERNATE APPROACH. . .

Next: Problems. . .

Example: **Find** the logarithmic derivative of $x^7 + x^4 + 5x$.

Sol'n: $$\frac{d}{dx}[\text{la}(x^7 + x^4 + 5x)] = \frac{7x^6 + 4x^3 + 5}{x^7 + x^4 + 5x} \blacksquare$$

General Fact: $$\frac{d}{dx}[\text{la}(f(x))] = \frac{f'(x)}{f(x)}$$

Example: **Find** the logarithmic derivative of $x^4 - x^2 + 8x$.

Sol'n: $$\frac{d}{dx}[\text{la}(x^4 - x^2 + 8x)] = \frac{4x^3 - 2x + 8}{x^4 - x^2 + 8x} \blacksquare$$

Facts: $\frac{d}{dx}[\text{la}[(f(x))(g(x))]] = \left[\frac{d}{dx}[\text{la}(f(x))]\right] + \left[\frac{d}{dx}[\text{la}(g(x))]\right]$

$$\frac{d}{dx}\left[\text{la}\left[\frac{f(x)}{g(x)}\right]\right] = \left[\frac{d}{dx}[\text{la}(f(x))]\right] - \left[\frac{d}{dx}[\text{la}(g(x))]\right]$$

$$\frac{d}{dx}\left[\text{la}\left[(f(x))^{\boxed{k}}\right]\right] = k\left[\frac{d}{dx}[\text{la}(f(x))]\right]$$

$(f(x))^k$ exists

non-*e.g.*:
$f(x) = -1$
$k = 1/2$

Example: Find the logarithmic derivative of

$$\frac{(x^5+2x-3)\boxed{(2x^7-x^4+1)^{100}}}{x^3-2}$$

exists, $\forall x \in \mathbb{R}$

Sol'n: $$\left[\frac{5x^4+2}{x^5+2x-3}\right] + 100\left[\frac{14x^6-4x^3}{2x^7-x^4+1}\right] - \left[\frac{3x^2}{x^3-2}\right]$$

■

Example: Find the derivative of

$$\frac{(x^5+2x-3)(2x^7-x^4+1)^{100}}{x^3-2}$$

Sol'n: $$\left[\frac{(x^5+2x-3)(2x^7-x^4+1)^{100}}{x^3-2}\right]\times$$
$$\left(\left[\frac{5x^4+2}{x^5+2x-3}\right]+100\left[\frac{14x^6-4x^3}{2x^7-x^4+1}\right]-\left[\frac{3x^2}{x^3-2}\right]\right)$$ ■

Example: Find the logarithmic derivative of

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$$-\left[\frac{3x^2}{x^3-2}\right]$$ ■

Example: Find the derivative of

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Sol'n: $\left[\frac{(x^5+2x-3)(2x^7-x^4+1)^{100}}{x^3-2}\right] \times$

$$\left(\left[\frac{5x^4+2}{x^5+2x-3}\right] + 100\left[\frac{14x^6-4x^3}{2x^7-x^4+1}\right] - \left[\frac{3x^2}{x^3-2}\right]\right)$$ ■

(except if $x^5+2x-3=0$
or if $2x^7-x^4+1=0$)

$$\frac{d}{dx}[f(x)] \underset{f(x)\neq 0}{=} [f(x)]\left[\frac{d}{dx}[\text{la}(f(x))]\right]$$

EXAMPLE: Find $\frac{d}{dx}\left[\mathrm{la}\left(\frac{2x+4}{\sqrt{x^3+5}}\right)\right]$.

Find $\frac{d}{dx}\left(\boxed{\frac{2x+4}{\sqrt{x^3+5}}}\right)$.

nonzero, because $x > -\sqrt[3]{5}$

Assume $x > -\sqrt[3]{5}$.

$$\frac{d}{dx}\left[\mathrm{la}\left(\frac{2x+4}{\sqrt{x^3+5}}\right)\right] = \left[\frac{2}{2x+4}\right] - \frac{1}{2}\left[\frac{3x^2}{x^3+5}\right]$$

$$\frac{d}{dx}\left(\frac{2x+4}{\sqrt{x^3+5}}\right) = \left(\frac{2x+4}{\sqrt{x^3+5}}\right)\left(\left[\frac{2}{2x+4}\right] - \frac{1}{2}\left[\frac{3x^2}{x^3+5}\right]\right)$$ ■

(no exceptions for $x > -\sqrt[3]{5}$)

EXAMPLE: Differentiate $G(t) = \ln \sqrt[3]{\dfrac{25 - t^2}{25 + t^2}}$.

$$G'(t) = \frac{1}{3}\left[\frac{-2t}{25 - t^2} - \frac{2t}{25 + t^2}\right]$$ ■

(untrue if $t > 5$)

(also untrue if $t < -5$)

NOTE: $$\frac{d}{dt}\left[\mathrm{la}\sqrt[3]{\frac{25 - t^2}{25 + t^2}}\right] = \frac{1}{3}\left[\frac{-2t}{25 - t^2} - \frac{2t}{25 + t^2}\right]$$

(la is better)

EXAMPLE: Use logarithmic differentiation to find the derivative of $y = \boxed{\sqrt[7]{\dfrac{x^4+2}{x^4+8}}}.\longleftarrow$ never zero

---

$$\frac{dy}{dx} = \sqrt[7]{\frac{x^4+2}{x^4+8}}\left(\frac{1}{7}\left[\frac{4x^3}{x^4+2} - \frac{4x^3}{x^4+8}\right]\right) \quad ■$$

(No exceptions.)

EXAMPLE: Use logarithmic differentiation to find the derivative of $y = \left[\sqrt[4]{x}\right]\boxed{\left[e^{x^6}\right]}\left[(x^3+2)^7\right]$.

log. deriv.?

$e > 0$, so e^{x^6} is always > 0

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undefined at $x = 0$

undefined at $x = 0$

$$\boxed{\frac{dy}{dx}} = \left[\sqrt[4]{x}\right]\left[e^{x^6}\right]\left[(x^3+2)^7\right]\left(\frac{1}{4}\cdot\boxed{\frac{1}{x}} + 6x^5 + 7\frac{3x^2}{x^3+2}\right) \blacksquare$$

exceptions?

$$\frac{d}{dx}\left[\mathrm{la}\left(e^{x^6}\right)\right] = \frac{d}{dx}\left[\ln\left(e^{x^6}\right)\right] = \frac{d}{dx}\left[x^6\right] = 6x^5$$

no exception

$$y = 0 \Rightarrow [(\underline{x = 0}) \text{ or } (\underline{x = -\sqrt[3]{2}})]$$

$\left[\sqrt[4]{x}\right]\left[e^{x^6}\right]\left[(x^3+2)^7\right]$ is undefined on $x < 0$.

$y = \left[\sqrt[4]{x}\right]\left[e^{x^6}\right]\left[(x^3+2)^7\right]$ is not diff. at $x = 0$.

EXAMPLE: Use logarithmic differentiation to find the derivative of $y = \left[\boxed{\sqrt[4]{x}}\right]\left[e^{x^6}\right]\left[(x^3+2)^7\right]$.

undefined at $x = -\sqrt[3]{2}$

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$$\boxed{\frac{dy}{dx}} = \left[\boxed{\sqrt[4]{x}}\right]\left[e^{x^6}\right]\left[(x^3+2)^7\right]\left(\frac{1}{4}\cdot\frac{1}{x} + 6x^5 + 7\frac{3x^2}{x^3+2}\right) \blacksquare$$

exceptions?

(no exceptions)

$$y = 0 \Rightarrow \left[\overset{\text{no exception}}{(x = 0)} \text{ or } \overset{\text{no exception}}{(\underline{x = -\sqrt[3]{2}})}\right]$$

$\left[\sqrt[4]{x}\right]\left[e^{x^6}\right]\left[(x^3+2)^7\right]$ is undefined on $x < 0$.

$y = \left[\sqrt[4]{x}\right]\left[e^{x^6}\right]\left[(x^3+2)^7\right]$ is not diff. at $x = 0$.

EXAMPLE: Differentiate $y = \dfrac{\boxed{x^{4/5}}\sqrt{x^2+8}}{(3x+7)^5}$.

no exception

$(y = 0 \Rightarrow x = 0)$

Common sol'n:

$$\frac{d}{dx} \longrightarrow \ln y = \tfrac{4}{5}\ln x + \tfrac{1}{2}\ln(x^2+8) - 5\ln(3x+7)$$

$$\frac{d}{dx} \qquad \frac{1}{y}\,\frac{dy}{dx} = \frac{4}{5}\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2+8} - 5\cdot\frac{3}{3x+7}$$

$$\frac{4}{5}\boxed{x^{-1/5}}$$

undefined at $x = 0$

$$\frac{dy}{dx} = y\left(\frac{4}{5}\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2+8} - 5\cdot\frac{3}{3x+7}\right)$$

$$\frac{dy}{dx} = \frac{x^{4/5}\sqrt{x^2+8}}{(3x+7)^5}\left(\frac{4}{5}\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2+8} - 5\cdot\frac{3}{3x+7}\right) \blacksquare$$

(argument not valid if $x \leq 0$)

My sol'n:

$$\boxed{\frac{dy}{dx}} = \frac{x^{4/5}\sqrt{x^2+8}}{(3x+7)^5}\left(\frac{4}{5}\cdot\frac{\boxed{}}{\boxed{x}} + \frac{1}{2}\cdot\frac{\boxed{}}{x^2+8} - 5\cdot\frac{\boxed{}}{3x+7}\right) \blacksquare$$

undefined at $x = 0$ (no exceptions) exceptions?

EXAMPLE: Find the derivative of $y = (5+\cos x)^{\ln x}$.

Assume $x > 0$.

$$\frac{dy}{dx} = \left[(5+\cos x)^{\ln x}\right]\left[\frac{d}{dx}\left[\text{la}\left((5+\cos x)^{\boxed{\ln x}}\right)\right]\right]$$

$$= \left[(5+\cos x)^{\ln x}\right]\left[\frac{d}{dx}\left[(\ln x)\,(\text{la}(5+\cos x))\right]\right]$$

$$= \left[(5+\cos x)^{\ln x}\right]\left[\left(\frac{1}{x}\right)(\boxed{\text{la}}(5+\cos x)) + (\ln x)\left(\frac{-\sin x}{5+\cos x}\right)\right]$$

$$= \left[(5+\cos x)^{\ln x}\right]\left[\left(\frac{1}{x}\right)(\boxed{\ln}(5+\cos x)) + (\ln x)\left(\frac{-\sin x}{5+\cos x}\right)\right]$$

🟩

(no exceptions for $x > 0$)

(also no exceptions for $x \le 0$)

EXAMPLE: Find the derivative of $y = (\ln x)^{\sin^2 x}$, $\underline{x > 1}$.

$$\frac{dy}{dx} = \left[(\ln x)^{\sin^2 x}\right]\left[\frac{d}{dx}\left[\mathrm{la}\left((\ln x)^{\boxed{\sin^2 x}}\right)\right]\right]$$

$(\ln x)^{\sin^2 x}$ exists?

☺

$$= \left[(\ln x)^{\sin^2 x}\right]\left[\frac{d}{dx}\left[\left(\sin^2 x\right)(\mathrm{la}(\ln x))\right]\right]$$

$$= \left[(\ln x)^{\sin^2 x}\right]\left[\left(\sin^{\boxed{2}} x\right)\left(\frac{1/x}{\ln x}\right) + (2(\sin x)(\cos x))\,(\boxed{\mathrm{la}}(\ln x))\right]$$

$$= \left[(\ln x)^{\sin^2 x}\right]\left[\left(\sin^2 x\right)\left(\frac{1/x}{\ln x}\right) + (2(\sin x)(\cos x))\,(\boxed{\ln}(\ln x))\right]$$

■

(no exceptions for $x > 1$)

(exercise: $x \leq 1$)

$\ln 1 = 0$
$\ln x > 0$

EXAMPLE:

Find the derivative of $y = \left(3+\sqrt{2x^2+x^4}\right)^x$.

$$\frac{dy}{dx} = \left[\left(3+\sqrt{2x^2+x^4}\right)^x\right]\left[\frac{d}{dx}\left(\mathrm{la}\left[\left(3+\sqrt{2x^2+x^4}\right)^{\boxed{x}}\right]\right)\right]$$

$$= \left[\left(3+\sqrt{2x^2+x^4}\right)^x\right]\left[\frac{d}{dx}\left(x\left[\mathrm{la}\left(3+\sqrt{2x^2+x^4}\right)\right]\right)\right]$$

$$= \left[\left(3+\sqrt{2x^2+x^4}\right)^x\right]\left[x\left(\frac{\frac{2x+2x^3}{\sqrt{2x^2+x^4}}}{3+\sqrt{2x^2+x^4}}\right)+(\cancel{1})\left(\mathrm{la}\left(3+\sqrt{2x^2+x^4}\right)\right)\right]$$

$$\frac{d}{dx}\left(3+\sqrt{2x^2+x^4}\right) = \frac{d}{dx}\left(3+(2x^2+x^4)^{1/2}\right)$$

$$= \boxed{\xcancel{0+}}\,\frac{1}{2}(2x^2+x^4)^{\boxed{-1/2}}(4x+4x^3)$$

$$= \cancel{\frac{1}{2}}\boxed{\frac{1}{\sqrt{2x^2+x^4}}}\left(\overset{2}{\cancel{4}}x+\overset{2}{\cancel{4}}x^3\right) = \frac{2x+2x^3}{\sqrt{2x^2+x^4}}$$

EXAMPLE:

Find the derivative of $y = \left(3+\sqrt{2x^2+x^4}\right)^x$.

$$\frac{dy}{dx} = \left[\left(3+\sqrt{2x^2+x^4}\right)^x\right]\left[\frac{d}{dx}\left(\mathrm{la}\left[\left(3+\sqrt{2x^2+x^4}\right)^x\right]\right)\right]$$

$$= \left[\left(3+\sqrt{2x^2+x^4}\right)^x\right]\left[\frac{d}{dx}\left(x\left[\mathrm{la}\left(3+\sqrt{2x^2+x^4}\right)\right]\right)\right]$$

$$=\left[\left(3+\sqrt{2x^2+x^4}\right)^x\right]\left[x\left(\frac{\frac{2x+2x^3}{\sqrt{2x^2+x^4}}}{3+\sqrt{2x^2+x^4}}\right)+(\cancel{1})\left(\boxed{\mathrm{la}}\left(3+\sqrt{2x^2+x^4}\right)\right)\right]$$

$$=\left[\left(3+\sqrt{2x^2+x^4}\right)^x\right]\left[x\left(\frac{\frac{4x+4x^3}{2\sqrt{2x^2+x^4}}}{3+\sqrt{2x^2+x^4}}\right)+\left(\boxed{\mathrm{ln}}\left(3+\sqrt{2x^2+x^4}\right)\right)\right] ■$$

(no exceptions)

SKILL
log properties

Whitman problems

§4.6, p. 74, #1-2,5

SKILL
deriv log

Whitman problems

§4.6, p. 74, #6-7

SKILL
log diff

Whitman problems

§4.7, p. 80, #1,6,9,15,19

SKILL
deriv w/ log

Whitman problems

§4.7, p. 80, #11-14,16-18,21

SKILL
gphs of log exprs

Whitman problems

§4.6, p. 74, #3-4,8,12-18

SKILL
solve log exprs

Whitman problems

§4.6, p. 74, #9-11

SKILL
deriv w/ exp

Whitman problems

§4.7, p. 80, #2-5,7-8,10

SKILL
tan line

Whitman problems

§4.7, p. 80, #20

STOP