CALCULUS
Indeterminate forms

"$1/(0^+) = \infty$"

"$1/(0^-) = -\infty$"

"$e^\infty = \infty$" "$e^{-\infty} = 0$"

"$\ln(\infty) = \infty$"

"$\sqrt{\infty} = \infty$"

"$(0^+)^\infty = 0$"

"$1^{\pm\infty}$ is indet."

"$(0^+)^0$, ∞^0 indet."

"$(-\infty) + \infty$ is indeterminate"

"$(-\infty) + (-\infty) = -\infty$"

"$\infty + \infty = \infty$"

"$\infty - \infty$ is indeterminate"

"$c > 0 \Rightarrow c \cdot (-\infty) = -\infty$"

"$c < 0 \Rightarrow c \cdot (-\infty) = \infty$"

"$c > 0 \Rightarrow c \cdot \infty = \infty$"

"$c < 0 \Rightarrow c \cdot \infty = -\infty$"

"$0 \cdot \infty$ and $0 \cdot (-\infty)$ are indeterminate"

"$0/0$, ∞/∞, $(-\infty)/\infty$, $\infty/(-\infty)$, $(-\infty)/(-\infty)$ all indeterminate"

"l'Hôpital indeterminate forms"

"$\infty \cdot \infty = \infty$"

"$\infty \cdot (-\infty) = -\infty$"

"$(-\infty) \cdot \infty = -\infty$"

"$(-\infty) \cdot (-\infty) = \infty$"

"$\infty + c = \infty$"

"$(-\infty) + c = -\infty$"

DETERMINATE AND INDETERMINATE FORMS

Spp

“$(-\infty)+\infty$ is indeterminate”

“$\infty-\infty$ is indeterminate”

“$1^{\pm\infty}$ is indet.”

“$1^{\pm\infty}$ is indet.”

“$(0^{+})^{0}$, ∞^{0} indet.”

“$0\cdot\infty$ and $0\cdot(-\infty)$ are indeterminate”

“$0/0$, ∞/∞, $(-\infty)/\infty$, $\infty/(-\infty)$, $(-\infty)/(-\infty)$ all indeterminate”

“l’Hôpital indeterminate forms”

INDETERMINATE FORMS

“$(-\infty) + \infty$ is indeterminate”

INDETERMINATE DIFFERENCES

“$\infty + (-\infty)$ is indeterminate”

“$\infty - \infty$ is indeterminate”

Then

Finally

INDETERMINATE POWERS

“$1^{\pm\infty}$ is indet.”

“$(0^+)^0$, ∞^0 indet.”

Next

INDETERMINATE PRODUCTS

“$0 \cdot \infty$ and $0 \cdot (-\infty)$ are indeterminate”

“$0/0$, ∞/∞, $(-\infty)/\infty$, $\infty/(-\infty)$, $(-\infty)/(-\infty)$ all indeterminate”

“l’Hôpital indeterminate forms”

Already discussed

INDETERMINATE PRODUCTS:

Given a problem of the form $\lim_{x\to a}[f(x)][g(x)]$,

if $\lim_{x\to a} f(x) = 0$ and if $\lim_{x\to a} g(x) = \pm\infty$

then the form is $[0][\pm\infty]$, which is indeterminate.

One approach that often works:

$$\lim_{x\to a}[f(x)][g(x)]$$

$$= \lim_{x\to a}\frac{f(x)}{1/[g(x)]} \quad \frac{0}{0} \text{ l'H?}$$

OR:

$$= \lim_{x\to a}\frac{g(x)}{1/[f(x)]} \quad \text{l'H? } \frac{\pm\infty}{\pm\infty} \text{ MAYBE}$$

Similar idea for: $\lim_{x\to a^+}$, $\lim_{x\to a^-}$, $\lim_{x\to\infty}$, $\lim_{x\to-\infty}$

INDETERMINATE PRODUCTS:

Given a problem of the form $\lim\,[f(x)][g(x)]$,

if $\lim f(x) = 0$ and if $\lim g(x) = \pm\infty$

then the form is $[0][\pm\infty]$, which is indeterminate.

One approach that often works:

$$\lim\,[f(x)][g(x)]$$

$$\lim \frac{f(x)}{1/[g(x)]} \qquad \text{OR:} \qquad \lim \frac{g(x)}{1/[f(x)]}$$

$\frac{0}{0}$ l'H?

l'H? $\frac{\pm\infty}{\pm\infty}$ MAYBE

lim stands for one of: $\lim_{x\to a}$, $\lim_{x\to a^+}$, $\lim_{x\to a^-}$, $\lim_{x\to\infty}$, $\lim_{x\to-\infty}$

cf. §4.8, EXAMPLE 4.15, p. 83: Find $\lim_{x\to 0^+}\left[x^2(\ln x)\right]$.

First step: Check the form.

$(0)(-\infty)$ INDETERMINATE PRODUCT

positive, as $x \to 0^+$

Answer: $\lim_{x\to 0^+}\left[x^2(\ln x)\right] = \lim_{x\to 0^+}\left[\frac{\ln x}{x^{-2}}\right]$ TRANSCENDENTAL

$\neq$

$\lim_{x\to 0^+}\left[\frac{x^2}{(\ln x)^{-1}}\right]$

Why not this??

tentative l'H

$= \lim_{x\to 0^+}\left[\frac{1/x}{-2x^{-3}}\right]$ RATIONAL

$= \lim_{x\to 0^+}\left[\frac{x^{\cancel{3}2}}{-2\cancel{x}}\right]$

$= \lim_{x\to 0^+}\left[\frac{x^2}{-2}\right] = 0$ ■

SKILL

$(0)(\pm\infty)$

INDETERMINATE DIFFERENCES:

lim stands for one of: $\lim\limits_{x\to a}$, $\lim\limits_{x\to a^+}$, $\lim\limits_{x\to a^-}$, $\lim\limits_{x\to\infty}$, $\lim\limits_{x\to-\infty}$

Given a problem of the form $\lim\,[f(x)] - [g(x)]$,

if $\lim f(x) = \infty$ and if $\lim g(x) = \infty$

then the form is $\infty - \infty$, which is indeterminate.

Try to do some algebra
(*e.g.*, common denominator,
rationalize numerator or denominator,
factoring out common factor)
to convert the problem to $0/0$ or $\pm\infty/\pm\infty$.

SKILL
$\infty - \infty$

$\infty + (-\infty)$

EXAMPLE: Find $\lim_{x \to \pi^-} ((\csc x) + (\cot x))$.

First step: Check the form.

$$\lim_{x \to \pi^-} \left(\frac{1}{\underset{0^+}{\sin x}} + \frac{\overset{-1}{\cos x}}{\underset{0^+}{\sin x}} \right) = \lim_{x \to \pi^-} \left(\frac{1 + \cos x}{\sin x} \right) \qquad \frac{0}{0}$$

$$\overset{\text{tentative l'H}}{=} \lim_{x \to \pi^-} \left(\frac{-\sin x}{\cos x} \right)$$

REWRITE
IN TERMS OF
$\sin x$ and $\cos x$

$$= \left[\frac{-\sin x}{\cos x} \right]_{x:\to\pi}$$

$$= \frac{-0}{-1} = 0 \blacksquare$$

INDETERMINATE POWERS:

Given a problem of the form $\lim\,[f(x)]^{[g(x)]}$ . . .

If $\lim f(x) = 0^+$ and if $\lim g(x) = 0$
then the form is $(0^+)^0$, which is indeterminate.

If $\lim f(x) = \infty$ and if $\lim g(x) = 0$
then the form is ∞^0, which is indeterminate.

If $\lim f(x) = 1$ and if $\lim g(x) = \infty$
then the form is 1^∞, which is indeterminate.

If $\lim f(x) = 1$ and if $\lim g(x) = -\infty$
then the form is $1^{-\infty}$, which is indeterminate.

If $\lim f(x) = 0^+$ and if $\lim g(x) = \infty$
then the form is $(0^+)^\infty$, which is DETERMINATE.

$$(0^+)^\infty = 0$$

INDETERMINATE POWERS:

Given a problem of the form $\lim\, [f(x)]^{[g(x)]}$ . . .

$(0^+)^0$ is indeterminate.

∞^0 is in$(0^+)^0$minate. is indeterminate.

1^∞ is indeterminate.

$1^{-\infty}$ is indeterminate.

∞^0 is indeterminate.

Standard approach to indeterminate powers:

$[f(x)]^{[g(x)]}$

1^∞ is indeterminate.

$1^{-\infty}$ is indeterminate.

INDETERMINATE POWERS:

Given a problem of the form $\lim [f(x)]^{[g(x)]}$ . . .

Colloquially:

$(0^+)^0$ is indeterminate. $\ln((0^+)^0) = 0(\ln(0^+))$

∞^0 is indeterminate. $\ln(\infty^0) = 0(\ln(\infty))$

1^∞ is indeterminate. $\ln(1^\infty) = \infty(\ln(1))$

$1^{-\infty}$ is indeterminate. $\ln(1^{-\infty}) = -\infty(\ln(1))$

Standard approach to indeterminate powers:

$$\lim \ln([f(x)]^{[g(x)]}) = \lim [g(x)][\ln(f(x))]$$

is an indeterminate product.

Do that indeterminate product problem, then exponentiate:

$$\exp(\lim \ln([f(x)]^{[g(x)]})) = \lim \exp(\ln([f(x)]^{[g(x)]}))$$

$$= \lim [f(x)]^{[g(x)]}$$

DON'T forget to exponentiate at the end!

EXAMPLE: Calculate $\lim_{x \to 0^+} (1 + (\sin(7x)))^{\cot x}$. 1^∞

$$\lim_{x \to 0^+} (1 + (\sin(7x)))^{\cot x} = e^7$$

SKILL
indet. power

$$\lim_{x \to 0^+} \ln[(1 + (\sin(7x)))^{\cot x}]$$

Next: another approach

ASYMPTOTICS? ASYMPTOTICS?

$$= \lim_{x \to 0^+} [\cot x](\ln[1 + (\sin(7x))])$$

positive, as $x \to 0^+$

$$= \lim_{x \to 0^+} \frac{\ln[1 + (\sin(7x))]}{\tan x}$$

tentative l'H

$$\stackrel{\text{l'H}}{=} \lim_{x \to 0^+} \frac{[1/[1 + (\sin(7x))]][\cos(7x)][7]}{\sec^2 x}$$

$$= \frac{[1/[1 + (0)]][1][7]}{1^2} = 7$$

DON'T forget to exponentiate at the end!

EXAMPLE: Calculate $\lim_{x\to 0^+} (1+(\sin(7x)))^{\cot x}$.

Fact: $p(x) \underset{x\to 0^+}{\sim} q(x) \underset{x\to 0^+}{\to} 0 \Rightarrow \ln[1+(p(x))] \underset{x\to 0^+}{\sim} q(x)$.

pf omitted

ln and 1+ cancel

(Works for $x\to a$, $x\to a^-$, $x\to a^+$, $x\to\infty$, $x\to -\infty$.)

$$\sin(7x) \underset{x\to 0^+}{\sim} 7x \underset{x\to 0^+}{\to} 0$$

Up to asymptotics, ln and 1+ cancel, and sin can be ignored.

$$\ln[1+(\sin(7x))] \underset{x\to 0^+}{\sim} 7x$$

$$\cot x = \frac{\cos x}{\sin x} \underset{x\to 0^+}{\sim} \frac{1}{x}$$

MULTIPLY

$\cos x \underset{x\to 0^+}{\sim} 1$

$\sin x \sim x$

$$(\cot x)(\ln[1+(\sin(7x))]) \underset{x\to 0^+}{\sim} 7 \underset{x\to 0^+}{\to} 7$$

$$\| $$

$$\ln[(1+(\sin(7x)))^{\cot x}]$$

EXAMPLE: Calculate $\lim_{x\to 0^+} (1+(\sin(7x)))^{\cot x}$.

$$\ln\left[(1+(\sin(7x)))^{\cot x}\right] \underset{x \to 0^+}{\longrightarrow} 7$$

$$(\cot x)(\ln[1+(\sin(7x))]) \underset{x \to 0^+}{\sim} 7 \underset{x \to 0^+}{\longrightarrow} \overline{7}$$

$$||$$

$$\ln\left[(1+(\sin(7x)))^{\cot x}\right]$$

§4.8

EXAMPLE: Calculate $\lim_{x \to 0^+} (1 + (\sin(7x)))^{\cot x}$.

$$\ln\left[(1 + (\sin(7x)))^{\cot x}\right] \underset{x \to 0^+}{\longrightarrow} 7$$

$$(1 + (\sin(7x)))^{\cot x} \underset{x \to 0^+}{\longrightarrow} e^7 \blacksquare$$

SKILL
indet. power

DON'T forget to exponentiate at the end!

cf. §4.8, EXAMPLE 4.15, p. 83: Find $\lim_{x \to 0^+} \left[x^2 (\ln x)\right]$.

$$\lim_{x \to 0^+} \left[x^2 (\ln x)\right] = 0 \quad \blacksquare$$

EXAMPLE: Find $\lim_{x \to 0^+} x^{(x^2)}$.

$$\ln \left[x^{(x^2)}\right] = \left(x^2\right) (\ln[x]) \to 0, \text{ as } x \to 0^+$$

$$\lim_{x \to 0^+} x^{(x^2)} = e^0 = 1 \quad \blacksquare$$

SKILL
indet. power

DON'T forget to exponentiate at the end!

EXAMPLE: Find $\lim_{x\to 0} \frac{\sin(5x)}{\tan(9x)}$.

$$\lim_{x\to 0} \frac{\sin(5x)}{\tan(9x)} \overset{\text{tentative l'H}}{=} \lim_{x\to 0} \frac{[\cos(5x)][5]}{[\sec^2(9x)][9]}$$

REWRITE IN TERMS OF $\sin x$ and $\cos x$

$$= \lim_{x\to 0} \frac{[\cos(5x)][5]}{[1/(\cos^2(9x))][9]}$$

$$= \left[\frac{[\cos(5x)][5]}{[1/(\cos^2(9x))][9]}\right]_{x:\to 0}$$

$$= \frac{[1][5]}{[1/1][9]} = \frac{5}{9} \blacksquare$$

Next: another approach

EXAMPLE: Find $\lim_{x\to 0} \frac{\sin(5x)}{\tan(9x)}$.

SKILL
general limits

$5x \to 0$, as $x \to 0$

$x :\to 5x$

$\sin x \underset{x\to 0}{\sim} x$ | $\sin(5x) \underset{x\to 0}{\sim} 5x$

DIVIDE

$\cos x \underset{x\to 0}{\sim} 1$

DIVIDE

$\tan x \underset{x\to 0}{\sim} x$ | $\tan(9x) \underset{x\to 0}{\sim} 9x$

$x :\to 9x$

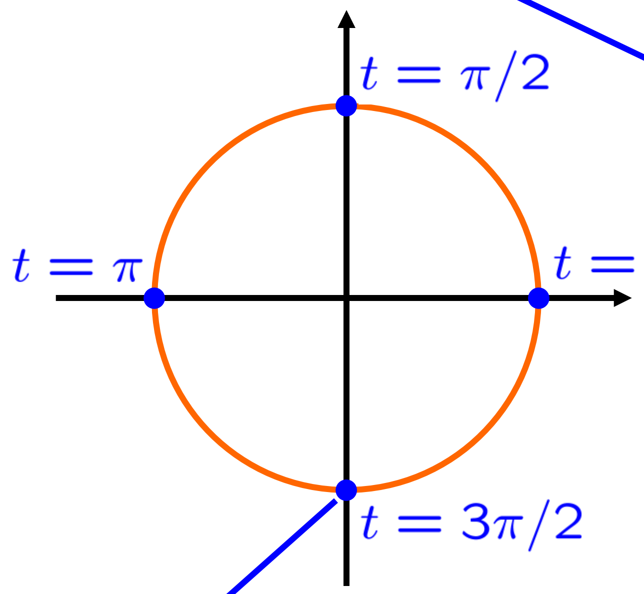
$9x \to 0$, as $x \to 0$

$$\frac{\sin(5x)}{\tan(9x)} \underset{x\to 0}{\sim} \frac{5}{9} \underset{x\to 0}{\to} \frac{5}{9}$$

EXAMPLE: Find $\lim_{\theta \to 3\pi/2} \frac{1 - \sin\theta}{\csc\theta}$. DETERMINATE FORM

SKILL general limits

$$\left[\frac{1 - \sin\theta}{\csc\theta}\right]_{\theta :\to 3\pi/2} = \frac{1 - (-1)}{-1} = -2 \quad \blacksquare$$



$$(\cos(3\pi/2), \sin(3\pi/2)) = (0, -1)$$

$$\sin(3\pi/2) = -1$$

$$\csc(3\pi/2) = \frac{1}{\sin(3\pi/2)} = \frac{1}{-1} = -1$$

EXAMPLE: Find $\lim_{x\to 2}\left[\frac{\ln(x/2)}{\sin(\pi x)}\right]$. $\frac{0}{0}$

SKILL
general limits

positive, as $x \to 2$

$$\lim_{x\to 2}\left[\frac{\ln(x/2)}{\sin(\pi x)}\right] \overset{\text{tentative l'H}}{=} \lim_{x\to 2}\left[\frac{[1/(x/2)][1/2]}{[\cos(\pi x)][\pi]}\right]$$

$$= \left[\frac{[1/(x/2)][1/2]}{[\cos(\pi x)][\pi]}\right]_{x:\to 2}$$

$\cos(2\pi) = \cos(0) = 1$

$$= \frac{[1/(2/2)][1/2]}{[\cos(2\pi)][\pi]}$$

$$= \frac{[1/1][1/2]}{[1][\pi]} = \frac{1}{2\pi}$$ ■

Next: another approach

EXAMPLE: Find $\lim_{x\to 2}\left[\frac{\ln(x/2)}{\sin(\pi x)}\right]$. $\frac{0}{0}$

SKILL
general limits

$$\lim_{x\to 2}\left[\frac{\ln(x/2)}{\sin(\pi x)}\right] = \lim_{t\to 0}\left[\frac{\ln[1+(t/2)]}{\sin(2\pi+\pi t)}\right]$$

sin is 2π-periodic

$$= \lim_{t\to 0}\left[\frac{\ln[1+(t/2)]}{\sin(\pi t)}\right]$$

ASYMPTOTICS?
ASYMPTOTICS?

$$= \lim_{t\to 0}\left[\frac{t/2}{\pi t}\right] = \lim_{t\to 0}\left[\frac{1/2}{\pi}\right]$$

$x :\to 2+t$

$(x\to 2) \;:\to\; (t\to 0)$

$x/2 :\to 1+(t/2)$

$\pi x :\to 2\pi+\pi t$

Fact: $p(t)\sim q(t)\to 0 \quad\Rightarrow\quad \ln[1+(p(t))]\sim q(t)$.

$t/2 \underset{t\to 0}{\sim} t/2 \underset{t\to 0}{\to} 0$

$\ln[1+(t/2)] \underset{t\to 0}{\sim} t/2$

$\sin u \underset{u\to 0}{\sim} u$

$\sin(\pi t) \underset{t\to 0}{\sim} \pi t$

EXAMPLE: Find $\lim_{x\to 2}\left[\dfrac{\ln(x/2)}{\sin(\pi x)}\right]$.

$\dfrac{0}{0}$

SKILL
general limits

$$\lim_{x\to 2}\left[\frac{\ln(x/2)}{\sin(\pi x)}\right] = \lim_{t\to 0}\left[\frac{\ln[1+(t/2)]}{\sin(2\pi+\pi t)}\right]$$

$$= \lim_{t\to 0}\left[\frac{\ln[1+(t/2)]}{\sin(\pi t)}\right]$$

$$= \lim_{t\to 0}\left[\frac{1\cancel{t}/2}{\pi\cancel{t}}\right] = \lim_{t\to 0}\left[\frac{1/2}{\pi}\right]$$

$$= \frac{1/2}{\pi} = \frac{1}{2\pi} \blacksquare$$

EXAMPLE: Find $\lim_{x\to-\infty} x^3e^x$. $(-\infty)\cdot 0$

$$\lim_{x\to-\infty} x^3e^x = \lim_{x\to-\infty} \frac{x^3}{e^{-x}} \qquad \frac{-\infty}{\infty}$$

$$\overset{\text{tentative l'H}}{=} \lim_{x\to-\infty} \frac{3x^2}{-e^{-x}} \qquad \frac{\infty}{-\infty}$$

$$\overset{\text{tentative l'H}}{=} \lim_{x\to-\infty} \frac{6x}{e^{-x}} \qquad \frac{-\infty}{\infty}$$

$$\overset{\text{tentative l'H}}{=} \lim_{x\to-\infty} \frac{6}{-e^{-x}} \qquad \frac{6}{-\infty}$$

DETERMINATE

$$= \quad 0$$

EXAMPLE: Find $\lim_{x\to\pi/4} (1 - \cot x)(\csc x)$.

DETERMINATE FORM

$$\lim_{x\to\pi/4} (1 - \cot x)(\csc x) = \lim_{x\to\pi/4} \frac{1 - \cot x}{\sin x}$$

REWRITE IN TERMS OF
$\sin x$ and $\cos x$

$$= \lim_{x\to\pi/4} \frac{1 - [(\cos x)/(\sin x)]}{\sin x}$$

$$= \left[\frac{1 - [(\cos x)/(\sin x)]}{\sin x}\right]_{x:\to\pi/4}$$

$\sin(\pi/4) = \sqrt{2}/2 = \cos(\pi/4)$

$$= \frac{1 - [(\sqrt{2}/2)/(\sqrt{2}/2)]}{\sqrt{2}/2}$$

$$= \frac{1-1}{\sqrt{2}/2} = 0$$ ■

EXAMPLE: Find $\lim_{x \to \pi/2} [(\sec x) - (\tan x)]$.

$\infty - \infty$, as $x \to (\pi/2)^-$

$(-\infty) - (-\infty)$, as $x \to (\pi/2)^+$

$$\lim_{x \to \pi/2} [(\sec x) - (\tan x)] = \lim_{x \to \pi/2} \left[\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right]$$

REWRITE IN TERMS OF
$\sin x$ and $\cos x$

$$= \lim_{x \to \pi/2} \frac{1 - \sin x}{\cos x} \qquad \frac{0}{0}$$

$\cos(\pi/2) = 0$

$\sin(\pi/2) = 1$

tentative l'H

$$\stackrel{\text{tentative l'H}}{=} \lim_{x \to \pi/2} \frac{+\cos x}{+\sin x}$$

$$= \left[\frac{\cos x}{\sin x}\right]_{x :\to \pi/2} = \frac{0}{1} = 0 \;\blacksquare$$

EXAMPLE: Find $\lim_{x\to\infty} x^{5/[7-(\ln x)]}$. ∞^0

$$\lim_{x\to\infty}\left[\ln\left(x^{5/[7-(\ln x)]}\right)\right] = \lim_{x\to\infty}\frac{5}{7-(\ln x)}[\ln(x)]$$

positive, as $x \to \infty$

$$= \lim_{x\to\infty}\frac{5[\ln x]}{7-(\ln x)} \qquad \frac{\infty}{-\infty}$$

tentative l'H

$$\stackrel{\text{l'H}}{=} \lim_{x\to\infty}\frac{5[1/x]}{-(1/x)1}$$

$$= \lim_{x\to\infty}\frac{5}{-1} = -5$$

$$\lim_{x\to\infty} x^{5/[7-(\ln x)]} = \exp\left[\lim_{x\to\infty}\left[\ln\left(x^{5/[7-(\ln x)]}\right)\right]\right]$$

$$= e^{-5}$$ ■

EXAMPLE: Find $\lim_{x\to\infty} (e^{2x}+x^3)^{1/x}$. ∞^0

SKILL
general limits

$$\lim_{x\to\infty}\left[\ln\left((e^{2x}+x^3)^{1/x}\right)\right] = \lim_{x\to\infty}\frac{1}{x}\left[\ln\left(e^{2x}+x^3\right)\right]$$

LOG DERIV

positive, as $x\to\infty$

$$= \lim_{x\to\infty}\frac{\ln\left(e^{2x}+x^3\right)}{x} \qquad \frac{\infty}{\infty}$$

$$\overset{\text{tentative l'H}}{=} \lim_{x\to\infty}\frac{(2e^{2x}+3x^2)/(e^{2x}+x^3)}{1} = \lim_{x\to\infty}\frac{2e^{2x}+3x^2}{e^{2x}+x^3} \qquad \frac{\infty}{\infty}$$

$$\overset{\text{tentative l'H}}{=} \lim_{x\to\infty}\frac{4e^{2x}+6x}{2e^{2x}+3x^2} \overset{\text{tentative l'H}}{=} \lim_{x\to\infty}\frac{8e^{2x}+6}{4e^{2x}+6x} \overset{\text{tentative l'H}}{=} \lim_{x\to\infty}\frac{16e^{2x}}{8e^{2x}+6}$$

$$\overset{\text{tentative l'H}}{=} \lim_{x\to\infty}\frac{32e^{2x}}{16e^{2x}} = \lim_{x\to\infty}\frac{32}{16} = 2$$

$$\lim_{x\to\infty}(e^{2x}+x^3)^{1/x} = \exp\left[\lim_{x\to\infty}\left[\ln\left((e^{2x}+x^3)^{1/x}\right)\right]\right]$$

$$= e^{2?}$$ ■

EXAMPLE: Find $\lim_{x\to 0^+} (\cos x)^{1/\sqrt{x}}$. 1^∞

$$?? = \lim_{x\to 0^+} \frac{1}{\sqrt{x}}[\ln(\cos x)]$$

LOG DERIV

positive, as $x \to 0^+$

$$= \lim_{x\to 0^+} \frac{\ln(\cos x)}{x^{1/2}} \qquad \frac{0}{0}$$

tentative l'H

$$= \lim_{x\to 0^+} \frac{(-\sin x)/(\cos x)}{(1/2)x^{-1/2}}$$

$$= \lim_{x\to 0^+} \frac{(-\sin x)(x^{1/2})}{(1/2)(\cos x)}$$

$$= \lim_{x\to 0^+} \frac{2(-\sin x)(\sqrt{x})}{\cos x}$$

$$= \left[\frac{2(-\sin x)(\sqrt{x})}{\cos x}\right]_{x:\to 0} = \frac{2(-0)(0)}{1} = 0$$

$$e^{??} = e^0 = 1$$

SKILL
general limits

Whitman problems
§4.8, p. 84, #1-6

SKILL
horizontal asymptotes

Whitman problems
§4.8, p. 84, #7

STOP