CALCULUS
Graphing problems

precalculus

labor saving device

A. Symmetry

(i) even function: $f(-x) = f(x)$
(ii) odd function: $f(-x) = -(f(x))$
(iii) periodic function: $f(x+p) = f(x)$

B. Intervals of Positivity or Negativity, and

(i) domain
(ii) x,y-intercepts
(iii) vertical, horizontal asymptotes

f

limits

C. Intervals of Increase or Decrease f'

D. Concavity and Points of Inflection f''

calculus

EXAMPLE: Use the checklist to sketch the curve $y = \dfrac{3x^2}{x^2 - 4}$.

A. Symmetry (over $[0, \infty)$; reflect thru y-axis)
even (y-axis symmetry)

(i) even function: $f(-x) = f(x)$
(ii) odd function: $f(-x) = -(f(x))$
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EXAMPLE: Use the checklist to sketch the curve $y = \frac{3x^2}{x^2 - 4}$.

A. Symmetry (over $[0, \infty)$; reflect thru y-axis) even (y-axis symmetry)

B. Intervals of Positivity or Negativity, and neg$(0, 2)$ pos$(2, \infty)$

(i) domain $\supseteq [0, \infty) \setminus \{2\}$

$\bullet(0, 0)$ (ii) x,y-intercepts

(iii) vertical, horizontal asymptotes $\bullet(2, -\infty | \infty)$ $\bullet(\infty, 3)$

$$y = \frac{3x^2}{x^2 - 4} = \frac{3x^2}{(x + 2)(x - 2)}$$

	0		2	
y	0^2	neg	U	pos$\rightarrow$3

EXAMPLE: Use the checklist to sketch the curve $y = \dfrac{3x^2}{x^2-4}$.

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$\bullet(0,0)$ (ii) x,y-intercepts

(iii) vertical, horizontal asymptotes $\bullet(2,-\infty|\infty)$ $\bullet(\infty,3)$

C. Intervals of Increase or Decrease

$\downarrow [0,2)$, $\downarrow (2,\infty)$

D. Concavity and Points of Inflection

$$\frac{dy}{dx} = \frac{(x^2-4)(6x)-(3x^2)(2x)}{(x^2-4)^2} = \frac{-24x}{(x-2)^2(x+2)^2}$$

	0		2	
dy/dx	0	neg	U^2	neg

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C. Intervals of Increase or Decrease

$\downarrow [0, 2)$, $\downarrow (2, \infty)$

D. Concavity and Points of Inflection

$$\frac{dy}{dx} = \frac{(x^2 - 4)(6x) - (3x^2)(2x)}{(x^2 - 4)^2} = \frac{-24x}{(x - 2)^2 (x + 2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2 - 4)^{\cancel{2}}(-24) - (-24x)(2\cancel{(x^2 - 4)}(2x))}{(x^2 - 4)^{\cancel{4}3}}$$

$$= \frac{(x^2 - 4)(-24) + (24x)(2(2x))}{(x^2 - 4)^3} = \frac{24[3x^2 + 4]}{(x^2 - 4)^3}$$

EXAMPLE: Use the checklist to sketch the curve $y = \dfrac{3x^2}{x^2-4}$.

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$\downarrow[0,2)$, $\downarrow(2,\infty)$

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$$\frac{d^2y}{dx^2} = \frac{24[3x^2+4]}{(x^2-4)^3} = \frac{24[3x^2+4]}{(x-2)^3(x+2)^3}$$

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D. Concavity and Points of Inflection

$\cap[0,2)$, $\cup(2,\infty)$

$$\frac{d^2y}{dx^2} = \frac{24[3x^2+4]}{(x^2-4)^3} = \frac{24[3x^2+4]}{(x-2)^3(x+2)^3}$$

d^2y/dx^2	neg	0	neg	2	pos
				U^3	

EXAMPLE: Use the checklist to sketch the curve $y = \dfrac{3x^2}{x^2-4}$.

(over $[0,\infty)$; reflect thru y-axis)

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domain $\supseteq [0,\infty)\backslash\{2\}$

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$\rightarrow$ $\rightarrow$

$\bullet(0,0),0)$

$\bullet(2,-\infty|\infty)$

$\bullet(\infty,3)$

neg$(0,2)$

pos$(2,\infty)$

neg$(0,2)$

pos$(2,\infty)$

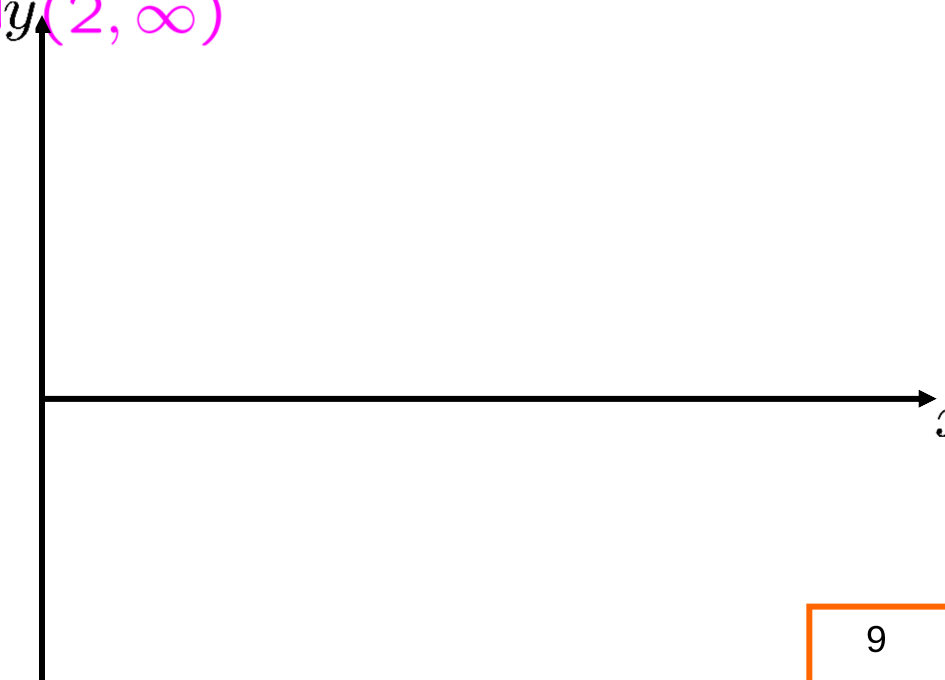
$\downarrow[0,2)$, $\downarrow(2,\infty)$

$\bullet(2,-\infty|\infty)$ $\bullet(\infty,3)$

$\cap[0,2)$, $\cup(2,\infty)$

$\downarrow[0,2)$, $\downarrow(2,\infty)$

$\cap[0,2)$, $\cup(2,\infty)$



$y = \dfrac{3x^2}{x^2-4}$

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(over $[0, \infty)$; reflect thru y-axis)

domain $\supseteq [0, \infty) \backslash \{2\}$

$\rightarrow$

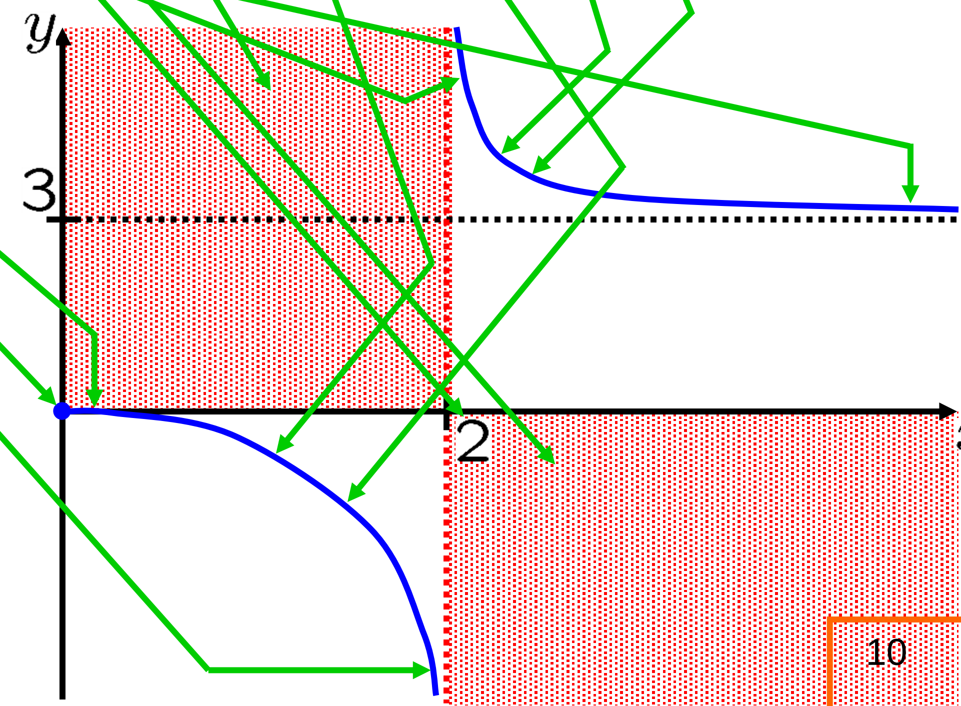
- $\bullet (0, 0)$
- $\bullet (2, -\infty | \infty)$
- $\bullet (\infty, 3)$

neg$(0, 2)$

pos$(2, \infty)$

$\downarrow [0, 2)$, $\downarrow (2, \infty)$

$\cap [0, 2)$, $\cup (2, \infty)$



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$\overrightarrow{\bullet(0,0)}$

$\bullet(2, -\infty | \infty)$

$\bullet(\infty, 3)$

neg$(0, 2)$

pos$(2, \infty)$

$\downarrow [0, 2)$, $\downarrow (2, \infty)$

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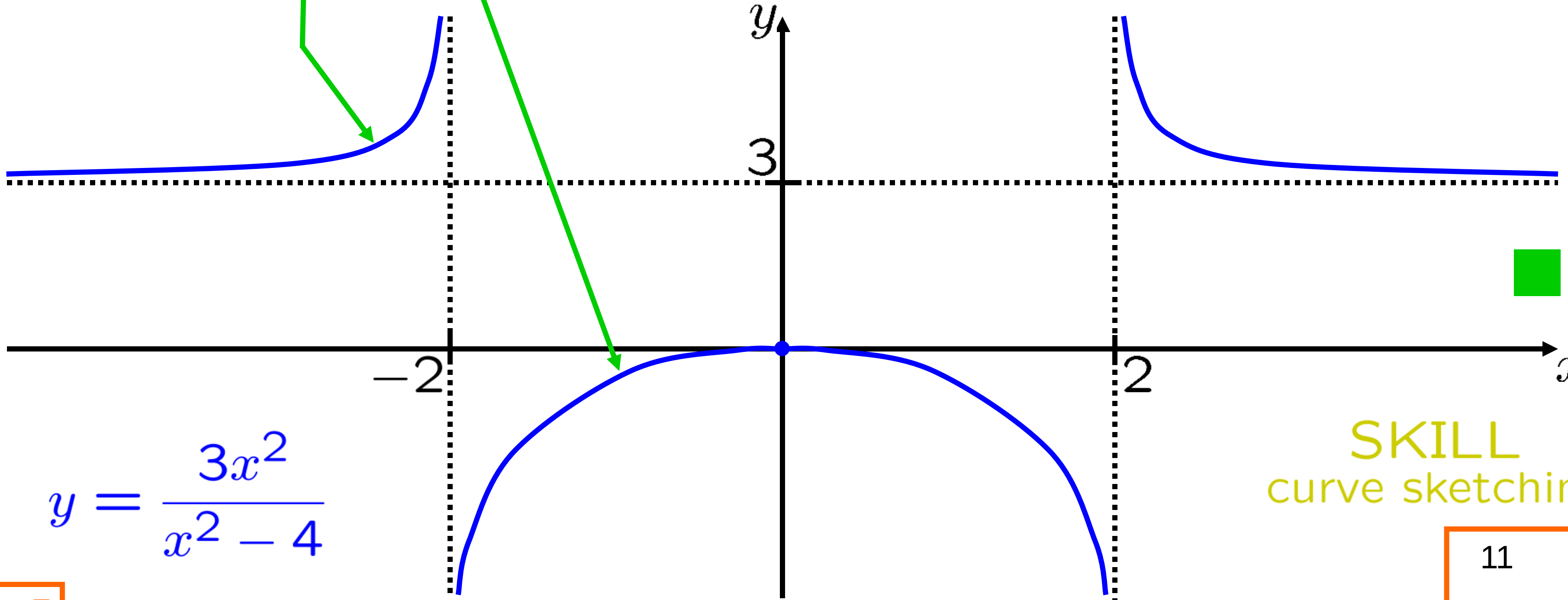

$y = \frac{3x^2}{x^2 - 4}$

SKILL

curve sketching

EXAMPLE: Sketch the graph of $y = \frac{2x^2}{\sqrt{x+3}}$.

A. Symmetry no symm

(i) even function: $f(-x) = f(x)$
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EXAMPLE: Sketch the graph of $y = \dfrac{2x^2}{\sqrt{x+3}}$. $\bullet(\infty, \infty)$

A. Symmetry no symm

B. Intervals of Positivity or Negativity, and

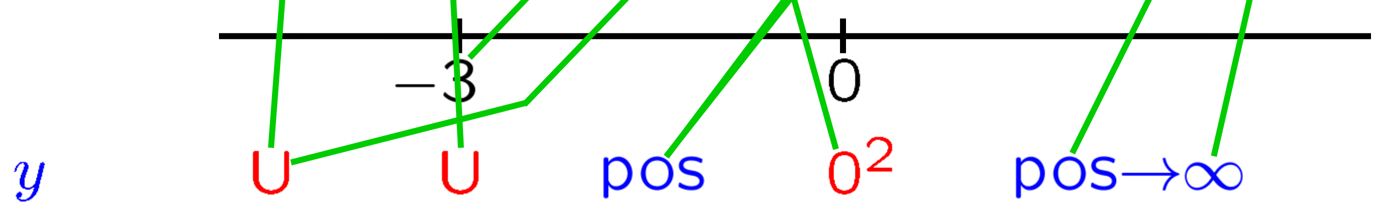
(i) domain $(-3, \infty)$

(ii) x,y-intercepts $\bullet(0,0)$

(iii) vertical, horizontal asymptotes $\bullet(-3, \text{U}|\infty)$

pos$(-3,0),(0,\infty)$

C. Intervals of Increase or Decrease



EXAMPLE: Sketch the graph of $y = \frac{2x^2}{\sqrt{x+3}}$.

$\bullet(\infty, \infty)$

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(ii) x,y-intercepts $\bullet(0, 0)$

(iii) vertical, horizontal asymptotes $\bullet(-3, \mathrm{U}|\infty)$

C. Intervals of Increase or Decrease

$$\frac{dy}{dx} = \frac{(x+3)^{1/2}(4x) - (2x^2)((1/2)(x+3)^{-1/2})}{(x+3)(x+3)^{1/2}}$$

$$= \frac{(x+3)(4x) - (x^2)}{(x+3)^{3/2}} = \frac{3x^2 + 12x}{(x+3)^{3/2}}$$

EXAMPLE: Sketch the graph of $y = \dfrac{2x^2}{\sqrt{x+3}}$. $\bullet(\infty, \infty)$

A. Symmetry no symm

B. Intervals of Positivity or Negativity, and

(i) domain $(-3, \infty)$ pos$(-3, 0), (0, \infty)$

(ii) x,y-intercepts $\bullet(0, 0)$

(iii) vertical, horizontal asymptotes $\bullet(-3, \text{U}|\infty)$

C. Intervals of Increase or Decrease

$$\frac{dy}{dx} = \frac{3x^2 + 12x}{(x+3)^{3/2}} = \frac{3x(x+4)}{(x+3)^{3/2}}$$

$$= \frac{3x^2 + 12x}{(x+3)^{3/2}}$$

EXAMPLE: Sketch the graph of $y = \dfrac{2x^2}{\sqrt{x+3}}$. $\bullet(\infty, \infty)$

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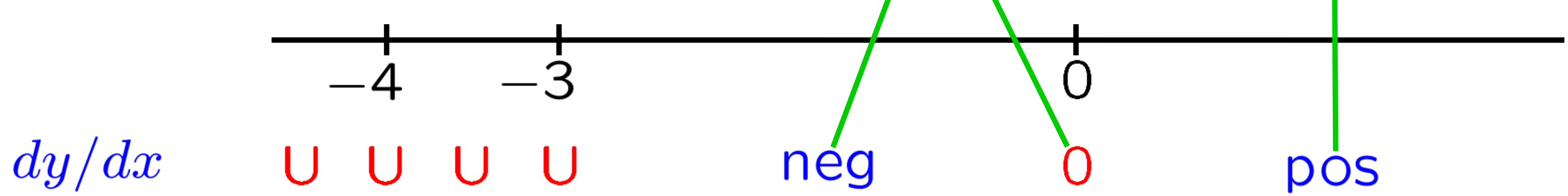
C. Intervals of Increase or Decrease $\downarrow (-3, 0]$, $\uparrow [0, \infty)$

D. Concavity and Points of Inflection

$$\frac{dy}{dx} = \frac{3x^2 + 12x}{(x+3)^{3/2}} = \frac{3x(x+4)}{(x+3)^{3/2}}$$



EXAMPLE: Sketch the graph of $y = \frac{2x^2}{\sqrt{x+3}}$. $\bullet(\infty,\infty)$

A. Symmetry no symm

B. Intervals of Positivity or Negativity, and

(i) domain $(-3,\infty)$ $\rightarrow$ pos$(-3,0),(0,\infty)$

(ii) x,y-intercepts $\bullet(0,0)$

(iii) vertical, horizontal asymptotes $\bullet(-3, \text{U}|\infty)$

C. Intervals of Increase or Decrease $\downarrow(-3,0]$, $\uparrow[0,\infty)$

D. Concavity and Points of Inflection

$$\frac{dy}{dx} = \frac{3x^2+12x}{(x+3)^{3/2}} = \frac{3x(x+4)}{(x+3)^{3/2}}$$

$$\frac{d^2y}{dx^2} = \frac{[(x+3)^{\cancel{3/2}}][6x+12] - [3x^2+12x][(3/2)\cancel{(x+3)^{1/2}})]}{(x+3)^{\cancel{6/2}\,5/2}}$$

$$= \frac{(x+3)(2x+4)(3) - (3x^2+12x)(3/2)}{(x+3)^{5/2}}$$

EXAMPLE: Sketch the graph of $y = \frac{2x^2}{\sqrt{x+3}}$. $\bullet(\infty, \infty)$

A. Symmetry no symm

B. Intervals of Positivity or Negativity, and

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C. Intervals of Increase or Decrease $\downarrow (-3, 0]$, $\uparrow [0, \infty)$

D. Concavity and Points of Inflection

$$\frac{d^2y}{dx^2} = \frac{(x+3)(2x+4)(3) - (3x^2+12x)(3/2)}{(x+3)^{5/2}}$$

$$\frac{d^2y}{dx^2} = \frac{(2x^2+10x+12)(2)(3/2) - (3x^2+12x)(3/2)}{(x+3)^{5/2}}$$

$$= \frac{(x+3)(2x+4)(3) - (3x^2+12x)(3/2)}{(x+3)^{5/2}}$$

$$= \frac{(x^2+8x+24)(3/2)}{(x+3)^{5/2}} = \frac{3(x^2+8x+24)}{2(x+3)^{5/2}}$$

EXAMPLE: Sketch the graph of $y = \frac{2x^2}{\sqrt{x+3}}$. •$(\infty, \infty)$

A. Symmetry no symm

B. Intervals of Positivity or Negativity, and

(i) domain $(-3, \infty)$ → pos $(-3, 0), (0, \infty)$

(ii) x,y-intercepts •$(0, 0)$

(iii) vertical, horizontal asymptotes •$(-3, U|\infty)$

C. Intervals of Increase or Decrease $\downarrow (-3, 0]$, $\uparrow [0, \infty)$

D. Concavity and Points of Inflection

$$\frac{d^2y}{dx^2} = \frac{3(x^2 + 8x + 24)}{2(x+3)^{5/2}}$$

discriminant: $8^2 - 4(1)(24)$
$(b^2 - 4ac)$

$$= \frac{3(x^2 + 8x + 24)}{2(x+3)^{5/2}}$$

EXAMPLE: Sketch the graph of $y = \frac{2x^2}{\sqrt{x+3}}$. $\bullet(\infty, \infty)$

A. Symmetry no symm

B. Intervals of Positivity or Negativity, and

(i) domain $(-3, \infty)$ $\to$ pos$(-3,0),(0,\infty)$

(ii) x,y-intercepts $\bullet(0,0)$

(iii) vertical, horizontal asymptotes $\bullet(-3, \text{U}|\infty)$

C. Intervals of Increase or Decrease $\downarrow (-3, 0]$, $\uparrow [0, \infty)$

D. Concavity and Points of Inflection $\cup(-3, \infty)$

always positive

$$\frac{d^2y}{dx^2} = \frac{3(\boxed{x^2 + 8x + 24})}{2(x+3)^{5/2}}$$

discriminant: $8^2 - 4(1)(24) < 0$

$(b^2 - 4ac)$

-3

d^2y/dx^2 U U pos

EXAMPLE: Sketch the graph of $y = \dfrac{2x^2}{\sqrt{x+3}}$.

$\bullet(\infty, \infty)$

domain $(-3, \infty)$

pos $(-3, 0), (0, \infty)$ $3, \infty)$

$\bullet(0, 0)$

$\bullet(-3, \text{U}|\infty)$

$\bullet(0, 0)$

$\downarrow (-3, 0], \uparrow [0, \infty)$

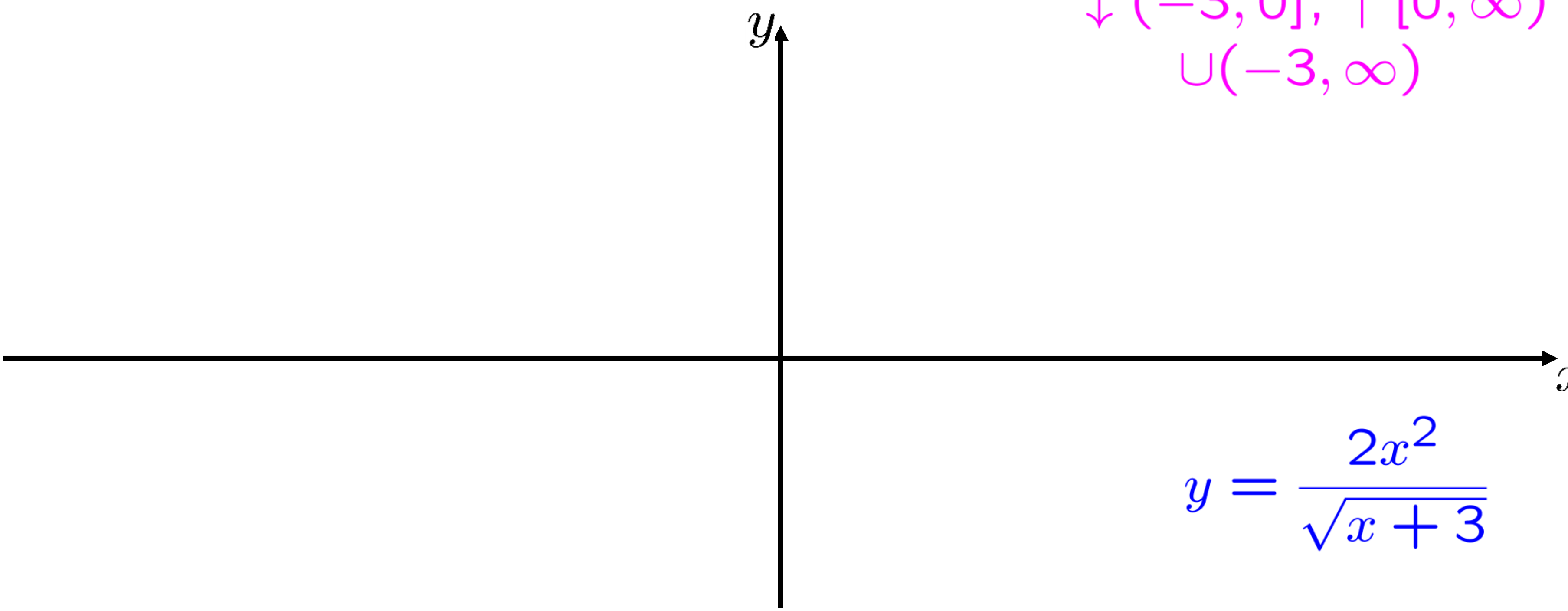
pos $(-3, 0), (0, \infty)$

$\bullet(\infty, \infty)$

$\bullet(-3, \text{U}|\infty)$

$\downarrow (-3, 0], \uparrow [0, \infty)$

$\cup(-3, \infty)$



$y = \dfrac{2x^2}{\sqrt{x+3}}$

EXAMPLE: Sketch the graph of $y = \dfrac{2x^2}{\sqrt{x+3}}$.

domain $(-3, \infty)$

pos $(-3, 0), (0, \infty)$

$\rightarrow$

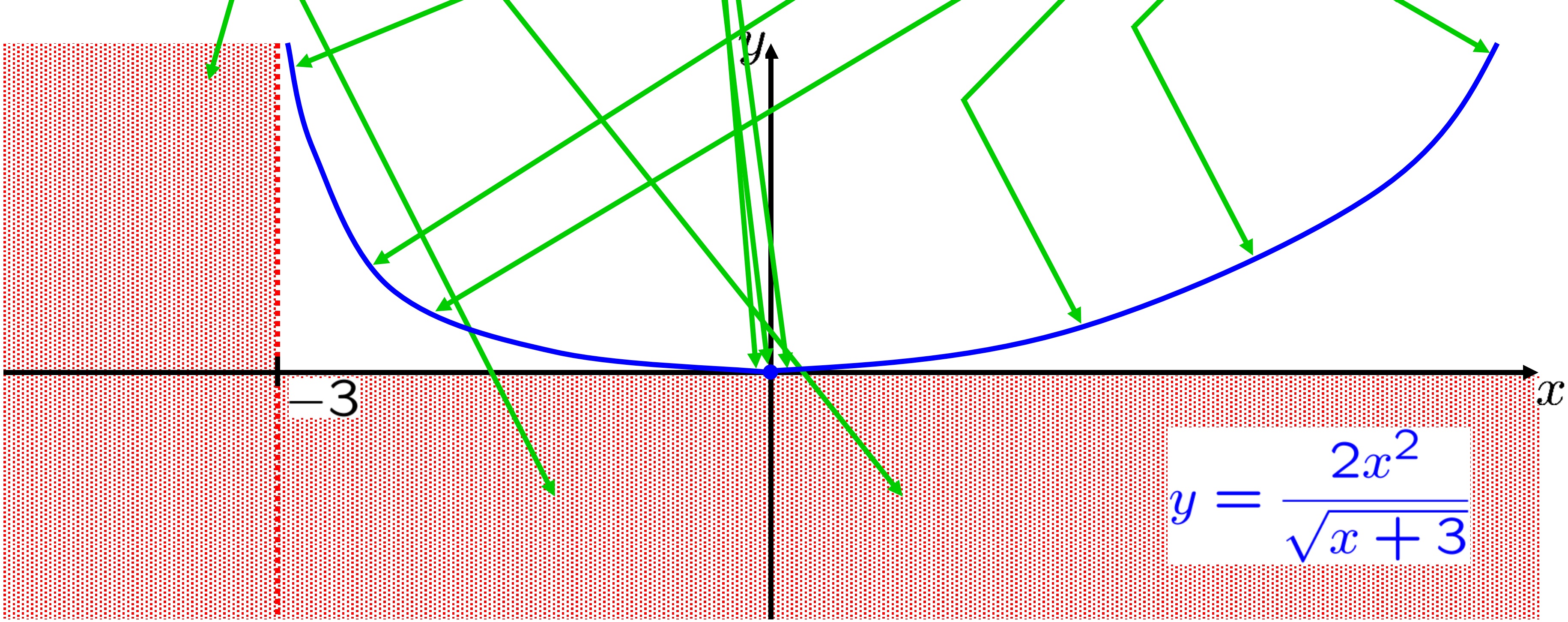
$\bullet (0, 0)$

$\bullet (-3, \text{U}|\infty)$

$\downarrow (-3, 0], \uparrow [0, \infty)$

$\cup (-3, \infty)$

$\bullet (\infty, \infty)$



EXAMPLE: Sketch the graph of $y = \frac{2x^2}{\sqrt{x+3}}$.

domain $(-3, \infty)$

pos $(-3, 0), (0, \infty)$

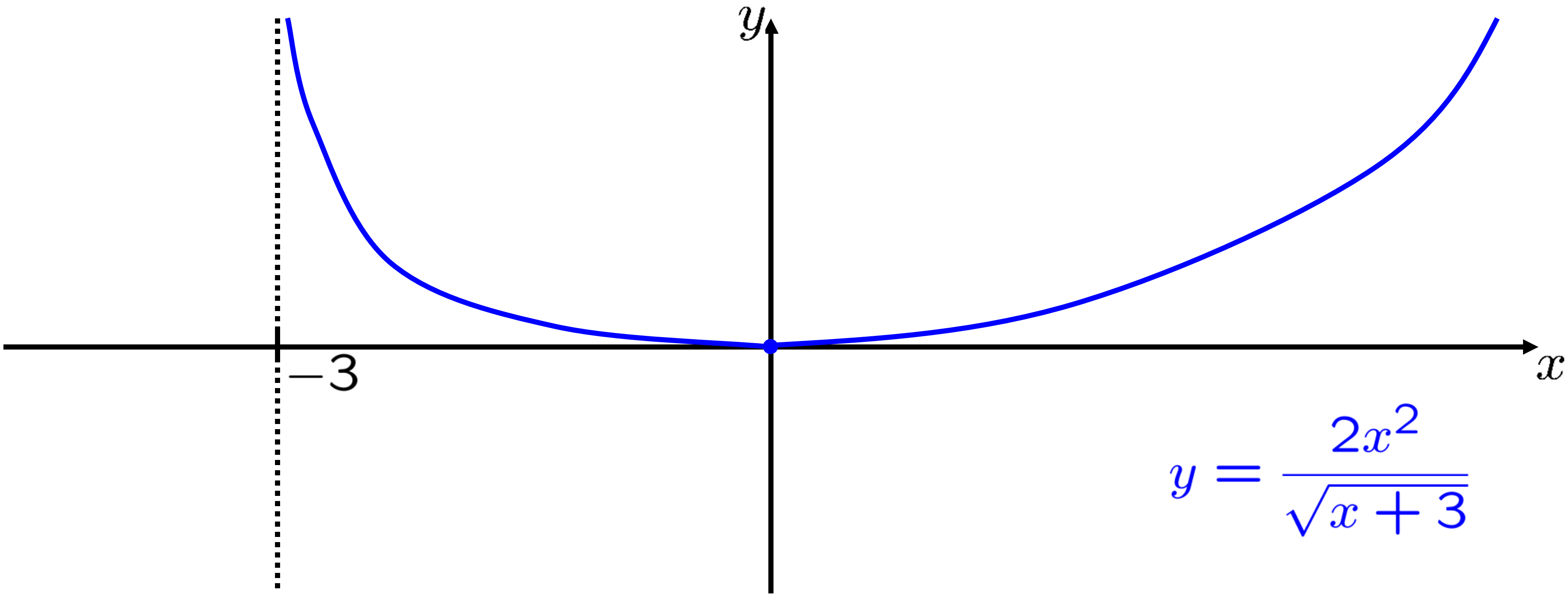
$\bullet \overrightarrow{(0, 0)}$

$\bullet (-3, \text{U}|\infty)$

$\downarrow (-3, 0], \uparrow [0, \infty)$

$\cup(-3, \infty)$

$\bullet(\infty, \infty)$



SKILL
curve sketching

EXAMPLE: Sketch the graph of $y = xe^{-x}$.

A. Symmetry no symm

(i) even function: $f(-x) = f(x)$

(ii) odd function: $f(-x) = -(f(x))$

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EXAMPLE: Sketch the graph of $y = xe^{-x}$.

A. Symmetry no symm

B. Intervals of Positivity or Negativity, and

(i) domain $\mathbb{R}$

(ii) x,y-intercepts $\bullet(0, 0)$

(iii) vertical, horizontal asymptotes $\bullet(\infty, 0)$

C. Intervals of Increase or Decrease

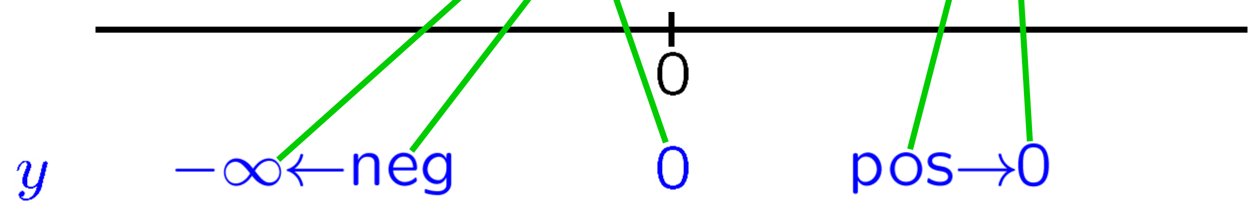
$\bullet(-\infty, -\infty)$

neg$(-\infty, 0)$

pos$(0, \infty)$

always positive

$y = x\boxed{e^{-x}}$



EXAMPLE: Sketch the graph of $y = xe^{-x}$.

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(iii) vertical, horizontal asymptotes $\bullet(\infty, 0)$

$\bullet(-\infty, -\infty)$

neg$(-\infty, 0)$

pos$(0, \infty)$

$\to \bullet(1, 1/e)$

C. Intervals of Increase or Decrease $\uparrow (-\infty, 1]$, $\downarrow [1, \infty)$

D. Concavity and Points of Inflection

$$y = xe^{-x}$$

always positive

$$\frac{dy}{dx} = e^{-x} - xe^{-x} = (1-x)e^{-x}$$

1

dy/dx pos 0 neg

EXAMPLE: Sketch the graph of $y = xe^{-x}$.

A. Symmetry no symm

B. Intervals of Positivity or Negativity, and

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(ii) x,y-intercepts $\bullet(0,0)$

(iii) vertical, horizontal asymptotes $\bullet(\infty, 0)$

C. Intervals of Increase or Decrease $\uparrow(-\infty, 1]$, $\downarrow[1, \infty)$

D. Concavity and Points of Inflection $\cap(-\infty, 2]$, $\cup[2, \infty)$

$\bullet(-\infty, -\infty)$

neg$(-\infty, 0)$

pos$(0, \infty)$

$\bullet(1, 1/e)$

infl$(2, 2/e^2)$

$$y = xe^{-x}$$

$$\frac{dy}{dx} = e^{-x} - xe^{-x} = (1-x)e^{-x}$$

$$\frac{d^2y}{dx^2} = -e^{-x} + (1-x)(-e^{-x}) = (x-2)e^{-x}$$

always positive: e^{-x}

2

d^2y/dx^2 neg 0 pos

EXAMPLE: Sketch the graph of $y = xe^{-x}$.

no symm

domain $\mathbb{R}$

•$(0,0)$

•$(\infty, 0)$

neg$(-\infty, 0)$

pos$(0, \infty)$

domain $\mathbb{R}$

•$(-\infty, -\infty)$

•$(0,0)$

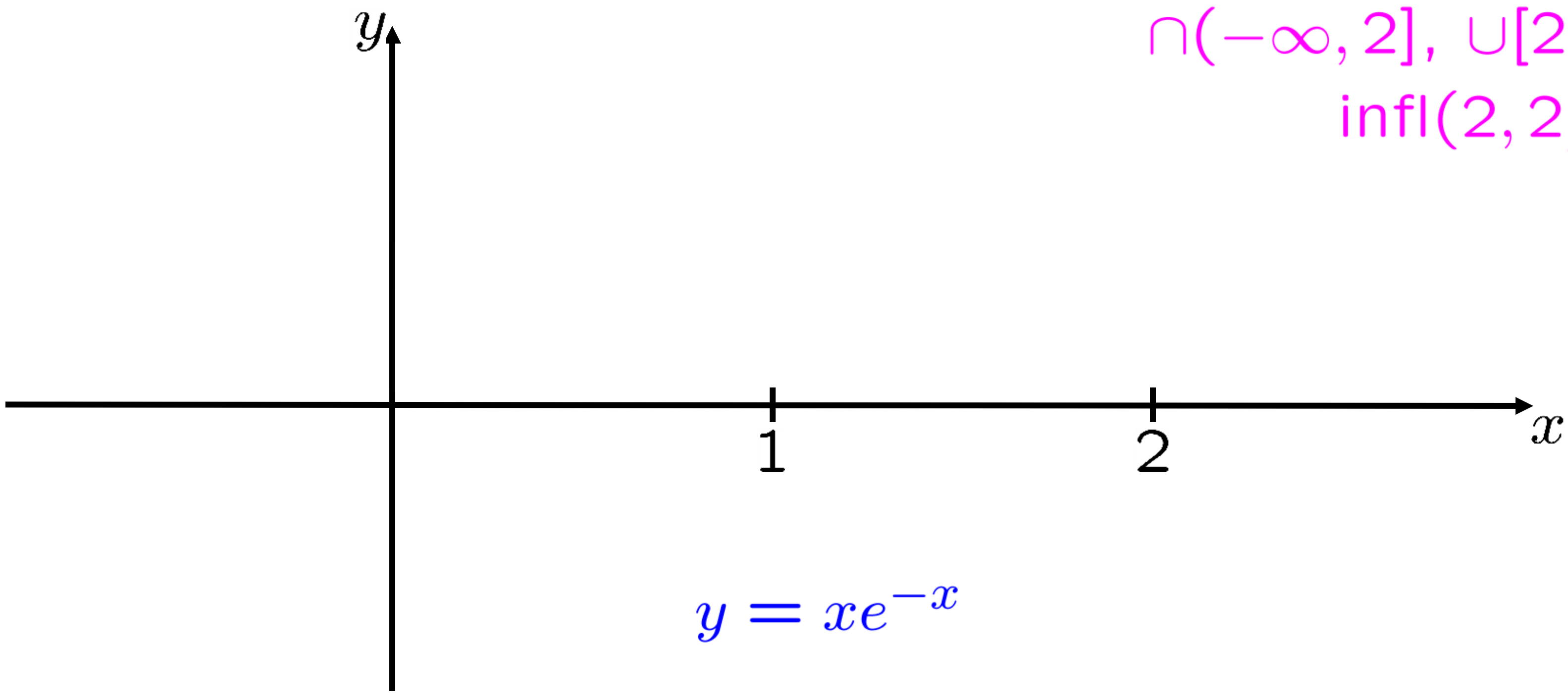
•$(\infty, 0)$

•$(1, 1/e)$

$\uparrow (-\infty, 1]$, $\downarrow [1, \infty)$

$\cap(-\infty, 2]$, $\cup[2, \infty)$

infl$(2, 2/e^2)$



EXAMPLE: Sketch the graph of $y = xe^{-x}$.

no symm

domain $\mathbb{R}$

$\bullet(0,0)$

$\bullet(\infty, 0)$

$\bullet(-\infty, -\infty)$

neg$(-\infty, 0)$

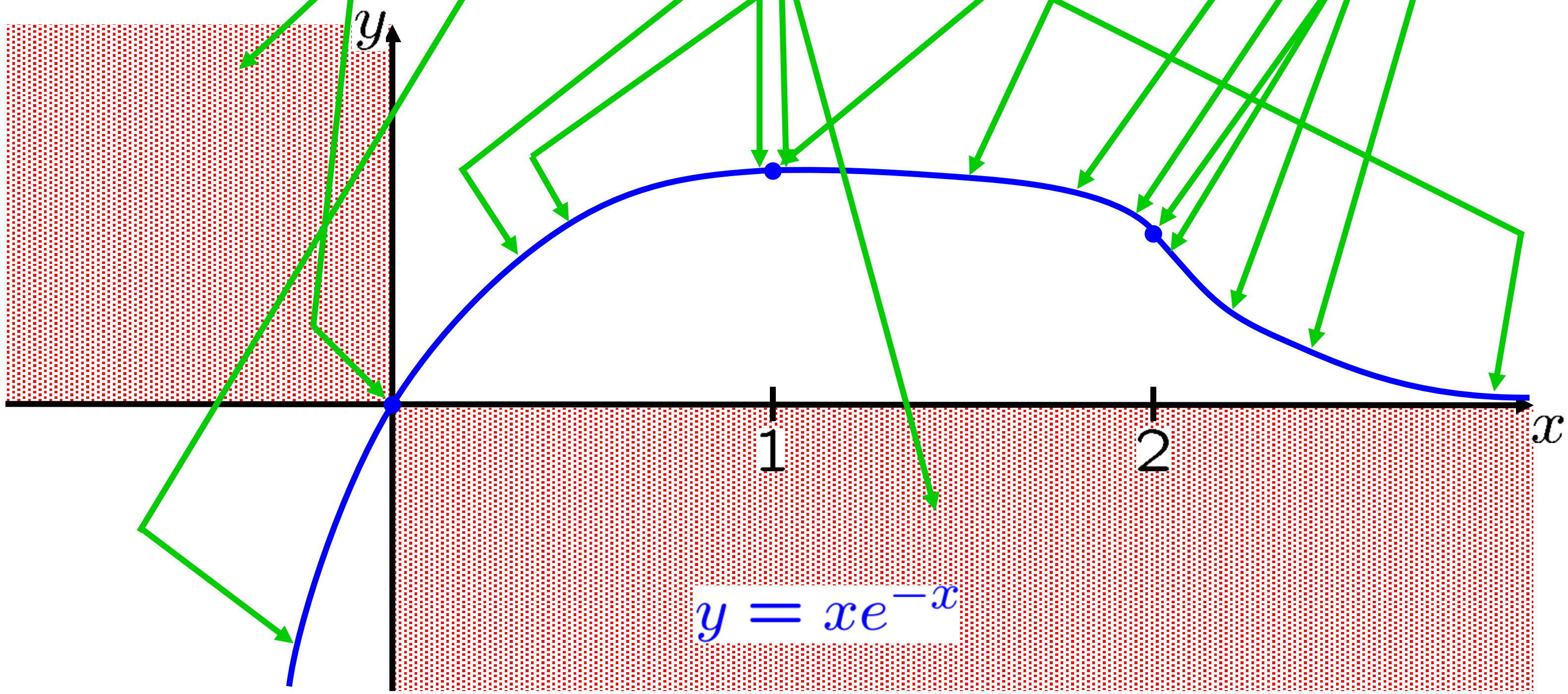
pos$(0, \infty)$

$\overrightarrow{\bullet(1, 1/e)}$

$\uparrow(-\infty, 1], \downarrow[1, \infty)$

$\cap(-\infty, 2], \cup[2, \infty)$

infl$(2, 2/e^2)$



SKILL

curve sketching

EXAMPLE: Sketch the graph of $y = \dfrac{\sin x}{4 + \cos x}$.

A. Symmetry 2π-periodic, odd (over $[0, \pi]$; reflect thru origin, repeat)

(i) even function: $f(-x) = f(x)$

(ii) odd function: $f(-x) = -(f(x))$

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A. Symmetry 2π-periodic, odd (over $[0, \pi]$; reflect thru origin, repeat)

B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0, \pi]$

(ii) x,y-intercepts •$(0, 0)$, •$(\pi, 0)$ pos$(0, \pi)$

(iii) vertical, horizontal asymptotes no asymptotes

C. Intervals of Increase or Decrease

$$y = \frac{\sin x}{\boxed{4 + \cos x}}$$ always positive

$\sin x$	0	pos	0
	0		π
y	0	pos	0

EXAMPLE: Sketch the graph of $y = \dfrac{\sin x}{4 + \cos x}$.

A. Symmetry 2π-periodic, odd (over $[0, \pi]$; reflect thru origin, repeat)

B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0, \pi]$ pos$(0, \pi)$

(ii) x,y-intercepts $\bullet(0, 0)$, $\bullet(\pi, 0)$

(iii) vertical, horizontal asymptotes no asymptotes

C. Intervals of Increase or Decrease

$$y = \frac{\sin x}{4 + \cos x}$$

$$\frac{dy}{dx} = \frac{[4 + \cos x][\cos x] - [\sin x][-\sin x]}{[4 + \cos x]^2}$$

$$= \frac{4\cos x + \cos^2 x + \sin^2 x}{[4 + \cos x]^2} = \frac{1 + 4\cos x}{[4 + \cos x]^2}$$

EXAMPLE: Sketch the graph of $y = \dfrac{\sin x}{4 + \cos x}$.

A. Symmetry 2π-periodic, odd (over $[0,\pi]$; reflect thru origin, repeat)

B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0,\pi]$ pos$(0,\pi)$

(ii) x,y-intercepts $\bullet(0,0)$, $\bullet(\pi,0)$

(iii) vertical, horizontal asymptotes no asymptotes

C. Intervals of Increase or Decrease

$$\frac{dy}{dx} = \frac{1 + 4\cos x}{[4 + \cos x]^2}$$ always positive

$$\frac{dy}{dx}$$

$$= \frac{1 + 4\cos x}{[4 + \cos x]^2}$$

EXAMPLE: Sketch the graph of $y = \dfrac{\sin x}{4 + \cos x}$.

A. Symmetry 2π-periodic, odd (over $[0, \pi]$; reflect thru origin, repeat)

B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0, \pi]$ pos$(0, \pi)$

(ii) x,y-intercepts $\bullet(0, 0)$, $\bullet(\pi, 0)$

(iii) vertical, horizontal asymptotes no asymptotes

C. Intervals of Increase or Decrease $\uparrow [0, \alpha], \downarrow [\alpha, \pi]$

D. Concavity and Points of Inflection

$\alpha \doteq 1.82$

$\beta \doteq 0.258$

$$\frac{dy}{dx} = \frac{1 + 4\boxed{\cos x}}{\boxed{[4 + \cos x]^2}}$$

$\cos x$: -1, for $x :\to \pi$; 1, for $x :\to 0$

$[4 + \cos x]^2$: always positive

$\to \bullet(\alpha, \beta)$

$$\beta := \frac{\sin \alpha}{4 + \cos \alpha} \doteq 0.258$$

	0		α		π
$1 + 4\cos x$		pos	0	neg	
dy/dx		pos	0	neg	

$$1 + 4\cos x = 0,\ x \in [0, \pi] \quad \Leftrightarrow \quad x = \underbrace{\arccos\left(-\tfrac{1}{4}\right)}_{\alpha} \doteq 1.82$$

EXAMPLE: Sketch the graph of $y = \dfrac{\sin x}{4 + \cos x}$.

A. Symmetry 2π-periodic, odd (over $[0, \pi]$; reflect thru origin, repeat)

B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0, \pi]$ pos$(0, \pi)$

(ii) x,y-intercepts $\bullet(0, 0)$, $\bullet(\pi, 0)$

(iii) vertical, horizontal asymptotes no asymptotes

C. Intervals of Increase or Decrease $\uparrow [0, \alpha], \downarrow [\alpha, \pi]$

D. Concavity and Points of Inflection

$\alpha \doteq 1.82$

$\beta \doteq 0.258$

$\bullet(\overrightarrow{\alpha, \beta})$

$$\frac{dy}{dx} = \frac{1 + 4\cos x}{[4 + \cos x]^2}$$

$$\frac{d^2y}{dx^2}$$

$$\|$$

$$\frac{([4 + \cos x]^2)(-4\sin x) - (1 + 4\cos x)(2[4 + \cos x][-\sin x])}{[4 + \cos x]^4}$$

$$\|$$

$$\frac{(4 + \cos x)(-4\sin x) - (1 + 4\cos x)(-2\sin x)}{[4 + \cos x]^3}$$

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D. Concavity and Points of Inflection $\alpha \doteq 1.82$, $\beta \doteq 0.258$

$$\frac{d^2y}{dx^2} = \frac{(4 + \cos x)(-4\sin x) - (1 + 4\cos x)(-2\sin x)}{[4 + \cos x]^3} \qquad \overrightarrow{\bullet(\alpha, \beta)}$$

$$\frac{d^2y}{dx^2}$$

$$= \frac{(-16\sin x - 4\sin x\cos x) - (-2\sin x - 8\sin x\cos x)}{[4 + \cos x]^3}$$

$$= \frac{-14\sin x + 4\sin x\cos x}{[4 + \cos x]^3} = \frac{(2\sin x)(-7 + 2\cos x)}{[4 + \cos x]^3}$$

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always positive

$\bullet(\overrightarrow{\alpha, \beta})$

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$\alpha \doteq 1.82$

D. Concavity and Points of Inflection $\cap [0,\pi]$

$\beta \doteq 0.258$

$$\frac{d^2y}{dx^2} = \frac{(2\sin x)(-7 + 2\cos x)}{[4+\cos x]^3}$$

$-7 + 2\cos x$: always negative

$[4+\cos x]^3$: always positive

$\bullet(\overrightarrow{\alpha,\beta})$

	0		π
$2\sin x$	0	pos	0
d^2y/dx^2	0	neg	0

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2π-periodic, odd (over $[0, \pi]$; reflect thru origin, repeat)

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$\bullet(0, 0)$, $\bullet(\pi, 0)$

$\bullet(0, 0)$, $\bullet$

pos$(0, \pi)$

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$\uparrow [0, \alpha], \downarrow [\alpha, \pi]$

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$\bullet(\overrightarrow{\alpha, \beta})$

$\uparrow [0, \alpha], \downarrow [\alpha, \pi]$

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$\bullet(\overrightarrow{\alpha, \beta})$

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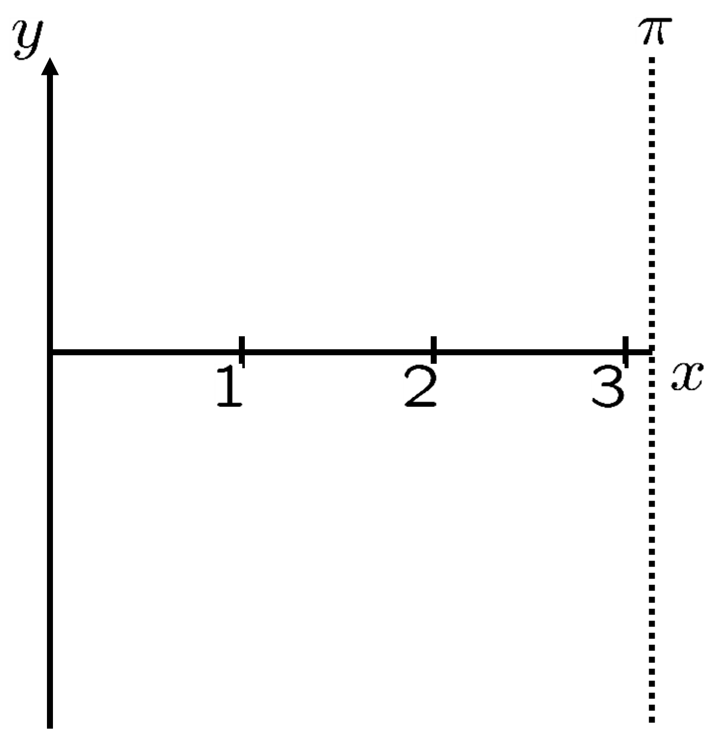

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$\alpha \doteq 1.82$

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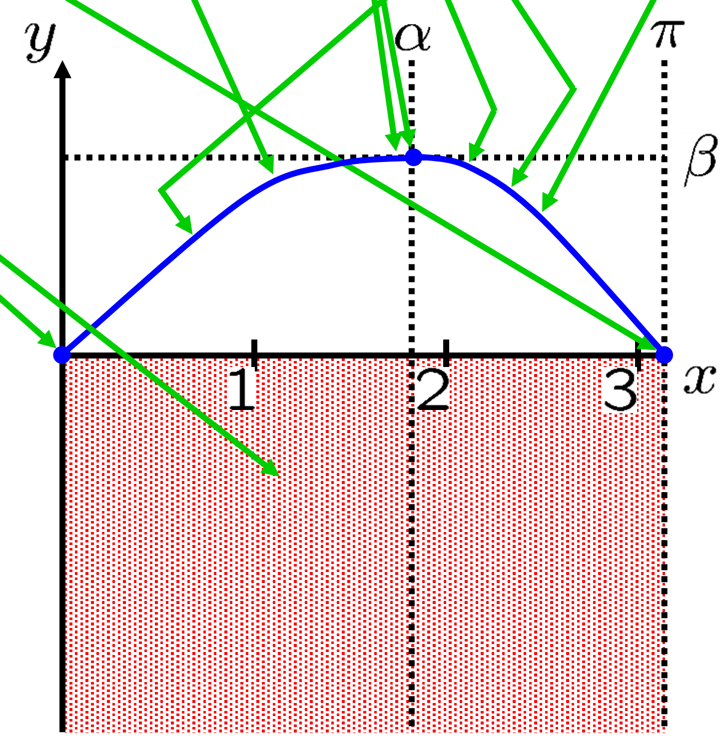
$\overrightarrow{\top}$

$\bullet(\alpha, \beta)$

$\cap [0, \pi]$



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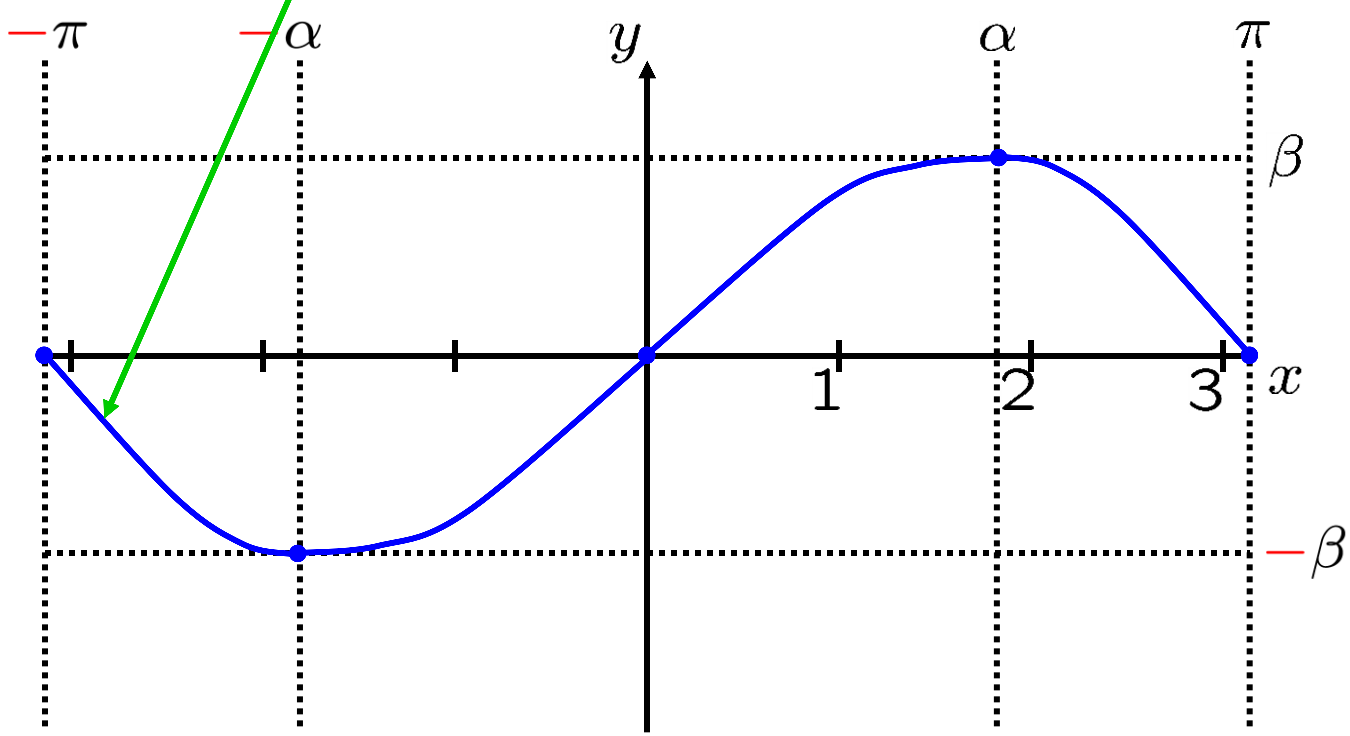

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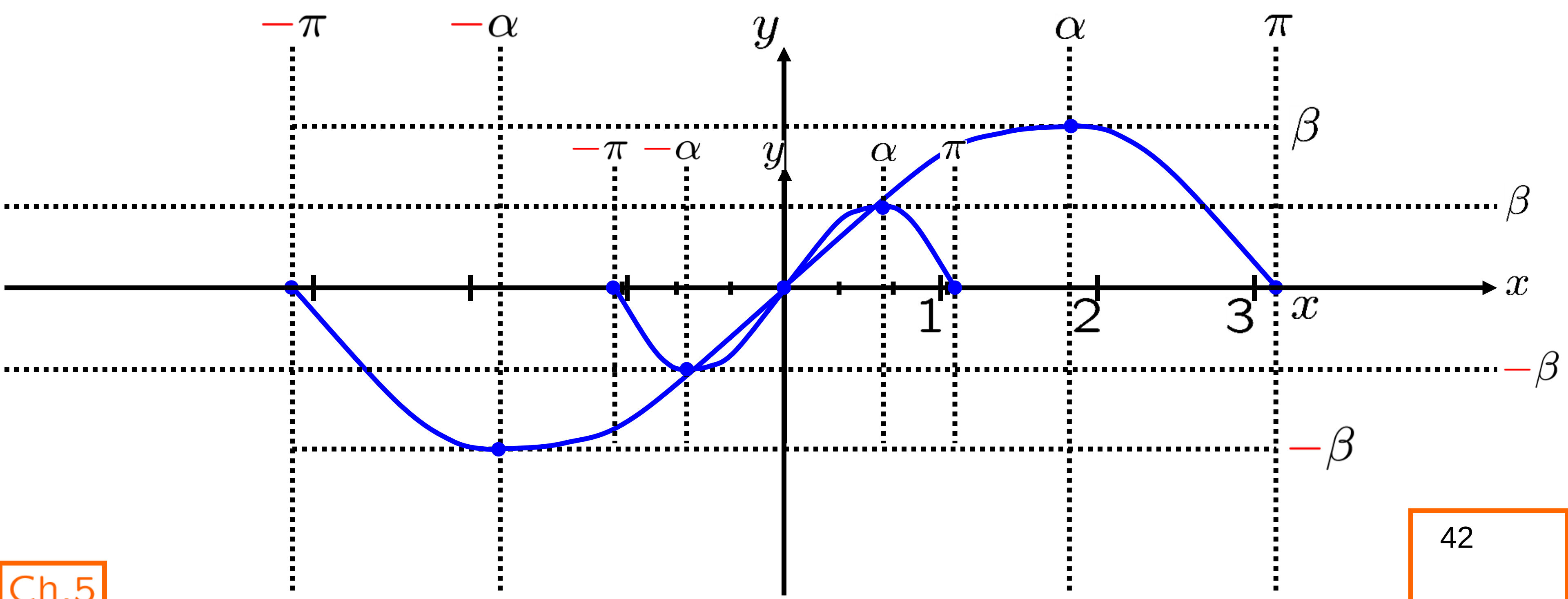
pos$(0,\pi)$

$\bullet(\alpha, \overrightarrow{\beta})$



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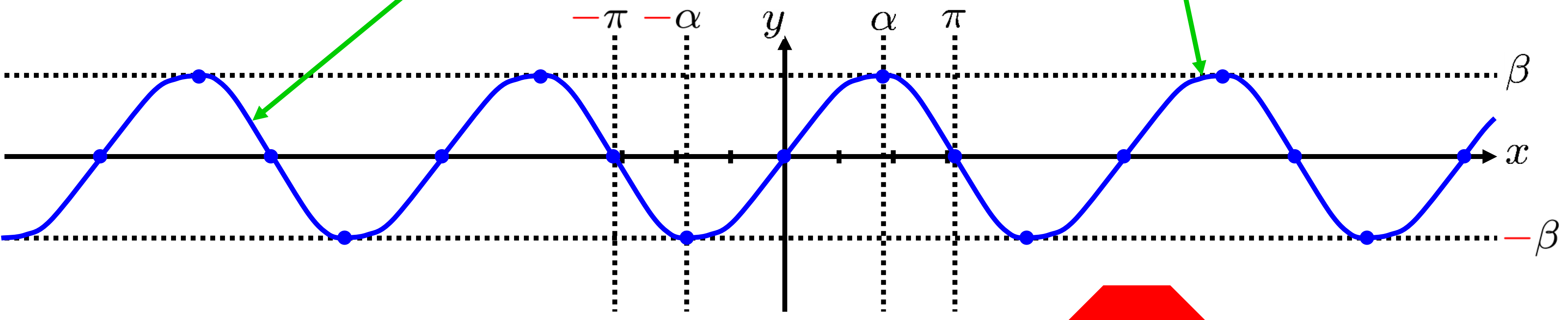

SKILL
curve sketching

STOP