CALCULUS
More graphing problems

EXAMPLE: Sketch the graph of $y = 2x - (\sin x)$.

A. Symmetry odd (over $[0, \infty)$; reflect through origin)

(i) even function: $f(-x) = f(x)$

(ii) odd function: $f(-x) = -(f(x))$

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EXAMPLE: Sketch the graph of $y = 2x - (\sin x)$.

A. Symmetry odd (over $[0, \infty)$; reflect through origin)

B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0, \infty)$

(ii) x,y-intercepts $\bullet(0, 0)$

(iii) vertical, horizontal asymptotes no asymptotes

pos$(0, \infty)$

C. Intervals of Increase or Decrease

$$[\sin x]_{x:\to 0} = 0 = [x]_{x:\to 0}$$

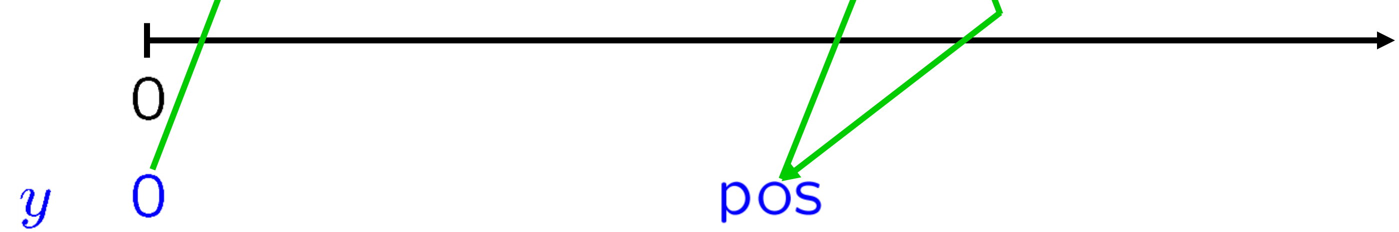
$$\forall x \geq 0,\ \frac{d}{dx}[\sin x] \leq \frac{d}{dx}[x]$$

$$\forall x \geq 0,\ \sin x \leq x$$

$$\forall x > 0,\ \qquad x < 2x$$

$$\forall x > 0,\ \sin x \leq x < 2x,$$ so $2x - (\sin x) > 0$.

$$y = 2x - (\sin x)$$



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(iii) vertical, horizontal asymptotes no asymptotes

pos$(0,\infty)$

C. Intervals of Increase or Decrease $\uparrow [0,\infty)$

D. Concavity and Points of Inflection

$$y = 2x - (\sin x)$$

$$\frac{dy}{dx} = 2 - (\cos x) > 0, \quad \forall x$$

0

dy/dx pos pos

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C. Intervals of Increase or Decrease ↑ $[0, \infty)$

D. Concavity and Points of Inflection

$$\frac{dy}{dx} = 2 - (\cos x)$$

$$\frac{d^2y}{dx^2} = \sin x$$

$$\frac{d}{dx} - (\cos x)$$

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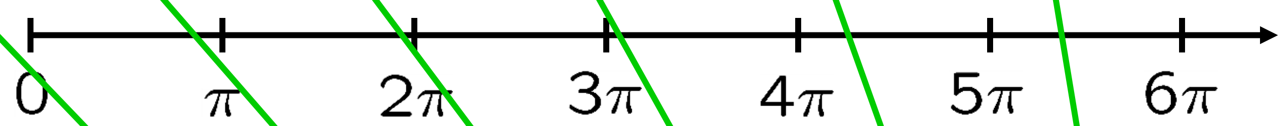
C. Intervals of Increase or Decrease $\uparrow [0,\infty)$

D. Concavity and Points of Inflection

$\cup[0,\pi]$, $\cap[\pi,2\pi]$, $\cup[2\pi,3\pi]$, $\cap[3\pi,4\pi]$, $\cup[4\pi,5\pi]$, $\cap[5\pi,6\pi]$, ...

$$\frac{dy}{dx} = 2 - (\cos x)$$

$$\frac{d^2y}{dx^2} = \sin x$$



d^2y/dx^2 0 pos 0 neg 0 pos 0 neg 0 pos 0 neg 0 ⋯

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pos$(0, \infty)$

$\bullet(0,0) \nearrow$ domain $\supseteq [0, \infty)$

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pos$(0, \infty)$

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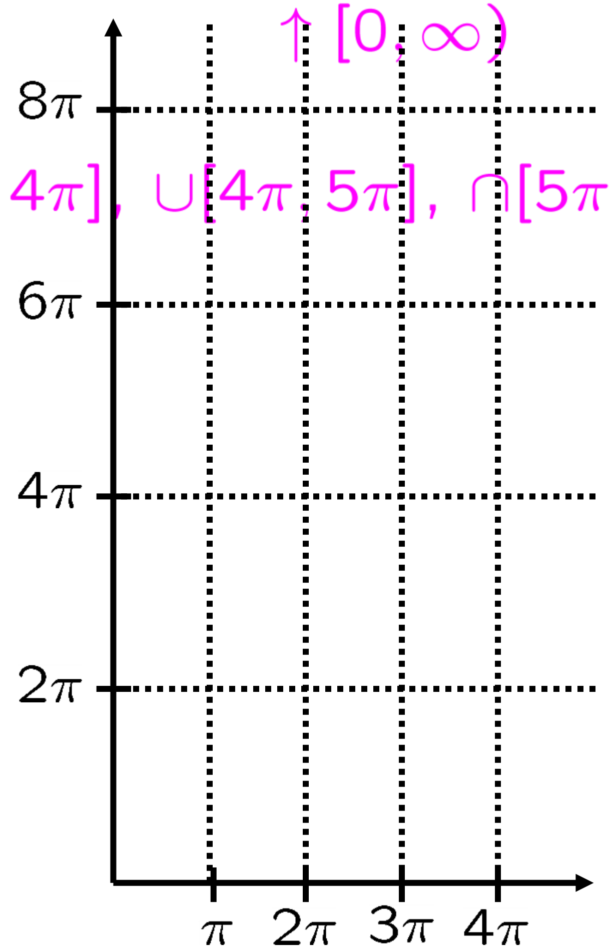

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$\bullet(\pi, 2\pi)$ $\bullet(2\pi, 4\pi)$

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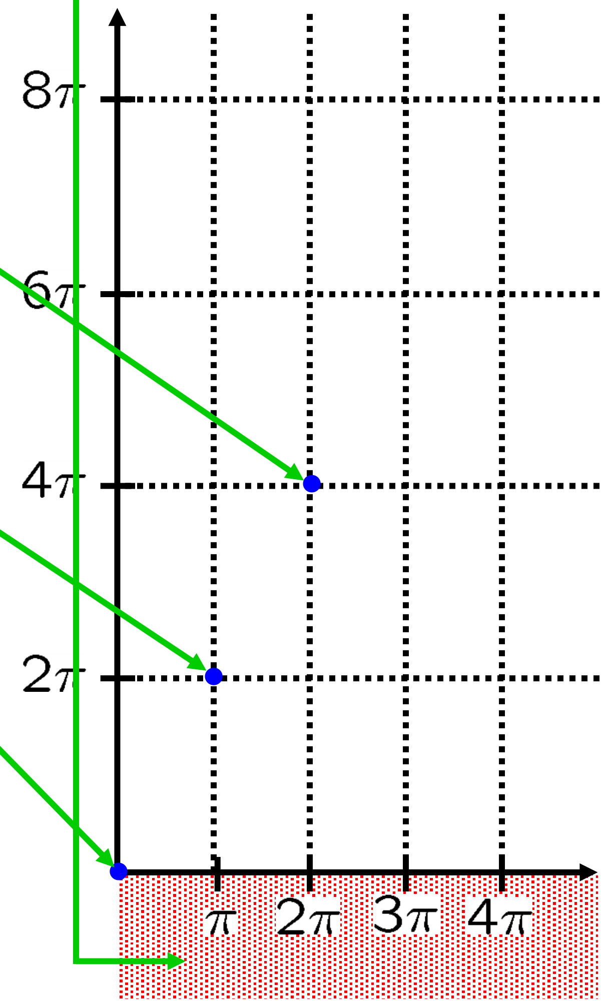

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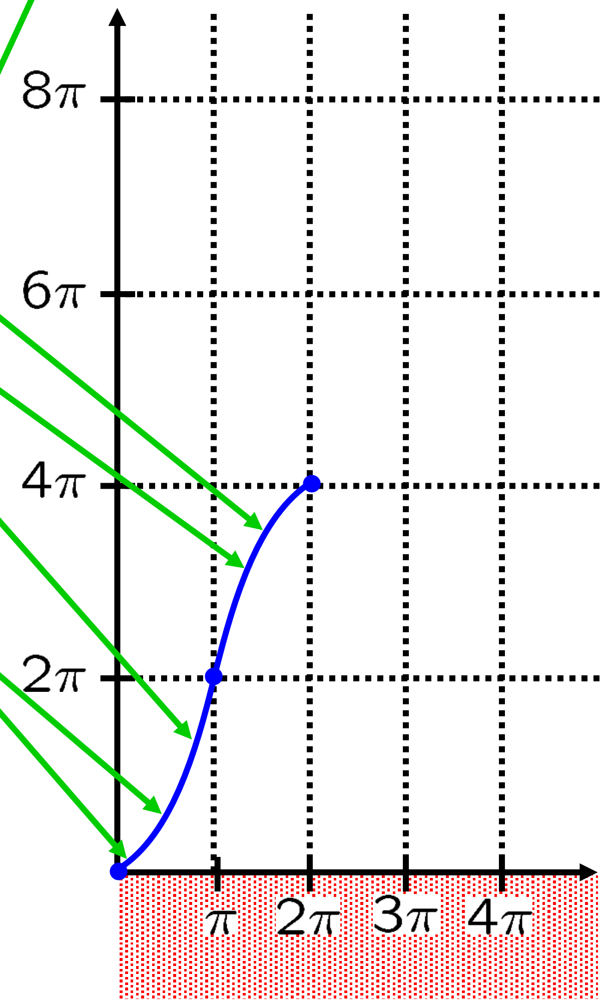

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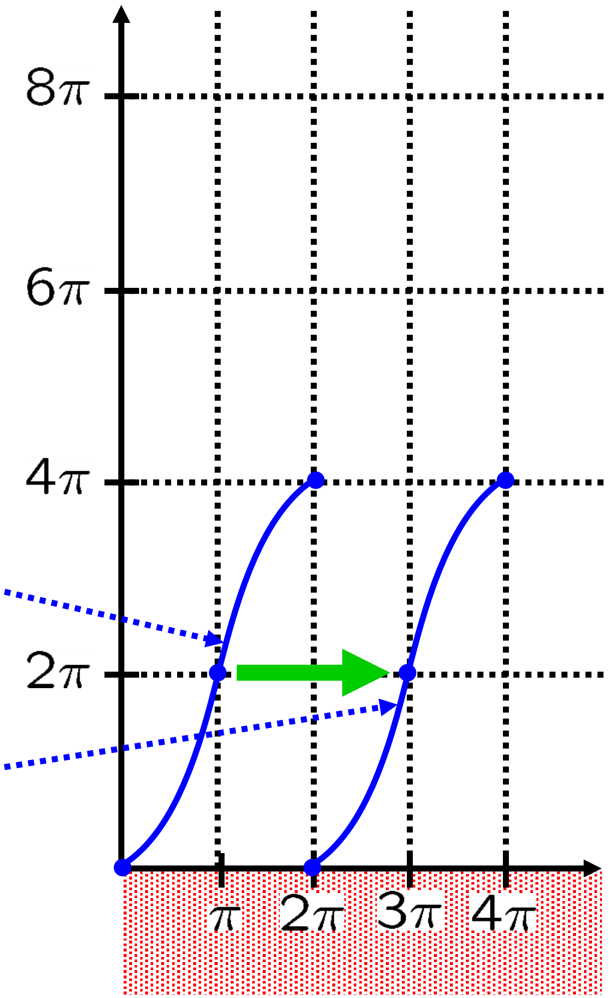
$\bullet(\pi, 2\pi)$ $\bullet(2\pi, 4\pi)$

$y = 2x - (\sin x)$

$x :\rightarrow x - 2\pi$

$y = 2(x - 2\pi) - (\sin(x - 2\pi))$

$= 2x - 4\pi - (\sin x)$



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$y - \cancel{4\pi} = 2x - \cancel{4\pi} - (\sin x)$

$y = 2x - (\sin x)$

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$y :\to y - 4\pi$

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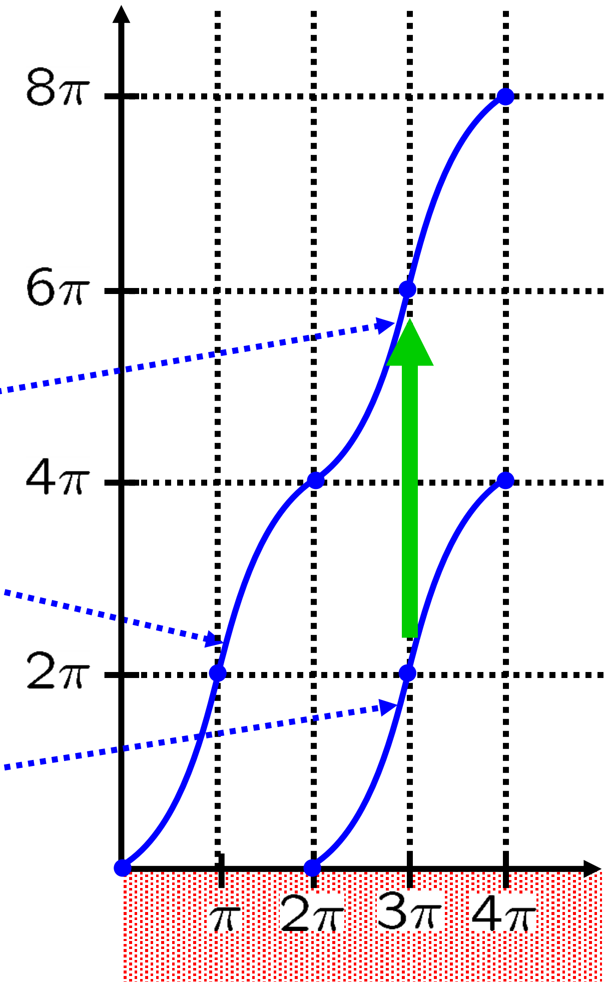

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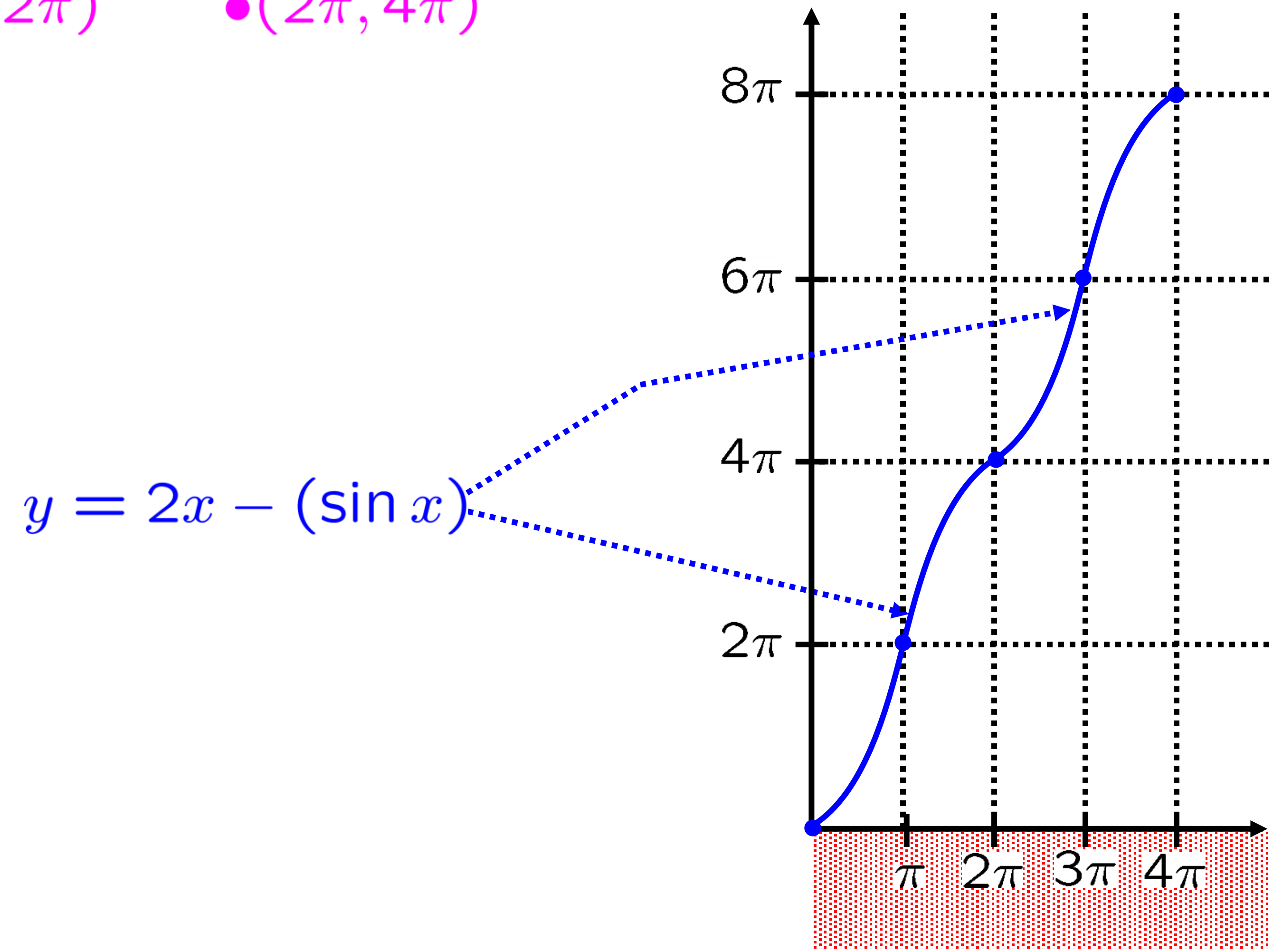

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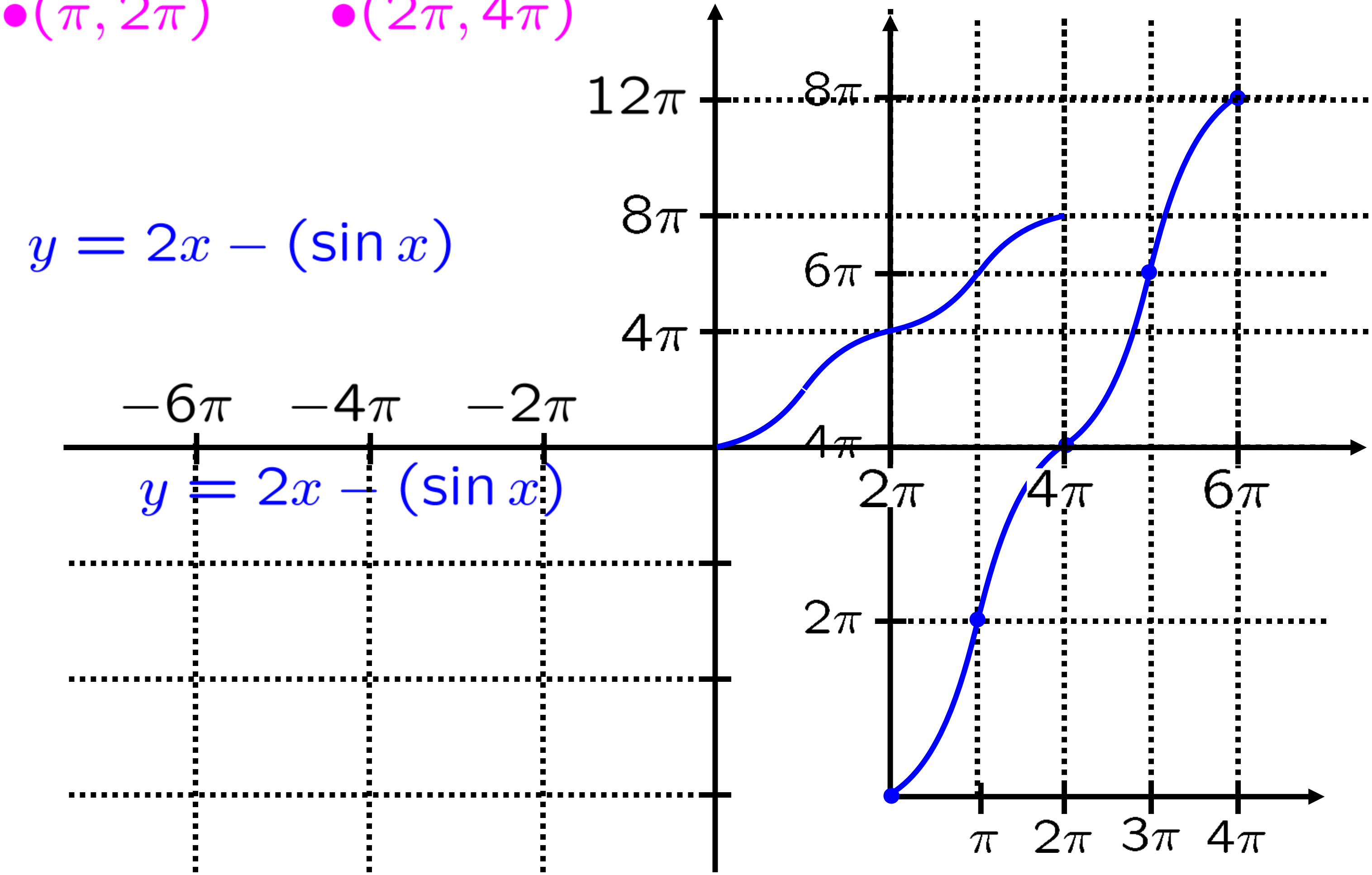

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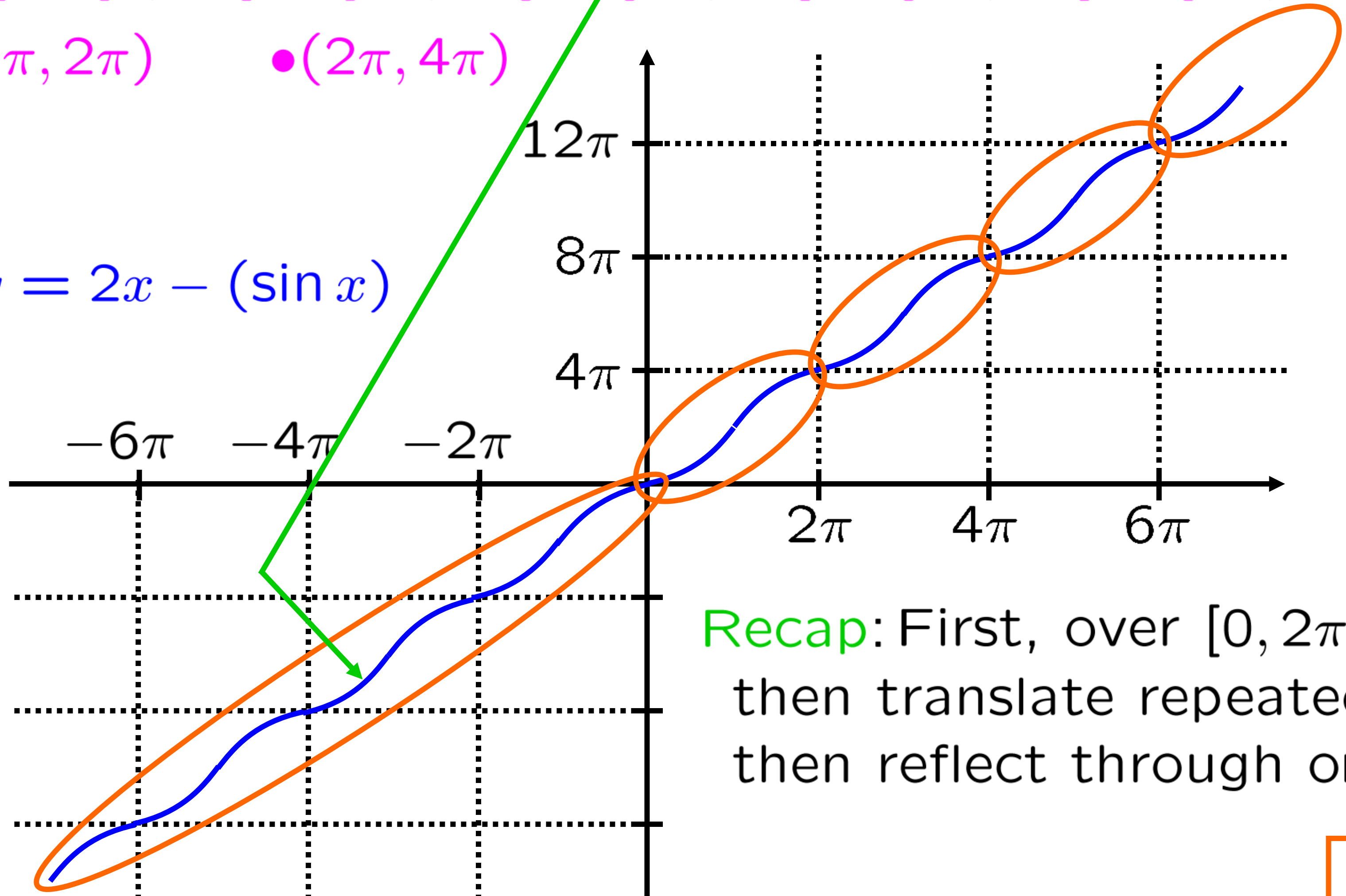

Recap: First, over $[0,2\pi]$, then translate repeatedly, then reflect through origin.

EXAMPLE: Sketch the graph of $y = 2x - (\sin x)$.

odd (over $[0,\infty)$; reflect through origin)

domain $\supseteq [0,\infty)$ pos$(0,\infty)$

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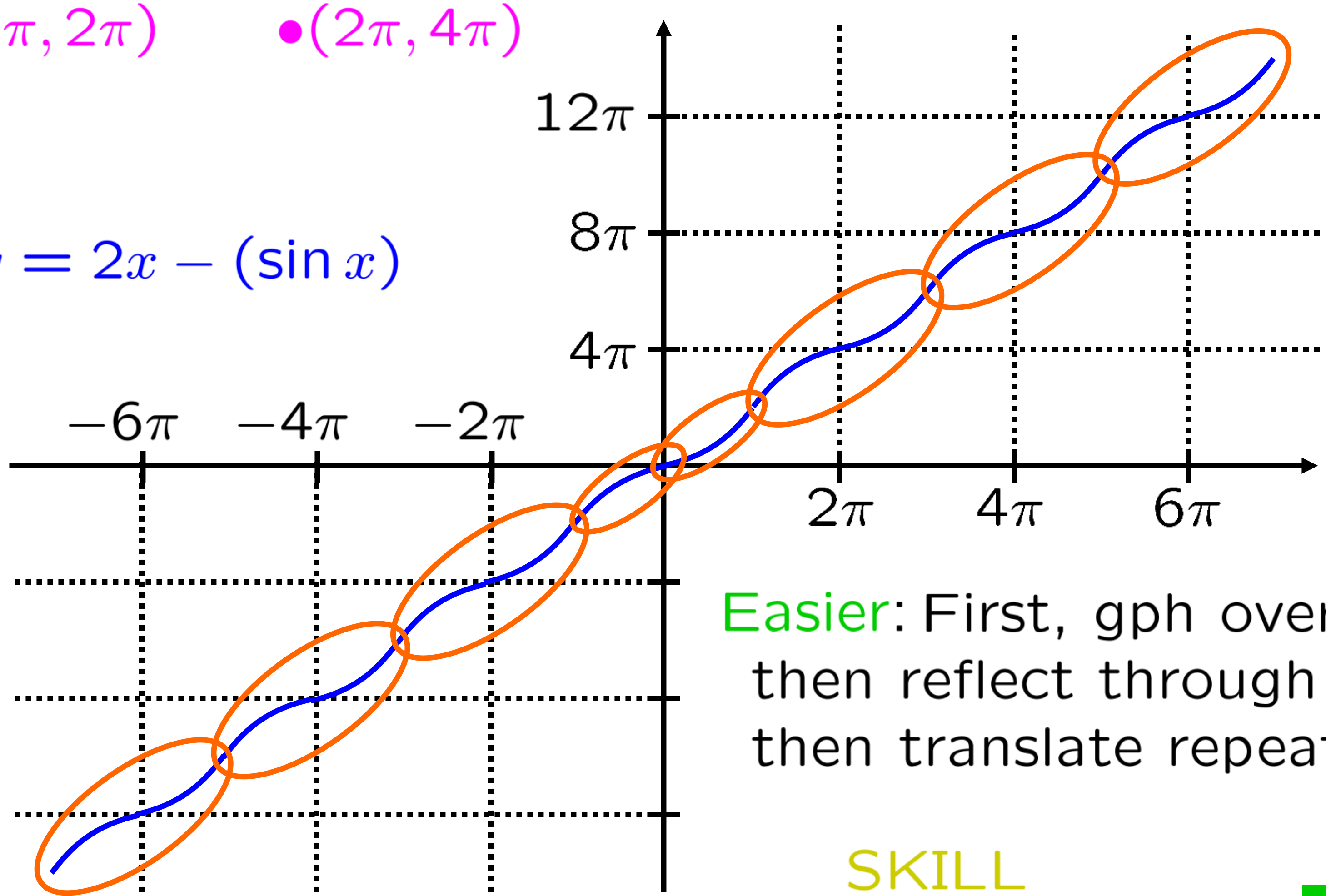

Easier: First, gph over $[0,\pi]$, then reflect through origin, then translate repeatedly.

SKILL
curve sketching

EXAMPLE: Sketch the graph of $y = \dfrac{x^3}{x^2 - 4}$.

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B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0, \infty) \setminus \{2\}$ — neg$(0, 2)$, pos$(2, \infty)$

(ii) x,y-intercepts $\bullet(0, 0)$ — $\bullet(\infty, \infty)$

(iii) vertical, horizontal asymptotes $\bullet(2, -\infty | \infty)$

C. Intervals of Increase or Decrease

$$y = \frac{x^3}{x^2 - 4} = \frac{x^3}{(x-2)(x+2)}$$

0^3 neg U pos$\to\infty$

EXAMPLE: Sketch the graph of $y = \frac{x^3}{x^2 - 4}$.

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C. Intervals of Increase or Decrease

$$y = \frac{x^3}{x^2 - 4}$$

$$\frac{dy}{dx} = \frac{(x^2-4)(3x^2) - (x^3)(2x)}{(x^2-4)^2} = \frac{(3x^4 - 12x^2) - (2x^4)}{(x^2-4)^2}$$

FACTOR OUT x^2

$$= \frac{x^4 - 12x^2}{(x^2-4)^2} = \frac{x^2(x^2-12)}{((x-2)(x+2))^2}$$

FACTOR

EXAMPLE: Sketch the graph of $y = \frac{x^3}{x^2-4}$.

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$$\frac{dy}{dx}$$

$$= \frac{x^4-12x^2}{(x^2-4)^2} = \frac{x^2(x^2-12)}{((x-2)(x+2))^2}$$

EXAMPLE: Sketch the graph of $y = \dfrac{x^3}{x^2 - 4}$. $x :\to \alpha$ $y :\to \beta$

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C. Intervals of Increase or Decrease $\alpha \approx 3.464$

$\bullet(\alpha, \beta)^{\to}$ $\downarrow [0, 2)$, $\downarrow (2, \alpha]$, $\uparrow [\alpha, \infty)$ $\beta \approx 5.196$

D. Concavity and Points of Inflection

FACTOR

$$\frac{dy}{dx} = \frac{x^4 - 12x^2}{(x^2 - 4)^2} = \frac{x^2(x^2 - 12)}{((x-2)(x+2))^2}$$

$$\alpha := \sqrt{12} \approx 3.464$$

$$= \frac{x^2(x - \alpha)(x + \alpha)}{(x-2)^2(x+2)^2}$$

$$\beta := \frac{\alpha^3}{\alpha^2 - 4} = \frac{\alpha^3}{8} \approx 5.196$$

0 2 α

0^2 neg U^2 neg 0 pos

EXAMPLE: Sketch the graph of $y = \dfrac{x^3}{x^2-4}$.

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C. Intervals of Increase or Decrease

$\alpha \approx 3.464$

$\beta \approx 5.196$

$\bullet(\alpha,\beta)^{\rightarrow}$ $\downarrow[0,2)$, $\downarrow(2,\alpha]$, $\uparrow[\alpha,\infty)$

D. Concavity and Points of Inflection

$$\frac{dy}{dx} = \frac{x^4-12x^2}{(x^2-4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2-4)^{\not 2}(4x^3-24x) - (x^4-12x^2)(2\cancel{(x^2-4)}(2x))}{(x^2-4)^{\not 4\,3}}$$

$$= \frac{(x^2-4)(4x^3-24x)-(x^4-12x^2)(4x)}{(x^2-4)^3}$$

EXAMPLE: Sketch the graph of $y = \dfrac{x^3}{x^2 - 4}$.

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C. Intervals of Increase or Decrease $\alpha \approx 3.464$

$\bullet (\alpha, \beta)^{\rightarrow}$ $\downarrow [0, 2)$, $\downarrow (2, \alpha]$, $\uparrow [\alpha, \infty)$ $\beta \approx 5.196$

D. Concavity and Points of Inflection FACTOR OUT $4x$

$$\frac{d^2y}{dx^2} = \frac{(x^2 - 4)(4x^3 - 24x) - (x^4 - 12x^2)(4x)}{(x^2 - 4)^3}$$

$$\frac{d^2y}{dx^2} = \frac{4x[(x^2 - 4)(x^2 - 6) - (x^4 - 12x^2)]}{(x^2 - 4)^3}$$

$$= \frac{(x^2 - 4)(4x^3 - 24x) - (x^4 - 12x^2)(4x)}{(x^2 - 4)^3}$$

EXAMPLE: Sketch the graph of $y = \dfrac{x^3}{x^2-4}$.

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D. Concavity and Points of Inflection

$$\frac{d^2y}{dx^2} = \frac{(x^2-4)(4x^3-24x)-(x^4-12x^2)(4x)}{(x^2-4)^3}$$

EXPAND

$$= \frac{4x[(x^2-4)(x^2-6)-(x^4-12x^2)]}{(x^2-4)^3}$$

$$= \frac{4x[(x^4-10x^2+24)-(x^4-12x^2)]}{(x^2-4)^3}$$

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D. Concavity and Points of Inflection

$$\frac{d^2y}{dx^2} = \frac{4x[(\cancel{x^4} - 10x^2 + 24) - (\cancel{x^4} - 12x^2)]}{(x^2-4)^3}$$

$$= \frac{4x[(x^4 - 10x^2 + 24) - (x^4 - 12x^2)]}{(x^2-4)^3}$$

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C. Intervals of Increase or Decrease — $\alpha \approx 3.464$, $\beta \approx 5.196$

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D. Concavity and Points of Inflection $\cap[0, 2)$, $\cup(2, \infty)$

$$\frac{d^2y}{dx^2} = \frac{4x[(\cancel{x^4} - 10x^2 + 24) - (\cancel{x^4} - 12x^2)]}{(x^2 - 4)^3}$$

FACTOR OUT 2 — always positive

$$= \frac{4x[2x^2 + 24]}{(x^2 - 4)^3} = \frac{4x[2(x^2 + 12)]}{((x - 2)(x + 2))^3} = \frac{8x(x^2 + 12)}{(x - 2)^3(x + 2)^3}$$

FACTOR

EXAMPLE: Sketch the graph of $y = \dfrac{x^3}{x^2-4}$.

domain $\supseteq [0,\infty)\backslash\{2\}$

odd (over $[0,\infty)$; reflect through origin)through origin)

neg$(0,2)$, pos$(2,\infty)$

$\downarrow [0,2)$, $\downarrow (2,\alpha]$, $\uparrow [\alpha,\infty)$

- $\bullet(0,0)^{\rightarrow}$
- $\bullet(\infty,\infty)$
- $\bullet(2,-\infty|\infty)$
- $\bullet(\alpha,\beta)^{\rightarrow}$

$\alpha \approx 3.464$

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$\cap[0,2)$, $\cup(2,\infty)$$2,\infty)$

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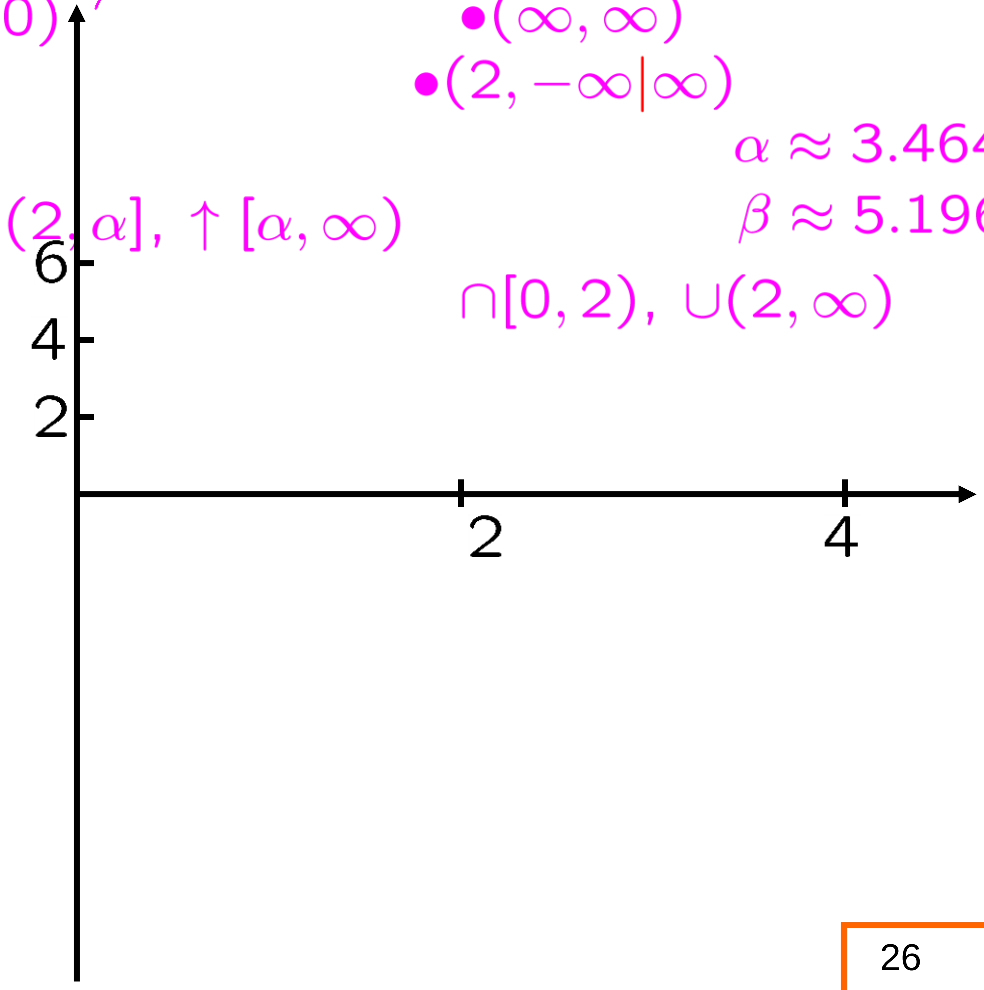

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domain $\supseteq [0, \infty) \backslash \{2\}$

neg$(0, 2)$, pos$(2, \infty)$

- $(0, 0)^{\rightarrow}$
- (∞, ∞)
- $(2, -\infty | \infty)$
- $(\alpha, \beta)^{\rightarrow}$

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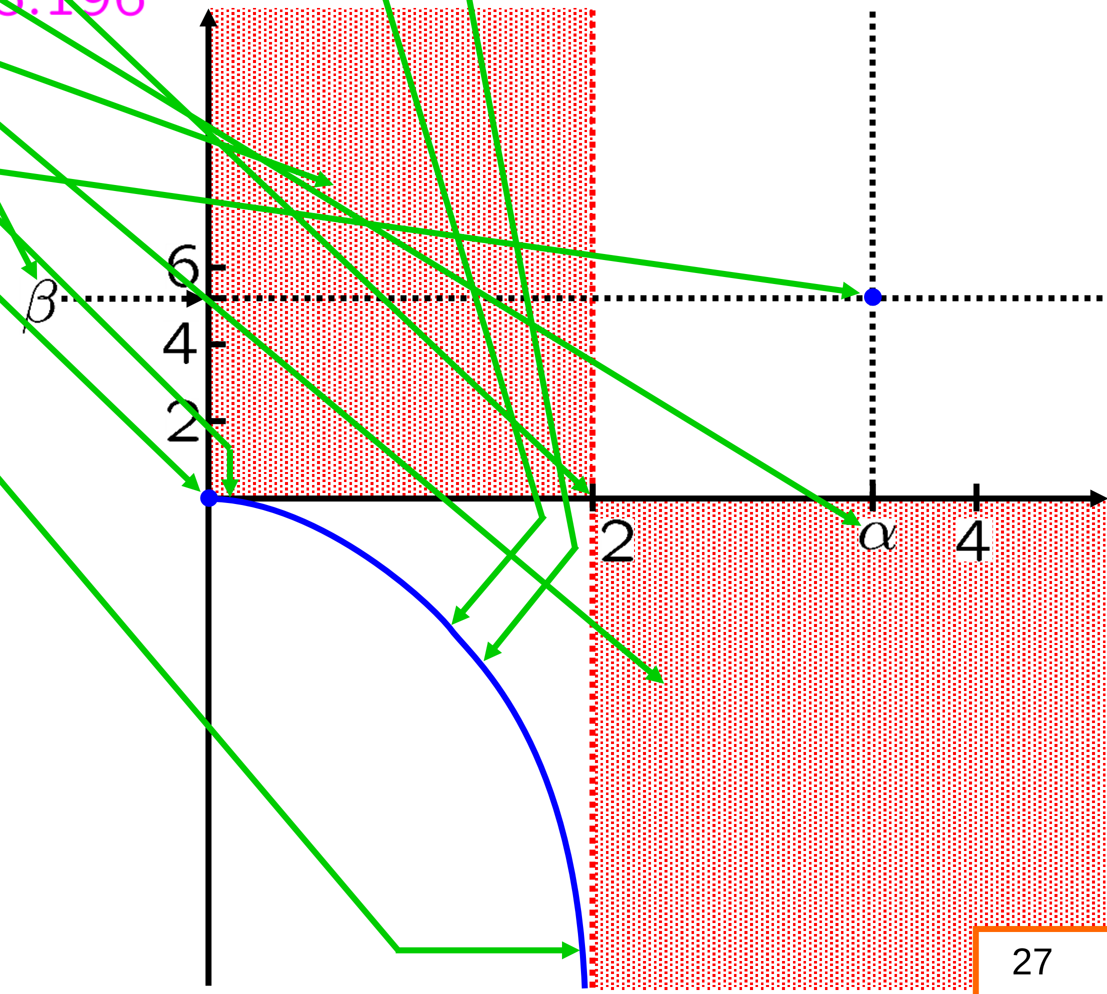

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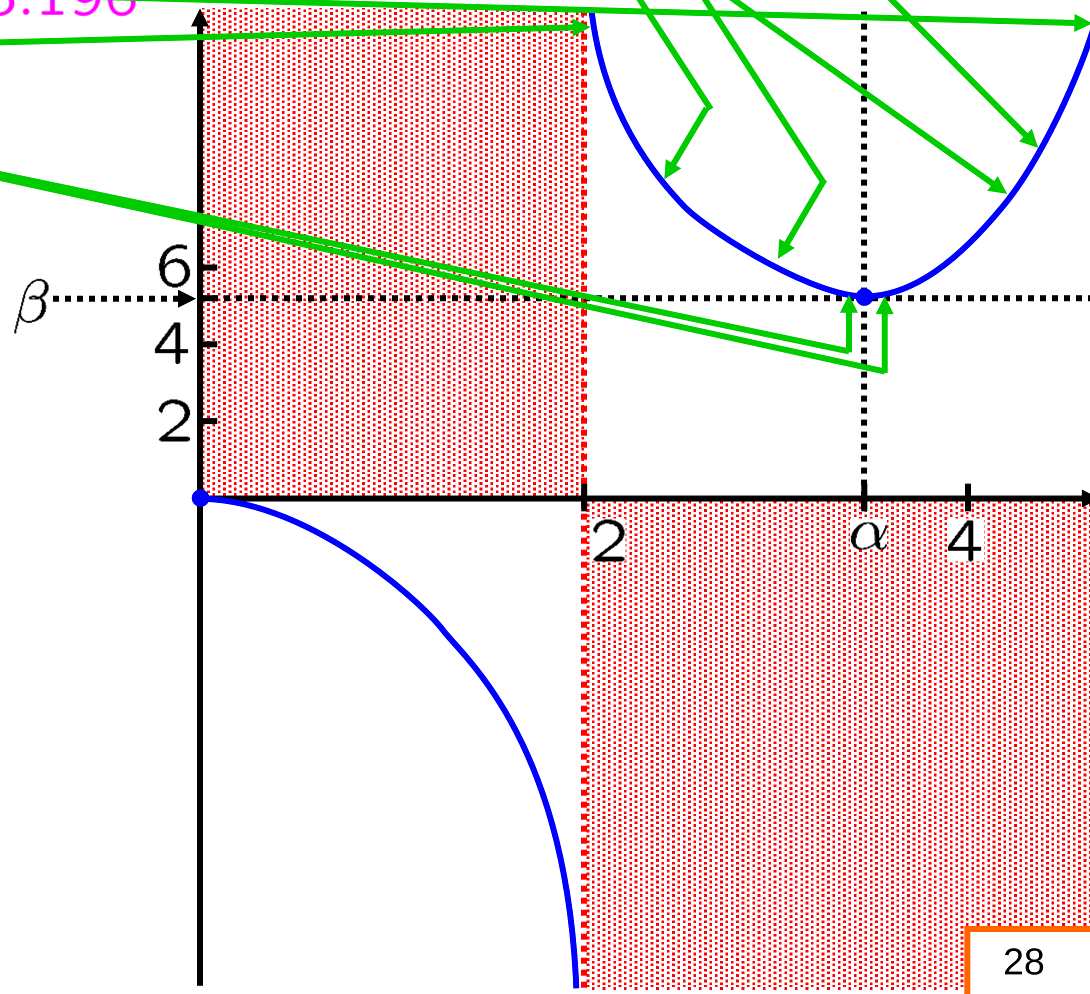

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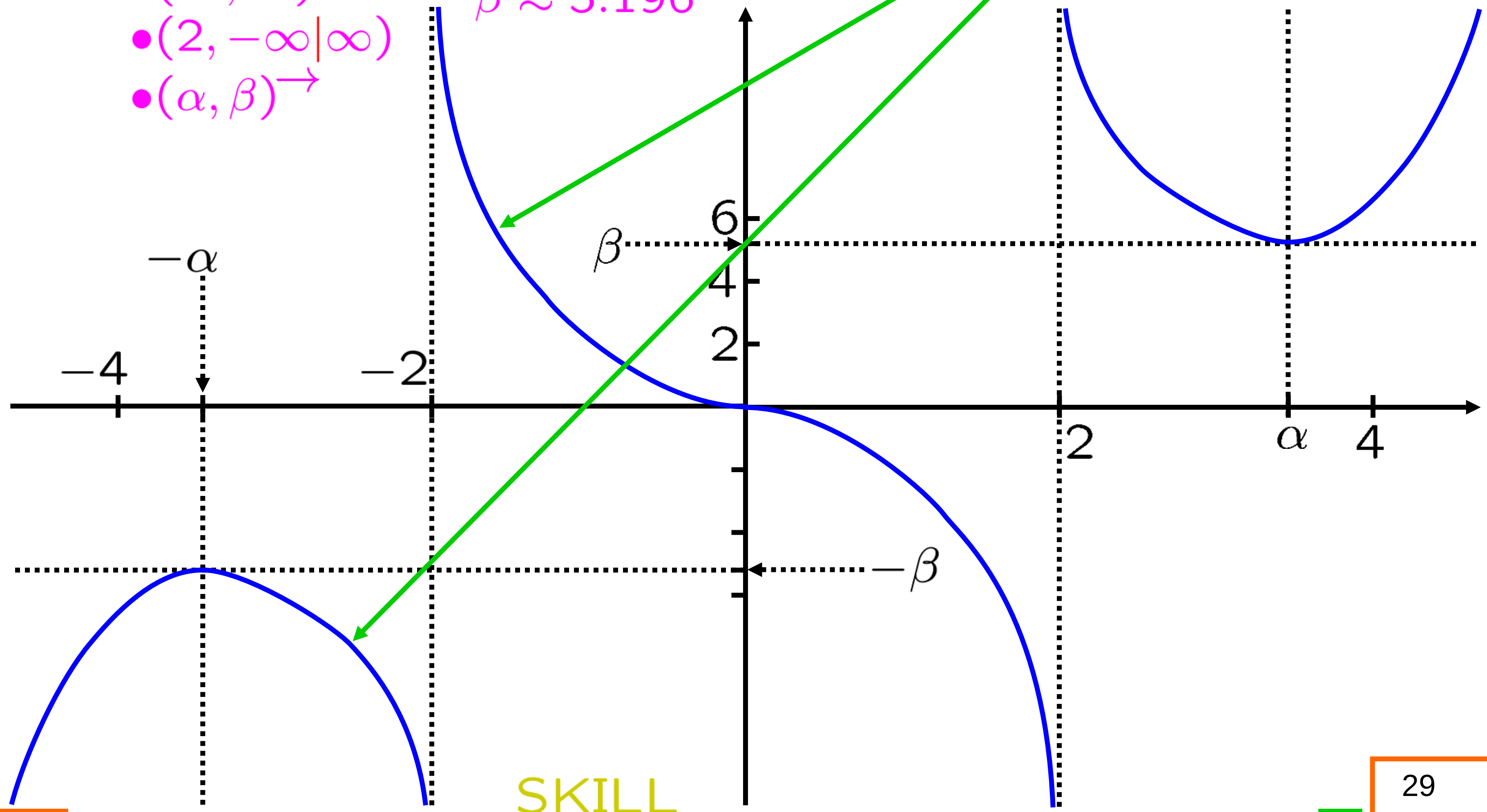
$\downarrow [0, 2)$, $\downarrow (2, \alpha]$, $\uparrow [\alpha, \infty)$

$\cap [0, 2)$, $\cup (2, \infty)$



SKILL
curve sketching

SKILL

curve sketching

Whitman problems

§5.5, p. 103–104, #1-32