CALCULUS
Even more graphing problems

EXAMPLE: Sketch the graph of $y = \ln(9 - x^2)$.

A. Symmetry (over $[0, \infty)$; reflect thru y-axis)
even (y-axis symmetry)

(i) even function: $f(-x) = f(x)$

(ii) odd function: $f(-x) = -(f(x))$

(iii) periodic function: $f(x + p) = f(x)$

EXAMPLE: Sketch the graph of $y = \ln(9 - x^2)$. $\ln 9 \approx 2.197$

$\sqrt{8} \approx 2.828$

A. Symmetry (over $[0,\infty)$; reflect thru y-axis)
even (y-axis symmetry)

B. Intervals of Positivity or Negativity, and pos$[0,\sqrt{8})$ neg$(\sqrt{8},3]$

(i) domain $\supseteq [0,3)$

(ii) x,y-intercepts •$(0, \ln 9)$ •$(\sqrt{8}, 0)$

(iii) vertical, horizontal asymptotes •$(3^-, -\infty)$

C. Intervals of Increase or Decrease

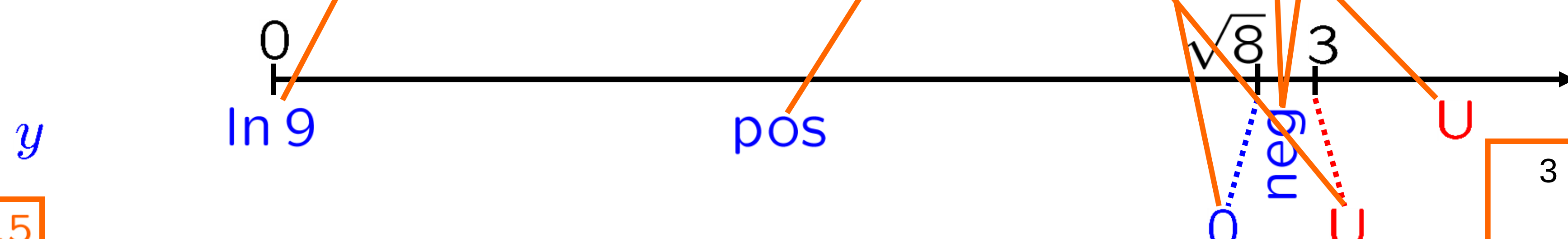
$y = \ln(9 - x^2)$

$\ln(9 - x^2) = 0$ and $x \geq 0$

$9 - x^2 = e^0 = 1$

$8 = 9 - 1 = x^2$

$x = \sqrt{8}$



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C. Intervals of Increase or Decrease $\downarrow [0, 3)$

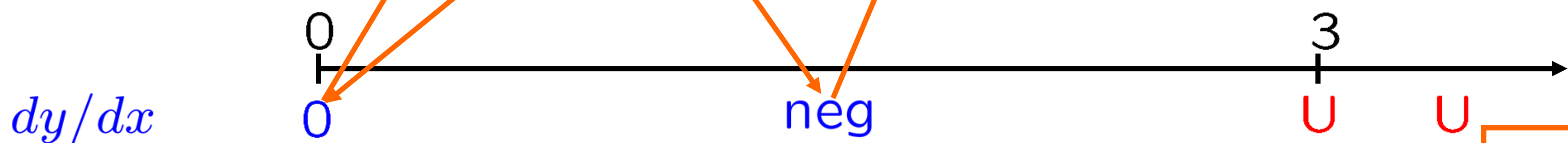
D. Concavity and Points of Inflection

$y = \ln(9 - x^2)$ on $0 \leq x < 3$

$$\frac{dy}{dx} = \frac{-2x}{9 - x^2}$$

$-2x$ ← neg on $0 < x$, 0 at $x = 0$

$9 - x^2$ ← pos on $0 \leq x < 3$



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$y = \ln(9 - x^2)$ on $0 \leq x < 3$

$$\frac{dy}{dx} = \frac{-2x}{9 - x^2}$$

$$\frac{d^2y}{dx^2} = \frac{(9 - x^2)(-2) - (-2x)(-2x)}{(9 - x^2)^2} = \frac{(-18 + 2x^2) - 4x^2}{(9 - x^2)^2}$$

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C. Intervals of Increase or Decrease $\downarrow [0, 3)$

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$y = \ln(9 - x^2)$ on $0 \leq x < 3$

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$$\frac{d^2y}{dx^2} = \frac{(-18 \boxed{+ 2x^2}) \boxed{- 4x^2}}{(9 - x^2)^2} = \frac{-18 \boxed{- 2x^2}}{(9 - x^2)^2} \frac{-18 + 2x^2) - 4x^2}{(9 - x^2)^2}$$

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neg

pos on $0 \le x < 3$

0 3

d^2y/dx^2 neg neg U U

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pos$[0, \sqrt{8})$

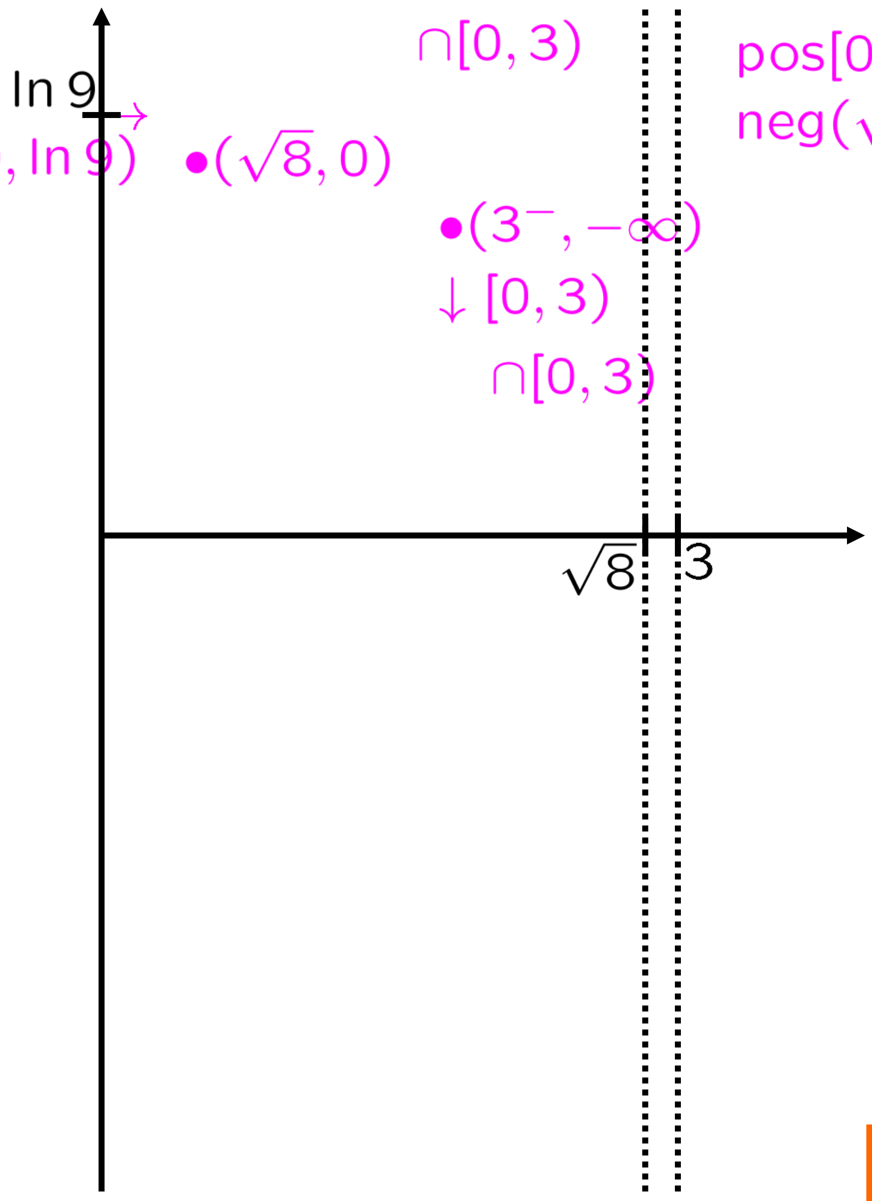
neg$(\sqrt{8}, 3]$

$\bullet(0, \ln 9)$ $\bullet(\sqrt{8}, 0)$

$\bullet(3^-, -\infty)$

$\downarrow [0, 3)$

$\cap [0, 3)$



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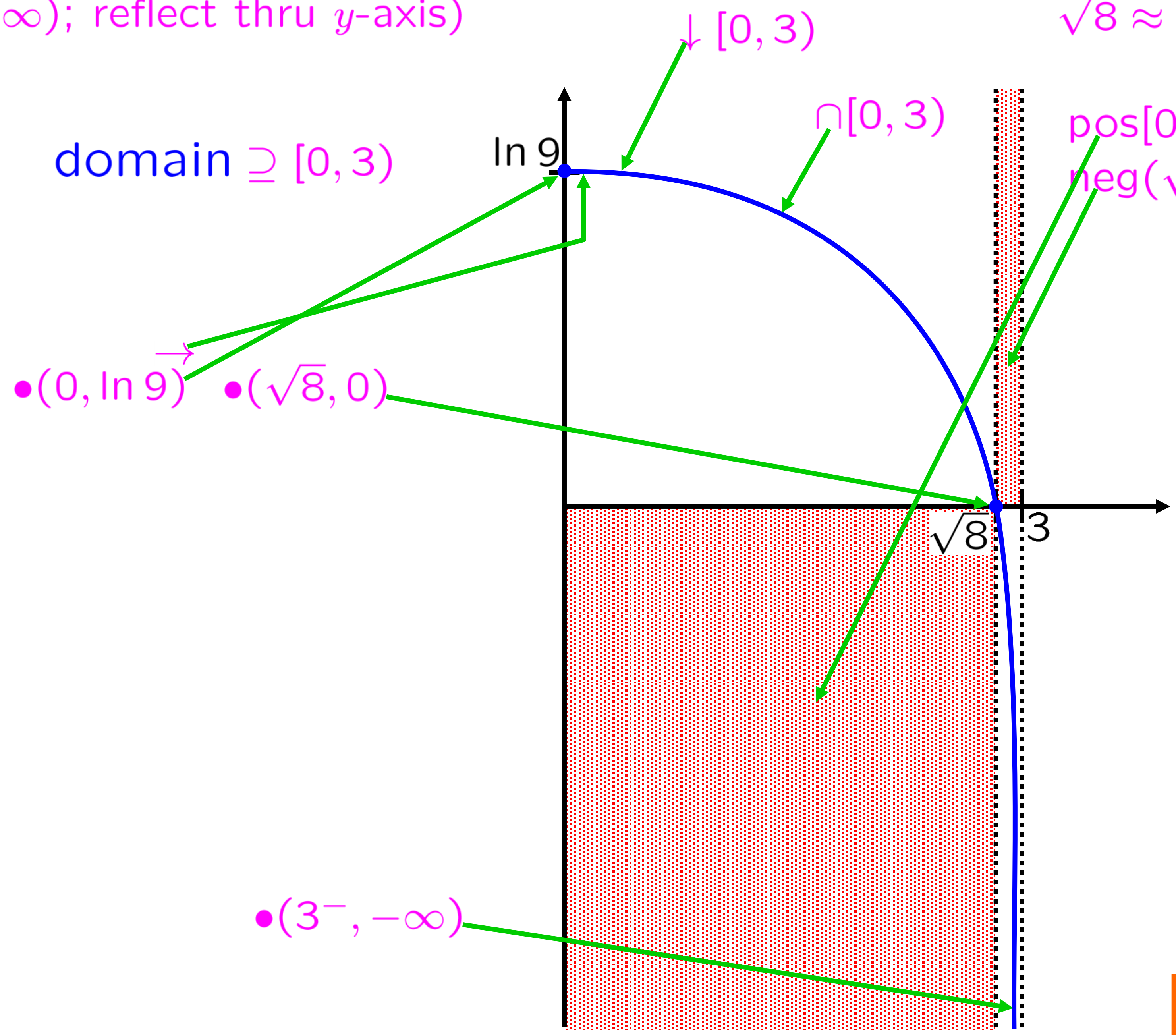

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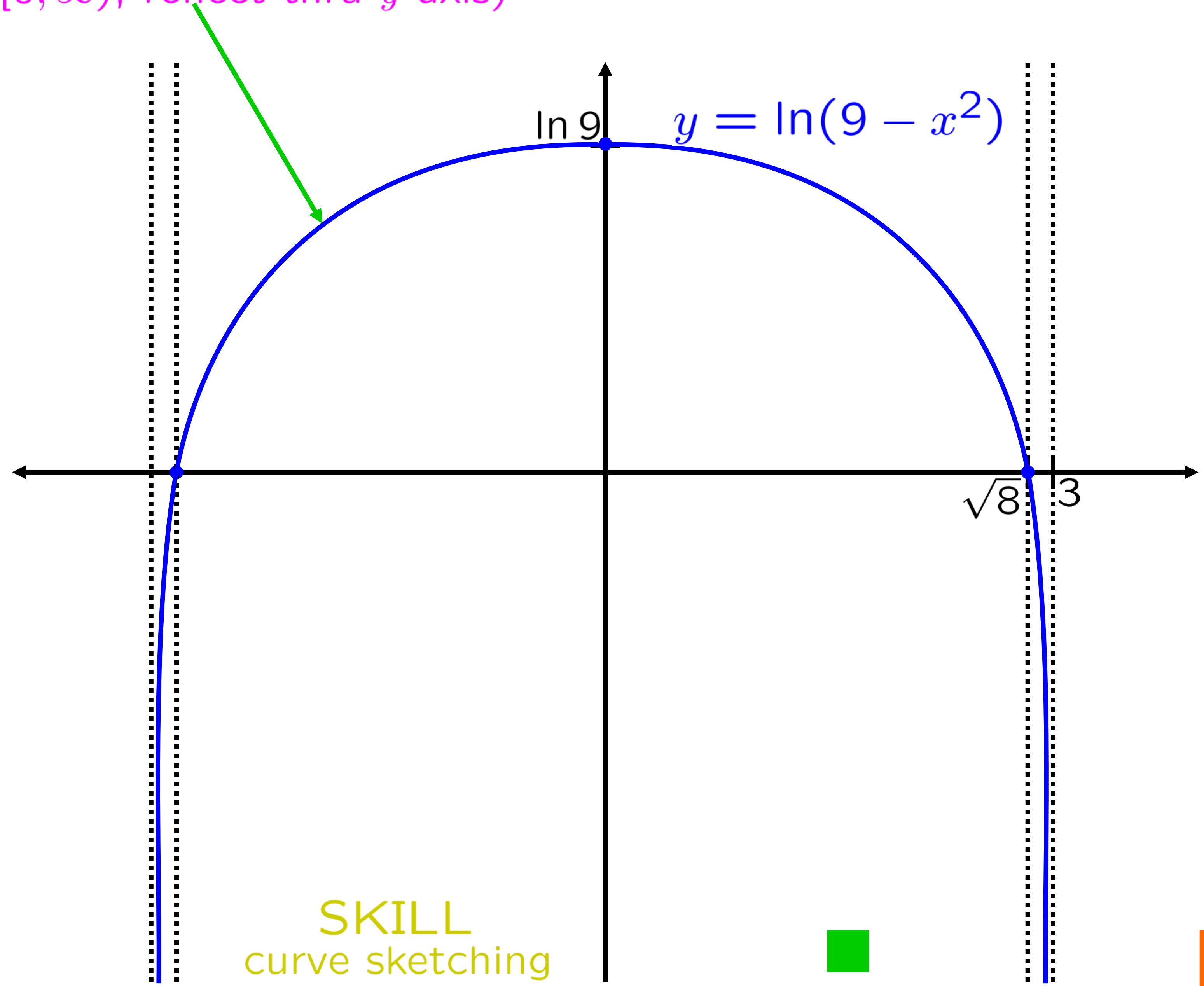

SKILL
curve sketching

EXAMPLE: Sketch the graph of $y = [\sin(2\pi x)] + [\cos(\pi x)]$.

A. Symmetry 2-periodic (over $[0, 2]$; then repeat)

(i) even function: $f(-x) = f(x)$

(ii) odd function: $f(-x) = -(f(x))$

(iii) periodic function: $f(x + p) = f(x)$

$\sin(x)$ is 2π-periodic in x.

$\sin(2\pi x)$ is 1-periodic in x.

$\cos(x)$ is 2π-periodic in x.

$\cos(\pi x)$ is 2-periodic in x.

$\sin(2\pi x)$ is 2-periodic in x.

$[\sin(2\pi x)] + [\cos(\pi x)]$ is 2-periodic in x.

EXAMPLE: Sketch the graph of $y = [\sin(2\pi x)] + [\cos(\pi x)]$.

A. Symmetry 2-periodic (over $[0, 2]$; then repeat)

B. Intervals of Positivity or Negativity, and

(i) domain

(ii) x,y-intercepts

(iii) vertical, horizontal asymptotes

$$y = [\sin(2\pi x)] + [\cos(\pi x)]$$

$[0 \leq x \leq 2]$ and $[[\sin(2\pi x)] + [\cos(\pi x)] = 0]$

$$2[\sin(\pi x)][\cos(\pi x)] = \sin(2\pi x) = -\cos(\pi x)$$

DOUBLE ANGLE

$\phi :\to \pi x$

$$2[\sin(\phi)][\cos(\phi)] = \sin(2\phi)$$ ← DOUBLE ANGLE FORMULA

$\psi :\to \phi$

$$\sin(\phi + \psi) = [\sin(\phi)][\cos(\psi)] + [\sin(\psi)][\cos(\phi)]$$

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B. Intervals of Positivity or Negativity, and

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$$y = [\sin(2\pi x)] + [\cos(\pi x)]$$

$$[0 \leq x \leq 2] \quad \text{and} \quad [[\sin(2\pi x)] + [\cos(\pi x)] = 0]$$

$$2[\sin(\pi x)][\cos(\pi x)] = \sin(2\pi x) = -\cos(\pi x)$$

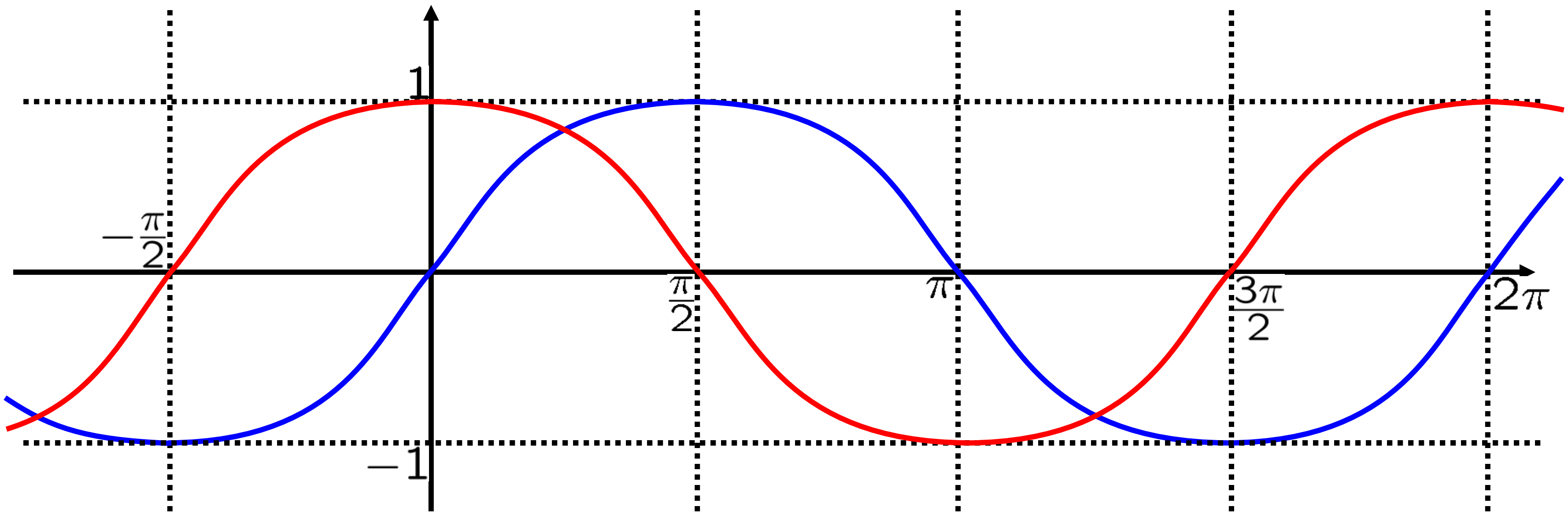
$$2[\sin(\pi x)][\cos(\pi x)] + [\cos(\pi x)] = 0$$

$$[2[\sin(\pi x)] + 1][\cos(\pi x)] = 0$$

$$[(2[\sin(\pi x)] + 1 = 0) \text{ or } (\cos(\pi x) = 0)]$$

$$[(\sin(\pi x) = -1/2) \text{ or } (\cos(\pi x) = 0)] \quad \text{and} \quad [0 \leq x \leq 2]$$

$y = \cos(x)$ $y = \sin(x)$



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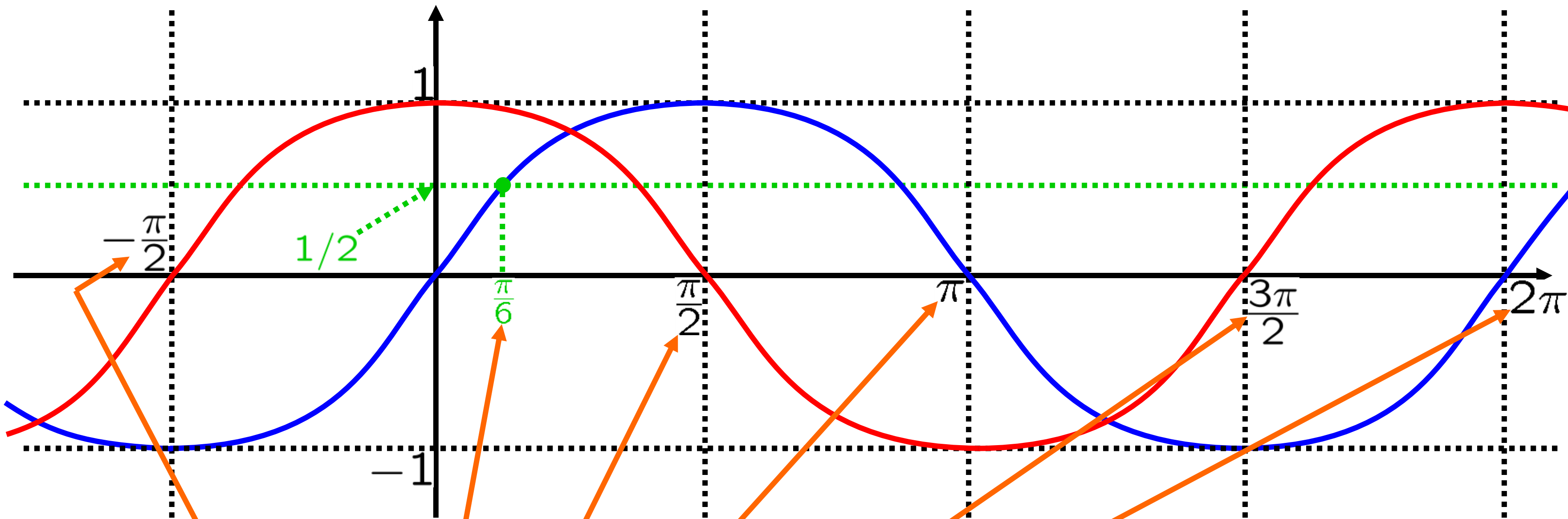
$x :\to \pi x$

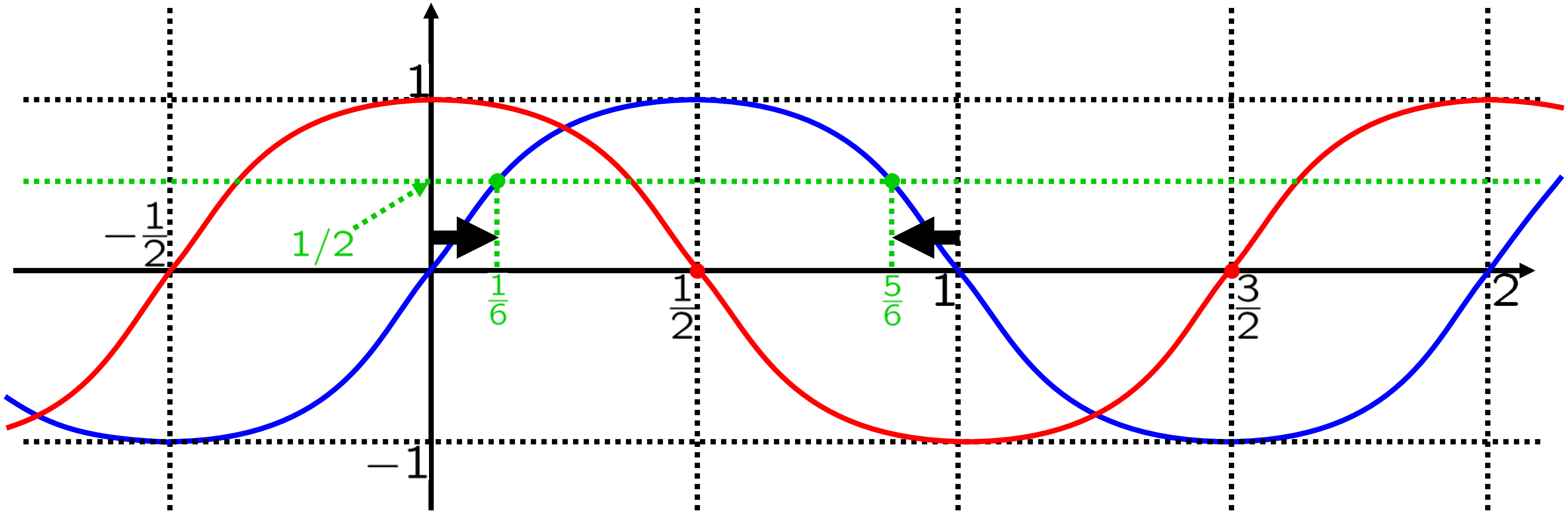
$y = \cos(x)$ $\qquad$ $y = \sin(x)$



$[(\sin(\pi x) = -1/2)$ or $(\cos(\pi x) = 0)]$ and $[0 \leq x \leq 2]$

DIVIDE BY π

$y = \cos(\pi x)$ $y = \sin(\pi x)$



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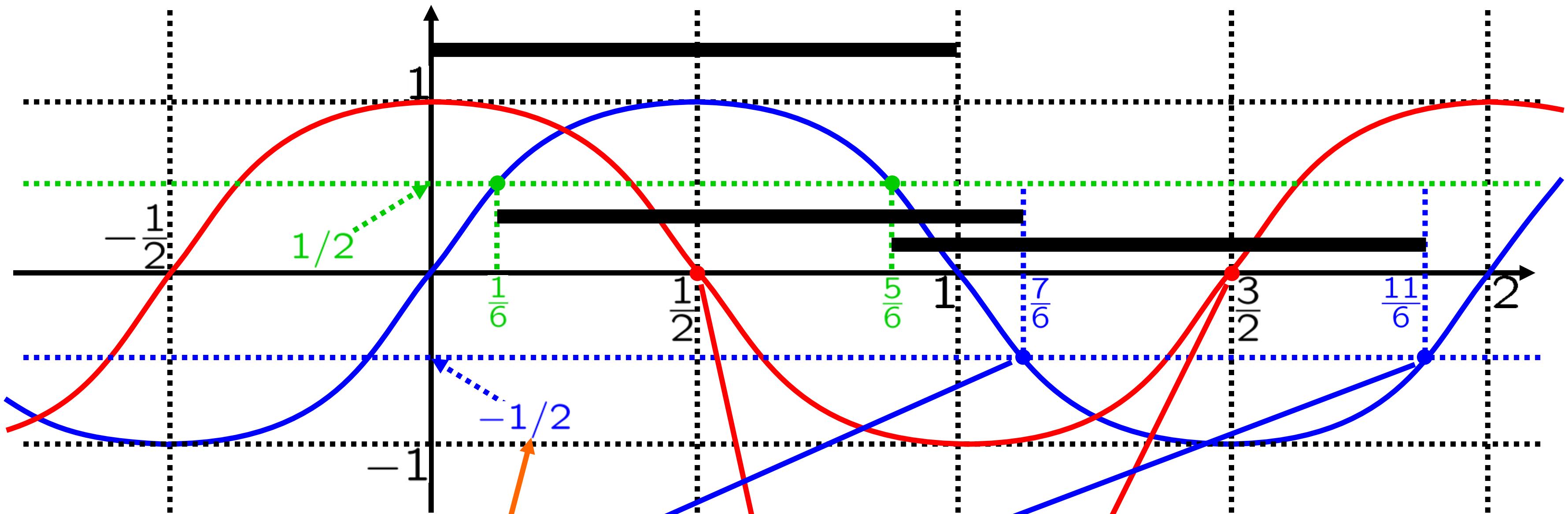

$[(\sin(\pi x) = -1/2)$ or $(\cos(\pi x) = 0)]$ $\quad$ and $\quad$ $[0 \leq x \leq 2]$

$x = \frac{7}{6}$ or $x = \frac{11}{6}$ or $x = \frac{1}{2}$ or $x = \frac{3}{2}$

$y = [\sin(2\pi x)] + [\cos(\pi x)]$

0 $\quad \frac{1}{2} \quad \frac{7}{6} \quad \frac{3}{2} \quad \frac{11}{6}$ $\quad$ 2

2-periodic

y 1 pos 0 neg 0 pos 0 neg 0 pos 1

EXAMPLE: Sketch the graph of $y = [\sin(2\pi x)] + [\cos(\pi x)]$.

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(i) domain $\supseteq [0, 2]$ $\bullet(\frac{1}{2}, 0)$, $\bullet(\frac{7}{6}, 0)$, $\bullet(\frac{3}{2}, 0)$, $\bullet(\frac{11}{6}, 0)$

$\bullet(0, 1)$ (ii) x,y-intercepts no asymptotes

(iii) vertical, horizontal asymptotes

C. Intervals of Increase or Decrease

pos$[0, \frac{1}{2})$, neg$(\frac{1}{2}, \frac{7}{6})$, pos$(\frac{7}{6}, \frac{3}{2})$, neg$(\frac{3}{2}, \frac{11}{6})$, pos$(\frac{11}{6}, 2]$

$$y = [\sin(2\pi x)] + [\cos(\pi x)]$$

$[0 \le x \le 2]$ and $[[\sin(2\pi x)] + [\cos(\pi x)] = 0]$

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$$y = [\sin(2\pi x)] + [\cos(\pi x)]$$

$$\frac{dy}{dx} = 2\pi[\cos(2\pi x)] - \pi[\sin(\pi x)]$$

$$0 = 2\pi[\cos(2\pi x)] - \pi[\sin(\pi x)]$$

Solve: $2[\cos(2\pi x)] = \sin(\pi x)$

$(0 \leq x \leq 2)$

$y = 2[\cos(2\pi x)]$

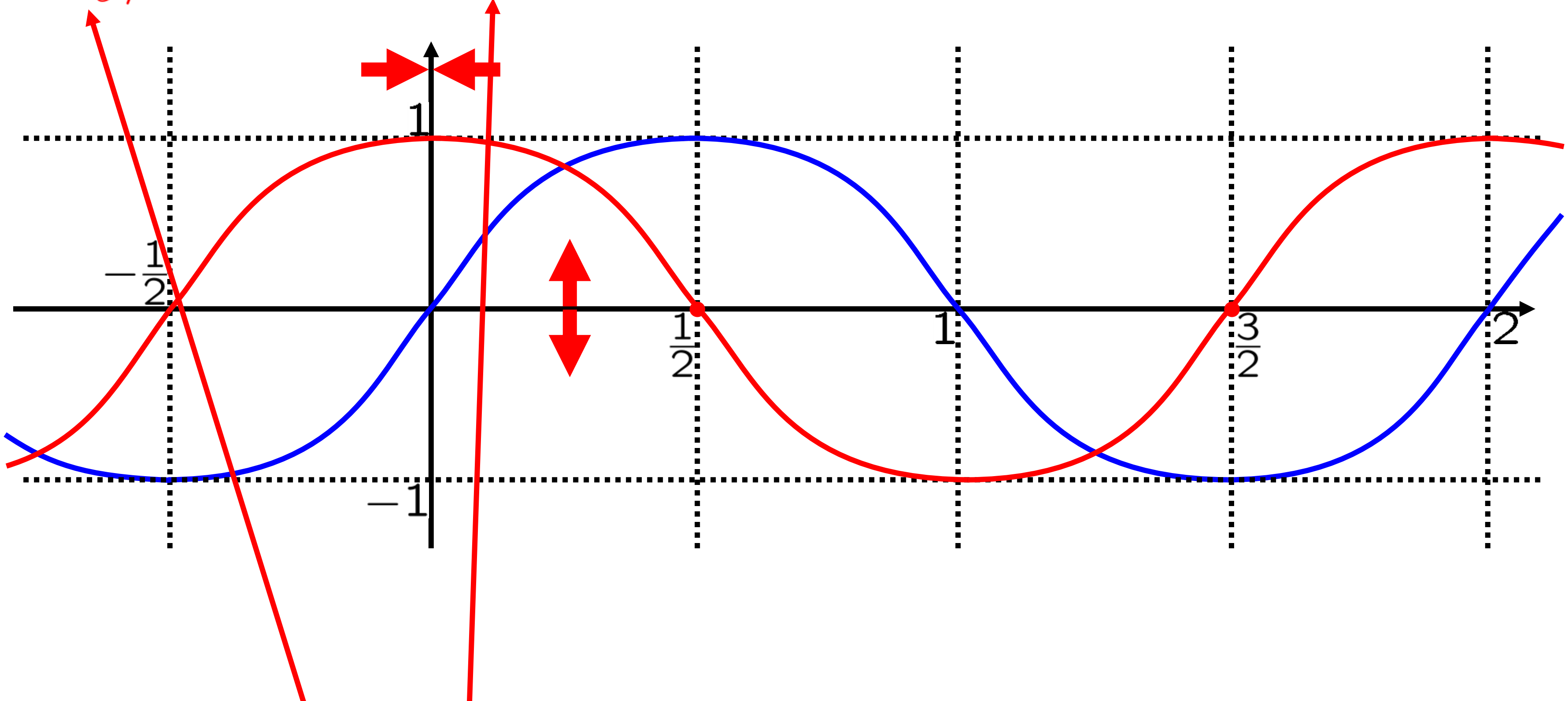
DOUBLE: $y = \cos(\pi x)$

$y :\to y/2$

$x :\to 2x$

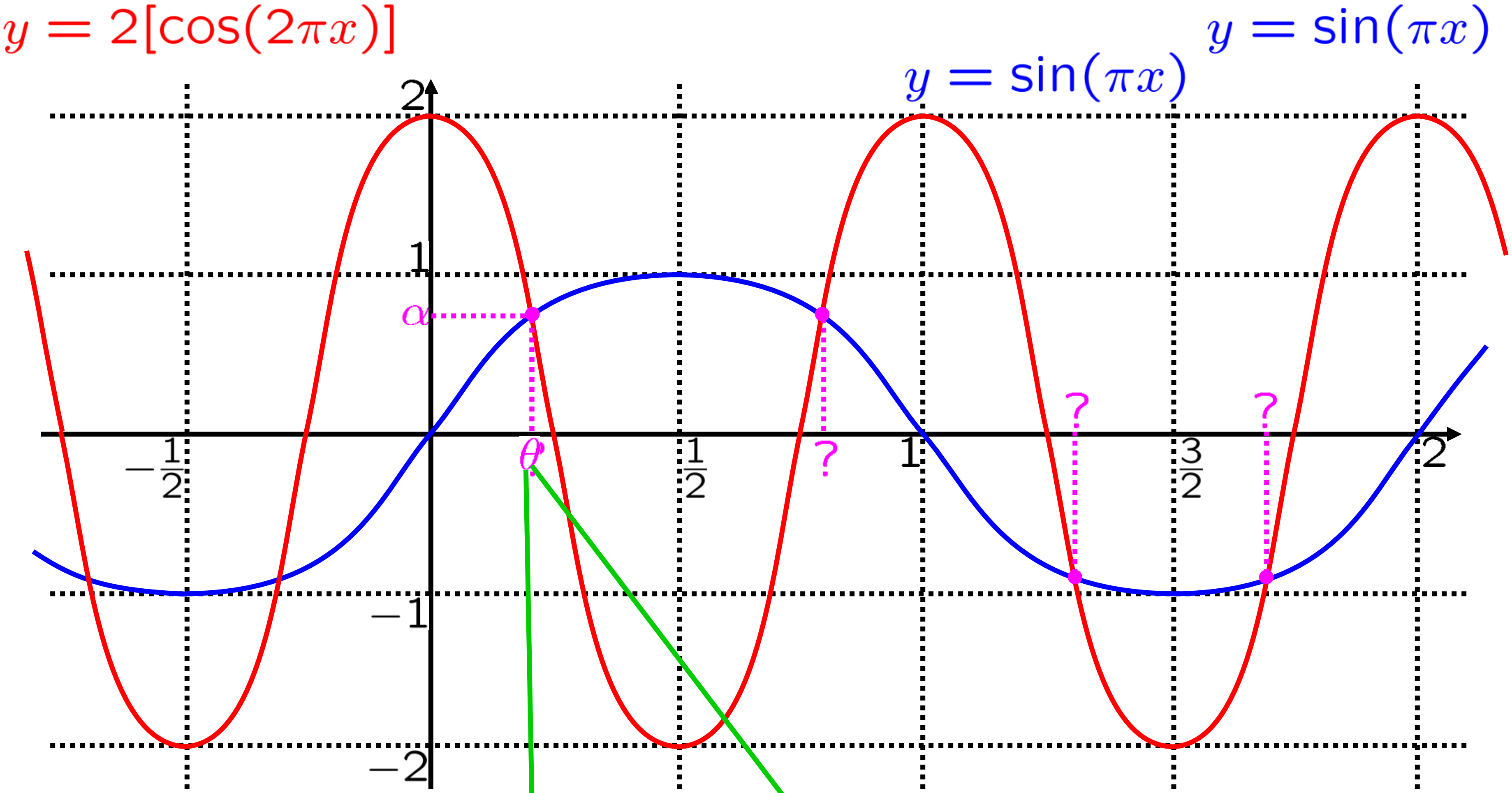
NO CHANGE TO BLUE GRAPH

$y = \sin(\pi x)$



Solve: $2[\cos(2\pi x)] = \sin(\pi x)$

$(0 \leq x \leq 2)$



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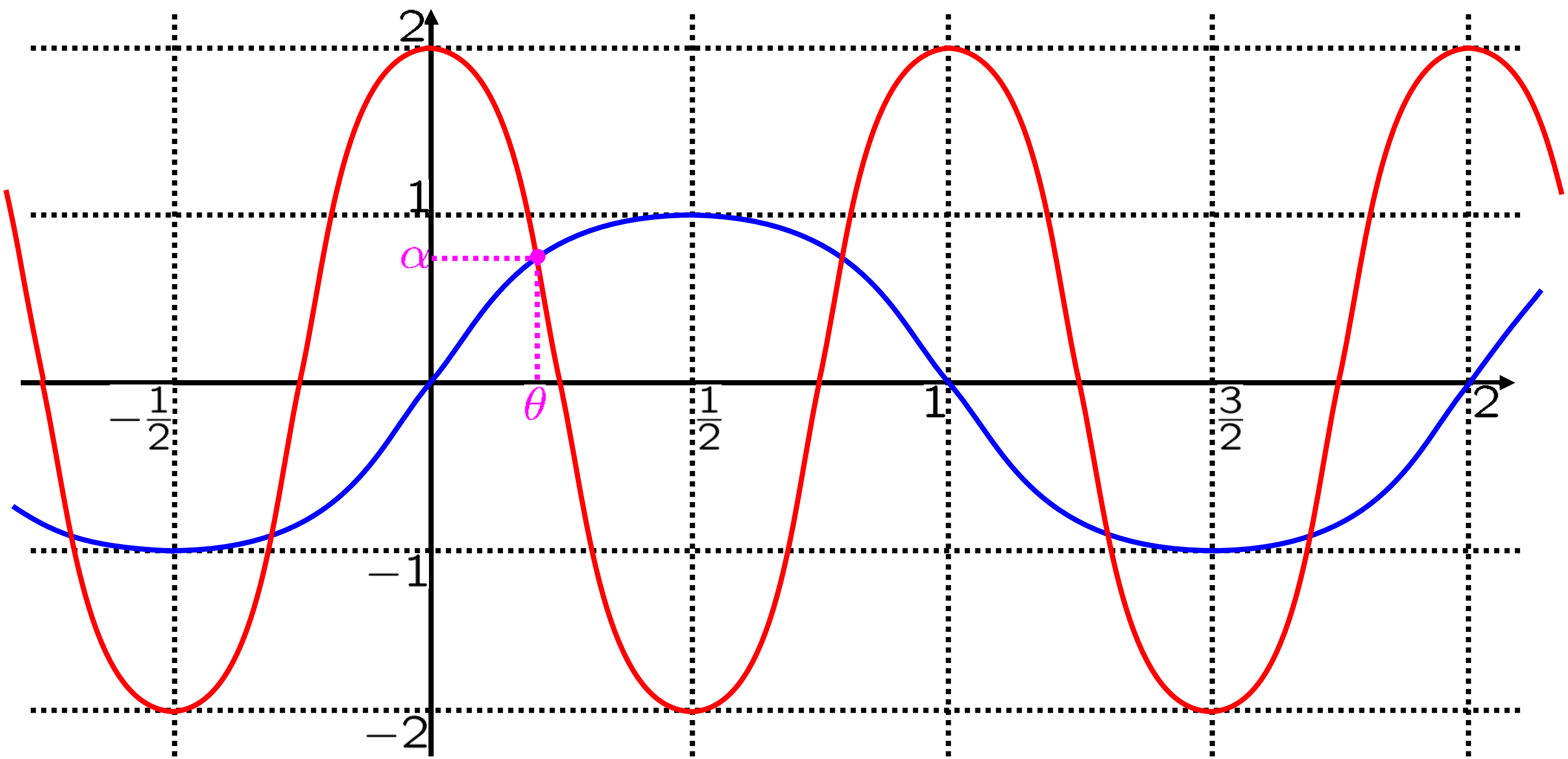
$(0 \leq x \leq 2)$

$y = 2[\cos(2\pi x)]$ $\qquad$ $y = \sin(\pi x)$



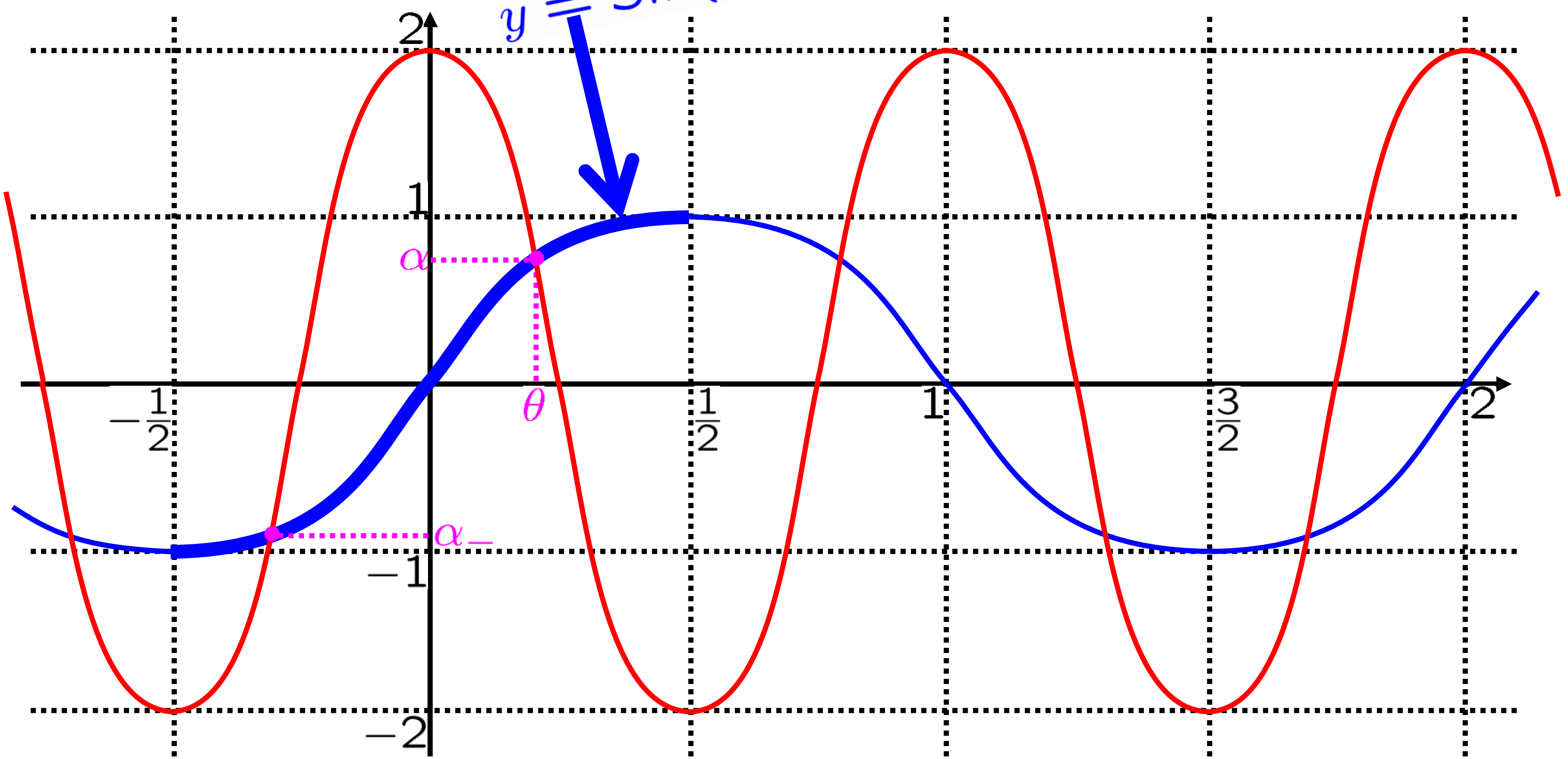
$$2[\cos(2\pi\theta)] = \sin(\pi\theta) =: \alpha$$

$$2[1 - 2\alpha^2] = \alpha$$

$$\sin^2(\pi\theta) = \alpha^2$$

$$\cos(2\pi\theta) = [\cos^2(\pi\theta)] - [\sin^2(\pi\theta)] = [1 - \alpha^2] - [\alpha^2] = 1 - 2\alpha^2$$

$$\cos^2(\pi\theta) = 1 - \alpha^2$$



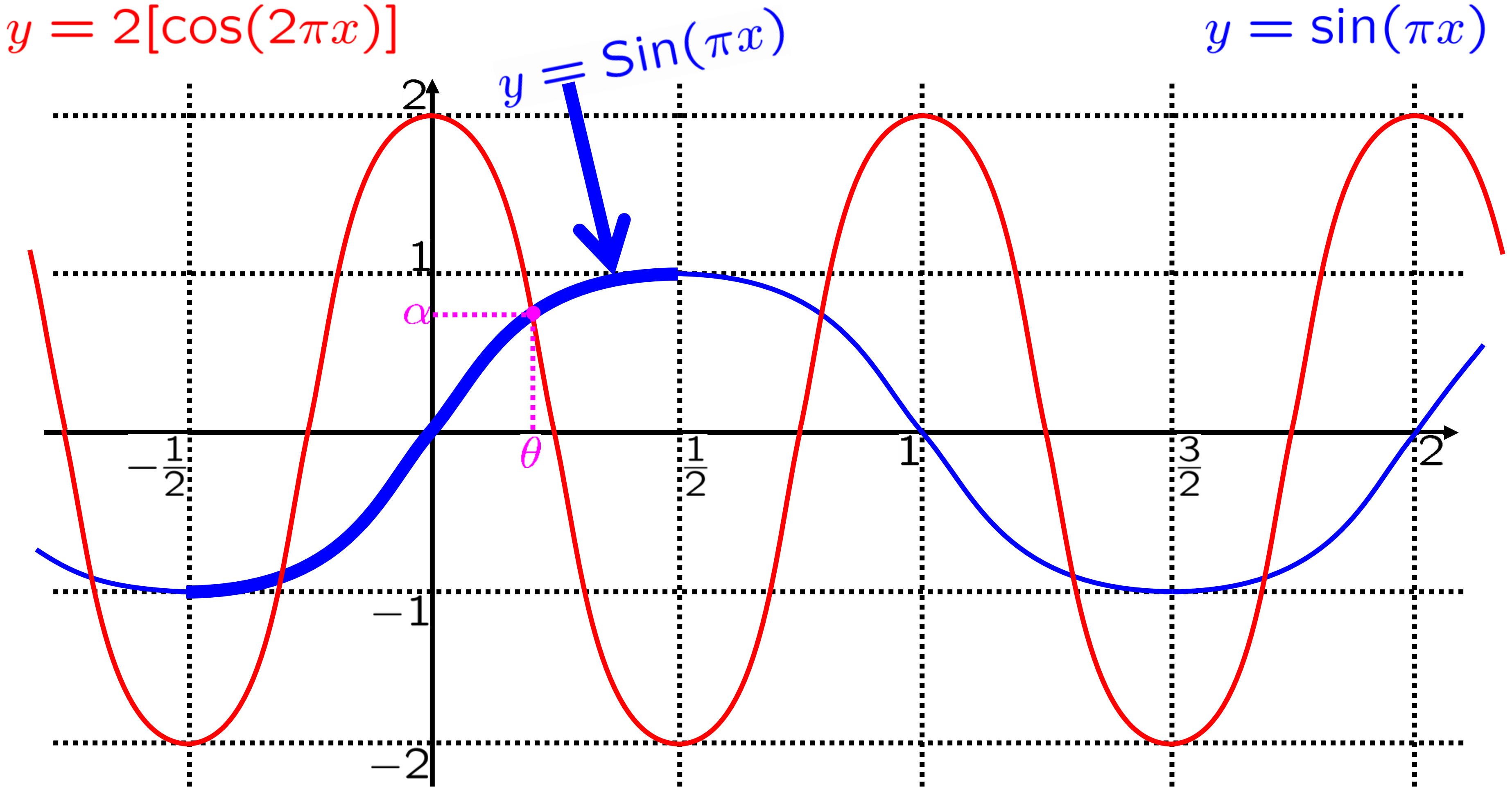
$$2[\cos(2\pi\theta)] = \sin(\pi\theta) =: \alpha$$

$$2[1 - 2\alpha^2] = \alpha$$

$$0 = 4\alpha^2 + \alpha - 2$$

$$\alpha = \frac{-1 + \sqrt{33}}{8}$$

$$\alpha_-, \alpha \in \left\{ \frac{-1 \pm \sqrt{1^2 - 4(4)(-2)}}{(2)(4)} \right\} = \left\{ \frac{-1 \pm \sqrt{33}}{8} \right\}$$



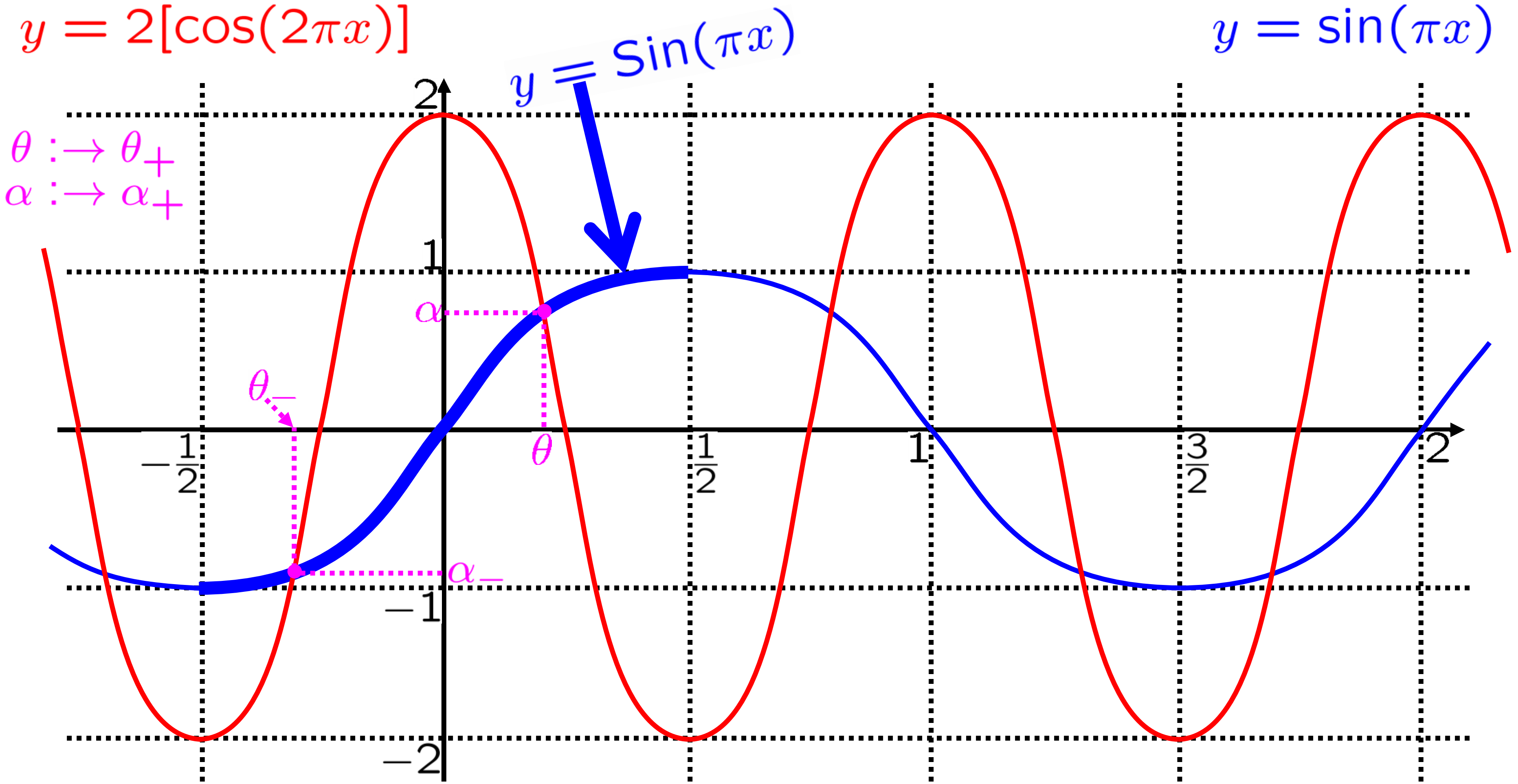
$$2[\cos(2\pi\theta)] = \sin(\pi\theta) =: \alpha$$

$$\alpha = \frac{-1+\sqrt{33}}{8}$$

$$\text{Sin}(\pi\theta) = \alpha$$

$$\pi\theta = \text{Sin}^{-1}(\alpha)$$

$$\theta = \frac{\text{Sin}^{-1}(\alpha)}{\pi}$$



Solve: $2[\cos(2\pi x)] = \sin(\pi x)$

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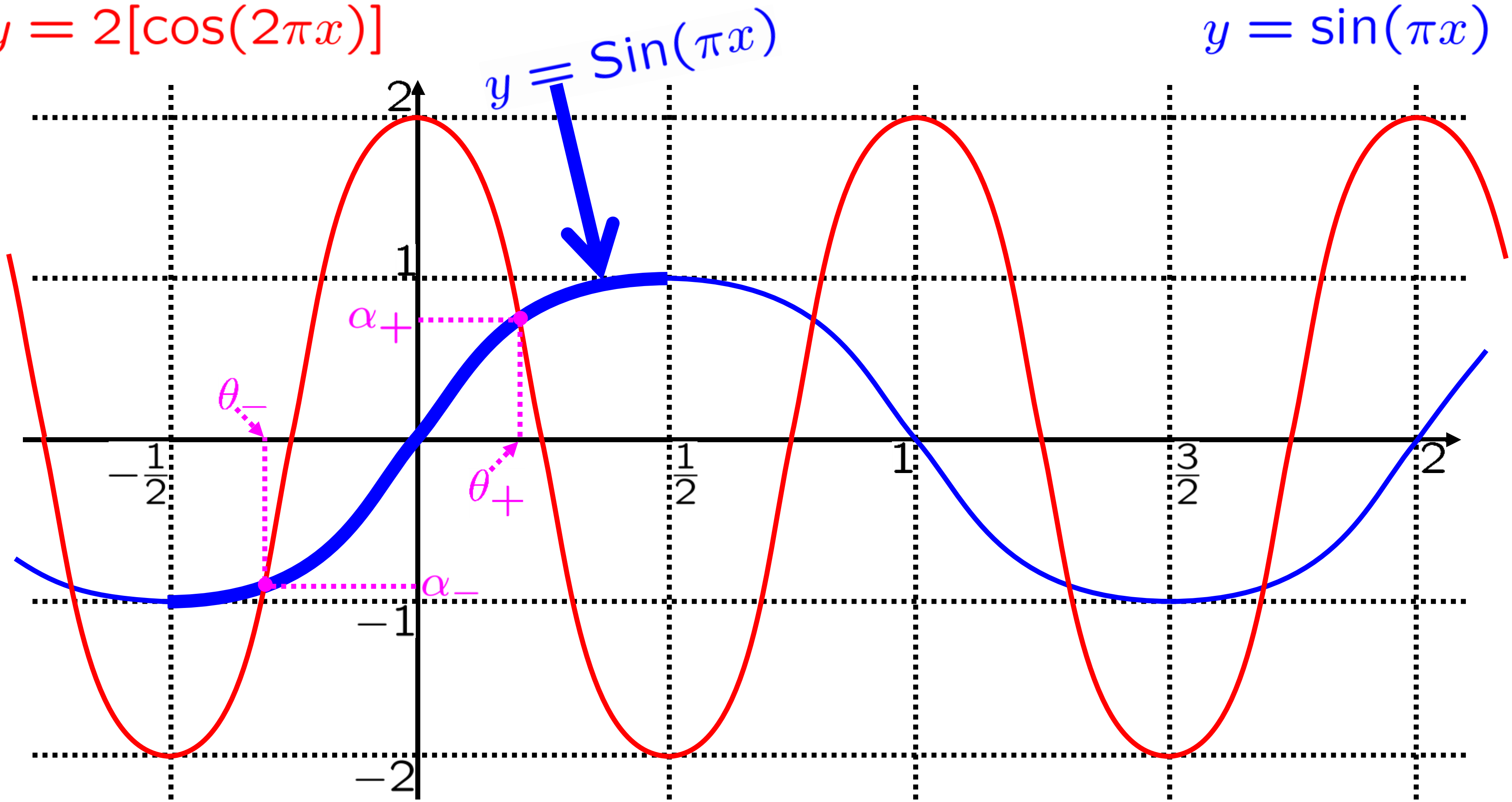
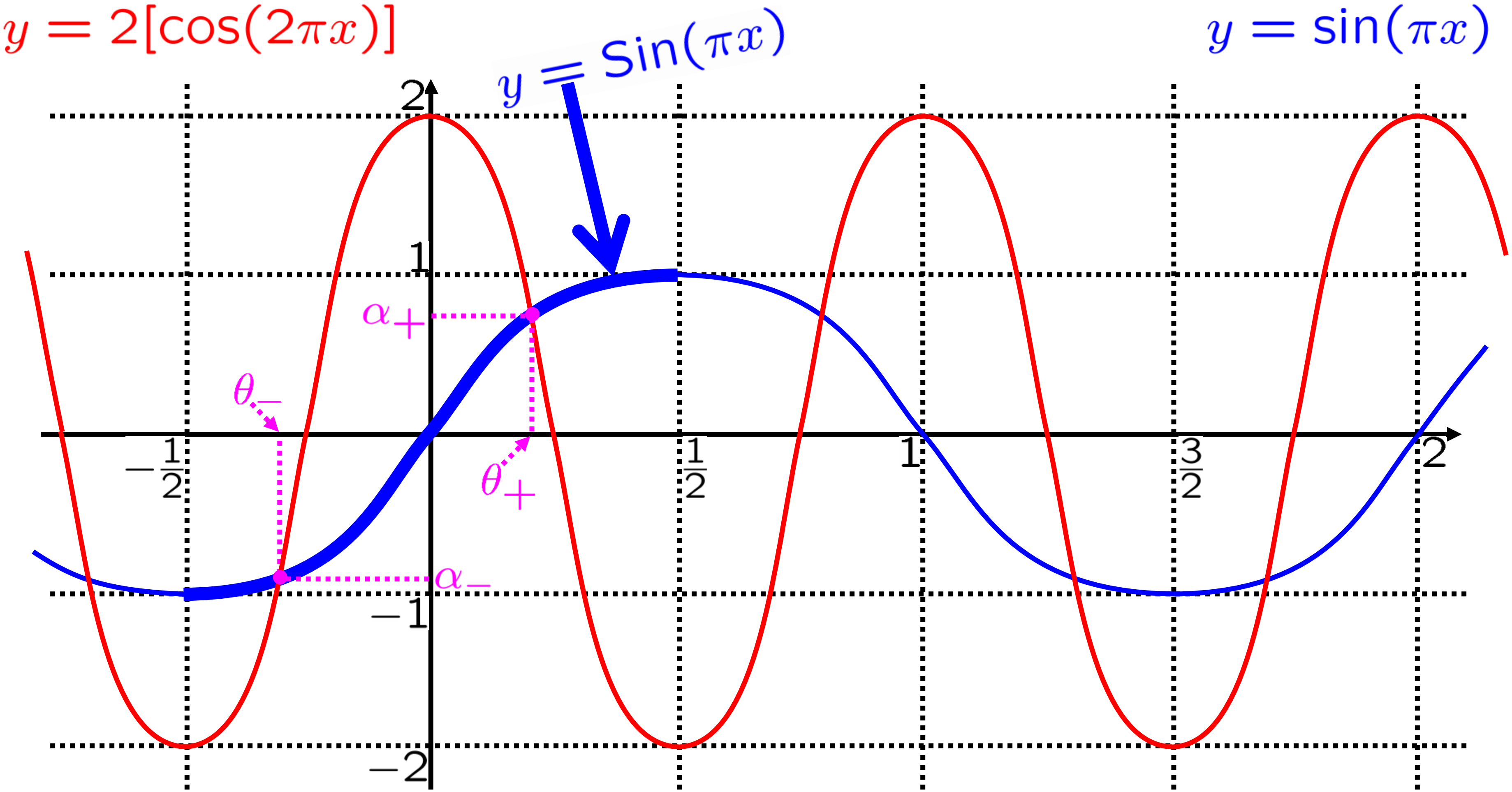


Solve: $2[\cos(2\pi x)] = \sin(\pi x)$

$$\alpha_{\oplus} = \frac{-1 \oplus \sqrt{33}}{8} \approx 0.593$$

$$\theta_{\oplus} = \frac{\text{Sin}^{-1}(\alpha_{\oplus})}{\pi} \approx 0.202$$

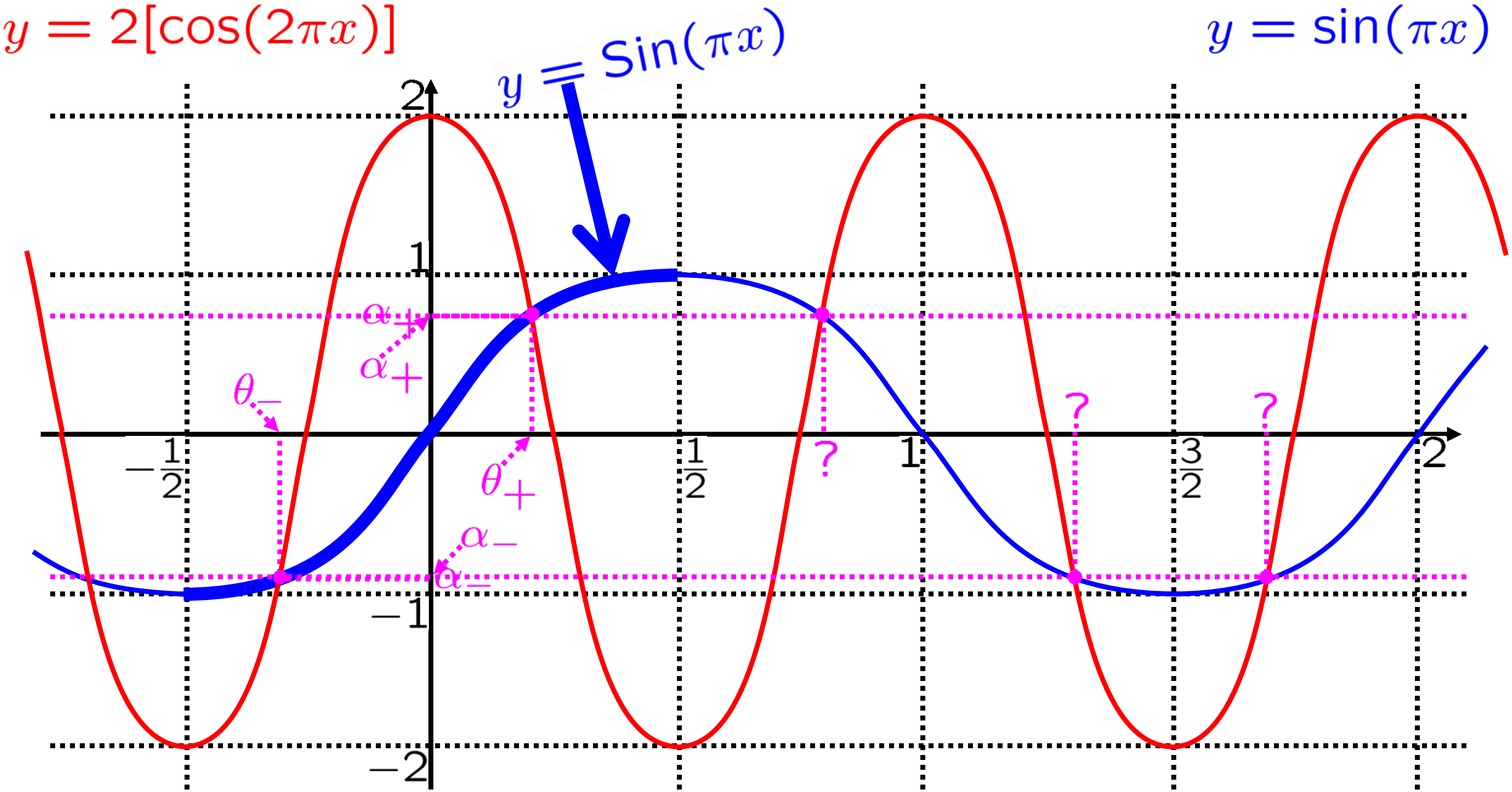
CHANGE TO $-$



Solve: $2[\cos(2\pi x)] = \sin(\pi x)$

$$\alpha_- = \frac{-1 - \sqrt{33}}{8} \approx -0.843$$

$$\theta_- = \frac{\text{Sin}^{-1}(\alpha_-)}{\pi} \approx -0.319$$



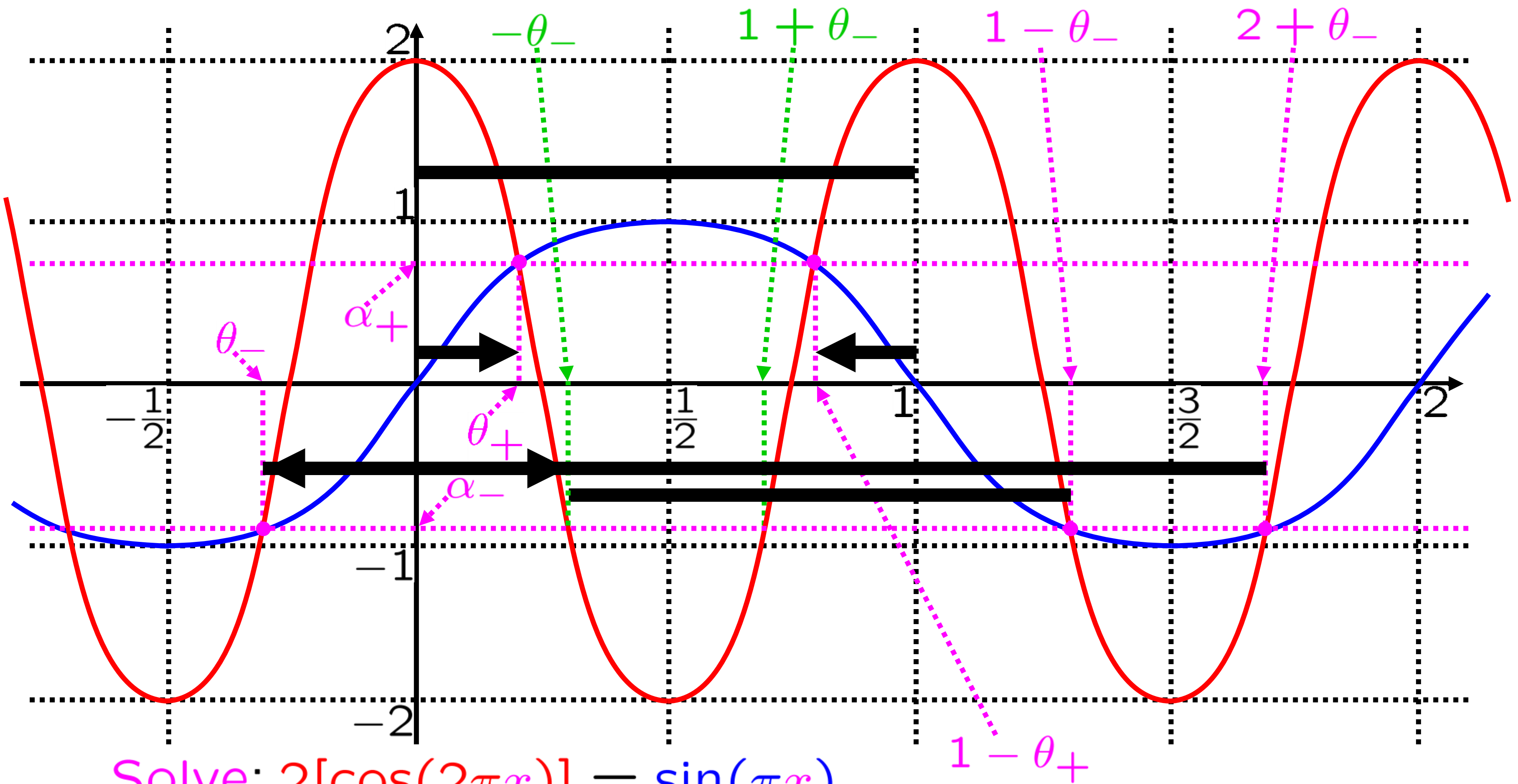
Solve: $2[\cos(2\pi x)] = \sin(\pi x)$

Solutions with $0 \leq x \leq 2$: $\theta_+, ?, ?, ?$

$$\alpha_\pm = \frac{-1 \pm \sqrt{33}}{8}$$

$$\theta_\pm = \frac{\mathrm{Sin}^{-1}(\alpha_\pm)}{\pi}$$

$y = 2[\cos(2\pi x)]$ $\qquad$ $y = \sin(\pi x)$



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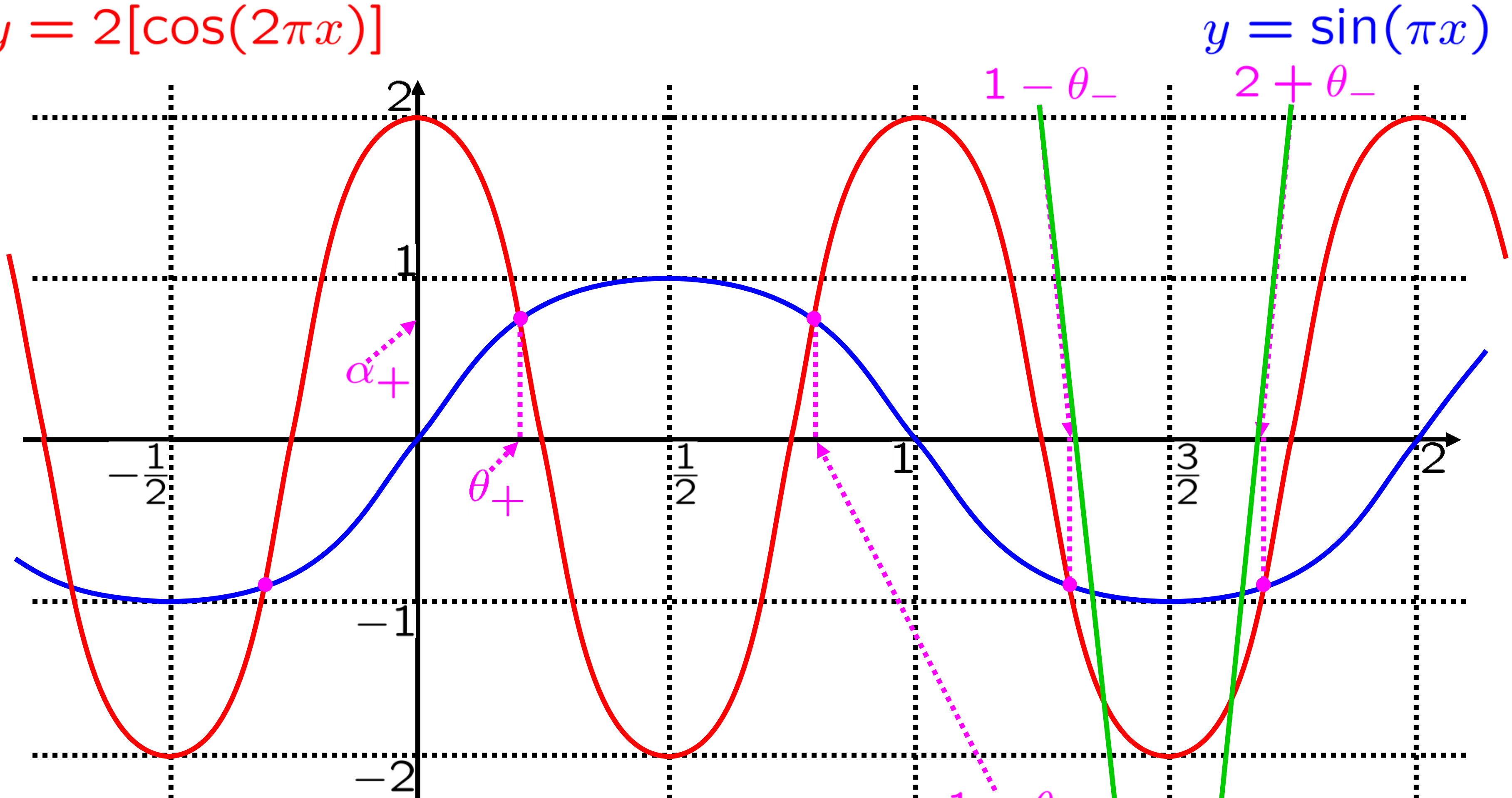

Solve: $2[\cos(2\pi x)] = \sin(\pi x)$

Solutions with $0 \le x \le 2$: $\theta_+, 1 - \theta_+, 1 - \theta_-, 2 + \theta_-$

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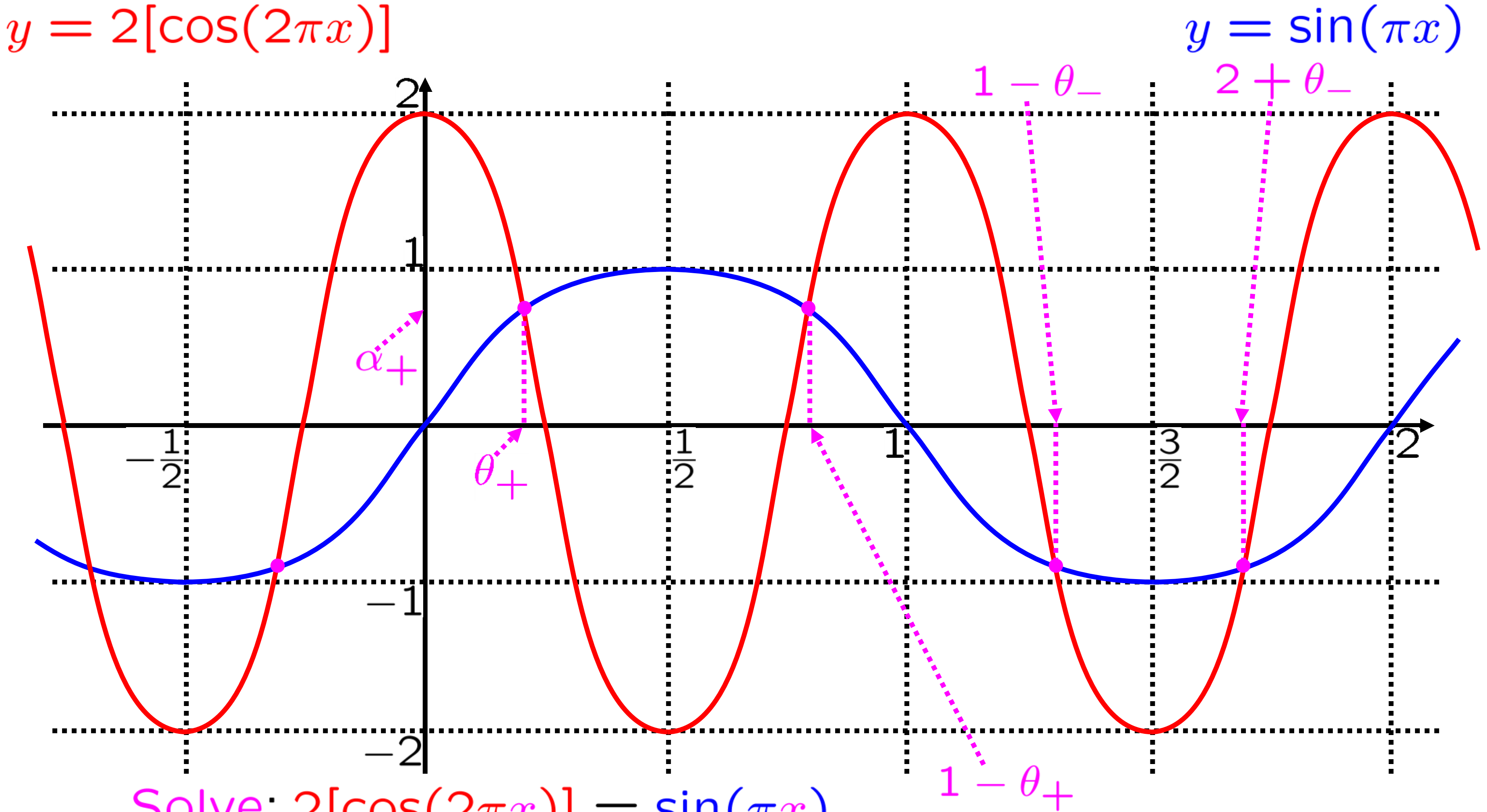

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$$\frac{dy}{dx} = 2\pi[\cos(2\pi x)] - \pi[\sin(\pi x)]$$

	0		θ_+		$1-\theta_+$		$1-\theta_-$		$2+\theta_-$		2
dy/dx		pos	0	neg	0	pos	0	neg	0	pos	

EXAMPLE: Sketch the graph of $y = [\sin(2\pi x)] + [\cos(\pi x)]$.

A. Symmetry 2-periodic (over $[0,2]$; then repeat)

B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0,2]$ $\bullet(\frac{1}{2},0)$, $\bullet(\frac{7}{6},0)$, $\bullet(\frac{3}{2},0)$, $\bullet(\frac{11}{6},0)$

$\bullet(0,1)$ (ii) x,y-intercepts

(iii) vertical, horizontal asymptotes — no asymptotes

pos$[0,\frac{1}{2})$, neg$(\frac{1}{2},\frac{7}{6})$, pos$(\frac{7}{6},\frac{3}{2})$, neg$(\frac{3}{2},\frac{11}{6})$, pos$(\frac{11}{6},2]$

C. Intervals of Increase or Decrease

$\uparrow [0,\theta_+]$, $\downarrow [\theta_+, 1-\theta_+]$, $\uparrow [1-\theta_+, 1-\theta_-]$, $\downarrow [1-\theta_-, 2+\theta_-]$, $\uparrow [2+\theta_-, 2]$

$$\alpha_\pm = \frac{-1 \pm \sqrt{33}}{8} \qquad \theta_\pm = \frac{\mathrm{Sin}^{-1}(\alpha_\pm)}{\pi}$$

D. Concavity and Points of Inflection

$$\frac{dy}{dx} = 2\pi[\cos(2\pi x)] - \pi[\sin(\pi x)]$$

x	0		θ_+		$1-\theta_+$		$1-\theta_-$		$2+\theta_-$		2
dy/dx		pos	0	neg	0	pos	0	neg	0	pos	

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no asymptotes

pos$[0,\frac{1}{2})$, neg$(\frac{1}{2},\frac{7}{6})$, pos$(\frac{7}{6},\frac{3}{2})$, neg$(\frac{3}{2},\frac{11}{6})$, pos$(\frac{11}{6},2]$

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$\uparrow [0,\theta_+]$, $\downarrow [\theta_+, 1-\theta_+]$, $\uparrow [1-\theta_+, 1-\theta_-]$, $\downarrow [1-\theta_-, 2+\theta_-]$, $\uparrow [2+\theta_-, 2]$

$$\alpha_\pm = \frac{-1 \pm \sqrt{33}}{8} \qquad \theta_\pm = \frac{\text{Sin}^{-1}(\alpha_\pm)}{\pi}$$

D. Concavity and Points of Inflection

$$0 = 4[\sin(2\pi x)] + [\cos(\pi x)]$$

$$0 = -4\pi^2[\sin(2\pi x)] - \pi^2[\cos(\pi x)]$$ DIVIDE BY $-\pi^2$

$$\frac{d^2y}{dx^2} = -4\pi^2[\sin(2\pi x)] - \pi^2[\cos(\pi x)]$$

$$\frac{dy}{dx} = 2\pi[\cos(2\pi x)] - \pi[\sin(\pi x)]$$

EXAMPLE: Sketch the graph of $y = [\sin(2\pi x)] + [\cos(\pi x)]$.

A. Symmetry 2-periodic (over $[0,2]$; then repeat)

B. Intervals of Positivity or Negativity, and

(i) domain $\supseteq [0,2]$

(ii) x,y-intercepts $\bullet(0,1)$ $\bullet(\frac{1}{2},0)$, $\bullet(\frac{7}{6},0)$, $\bullet(\frac{3}{2},0)$, $\bullet(\frac{11}{6},0)$

(iii) vertical, horizontal asymptotes — no asymptotes

pos$[0,\frac{1}{2})$, neg$(\frac{1}{2},\frac{7}{6})$, pos$(\frac{7}{6},\frac{3}{2})$, neg$(\frac{3}{2},\frac{11}{6})$, pos$(\frac{11}{6},2]$

C. Intervals of Increase or Decrease

$$\alpha_\pm = \frac{-1 \pm \sqrt{33}}{8} \qquad \theta_\pm = \frac{\mathrm{Sin}^{-1}(\alpha_\pm)}{\pi}$$

$\uparrow [0,\theta_+]$, $\downarrow [\theta_+, 1-\theta_+]$, $\uparrow [1-\theta_+, 1-\theta_-]$, $\downarrow [1-\theta_-, 2+\theta_-]$, $\uparrow [2+\theta_-, 2]$

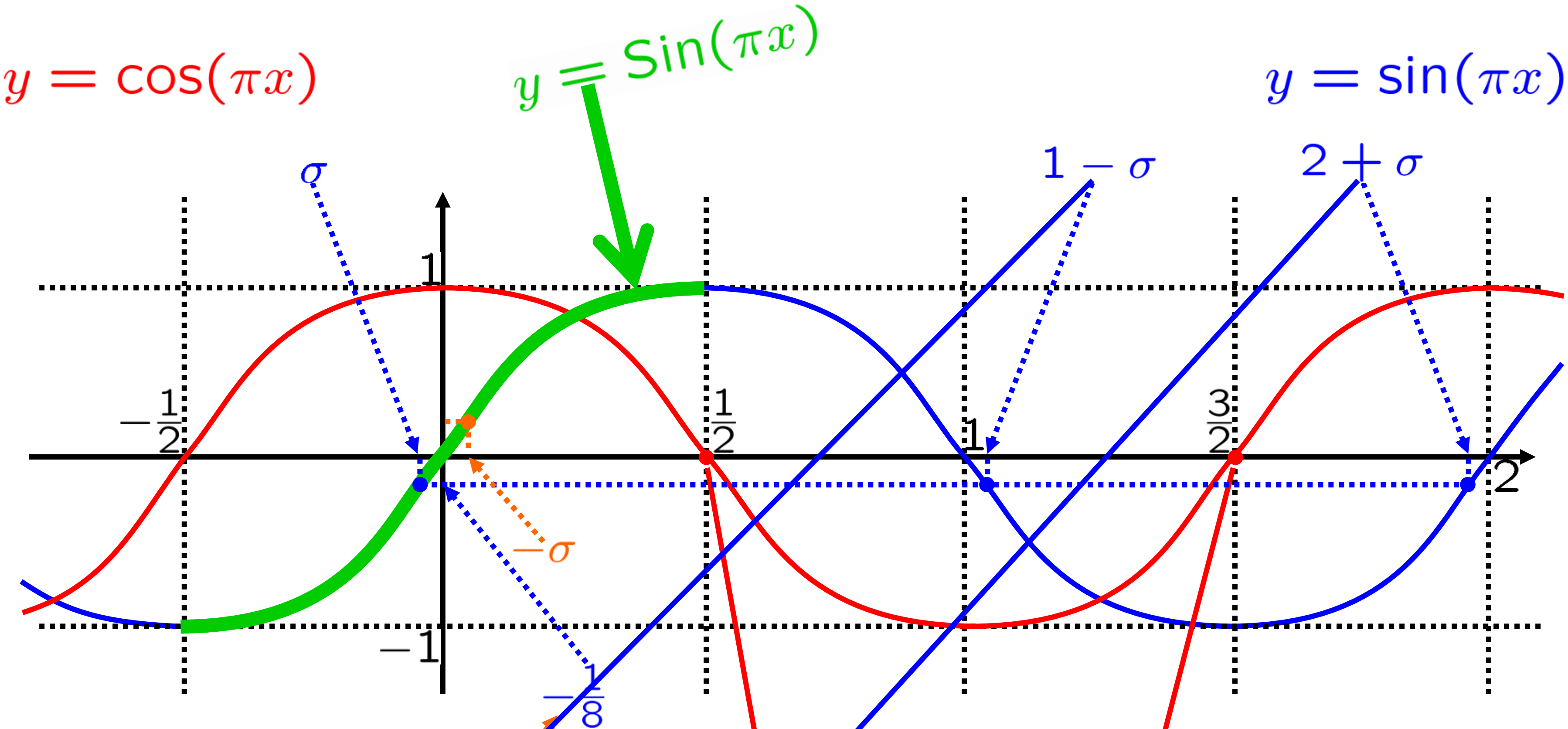
D. Concavity and Points of Inflection

$$0 = 4[\sin(2\pi x)] + [\cos(\pi x)]$$

$$0 = 4[2(\sin(\pi x))(\cos(\pi x))] + [\cos(\pi x)]$$
$$= 8[\sin(\pi x)][\cos(\pi x)] + [\cos(\pi x)]$$
$$= [8(\sin(\pi x)) + 1][\cos(\pi x)]$$

$0 = 8(\sin(\pi x)) + 1$ or $0 = \cos(\pi x)$

$[-\frac{1}{8} = \sin(\pi x)$ or $0 = \cos(\pi x)]$ and $[0 \le x \le 2]$



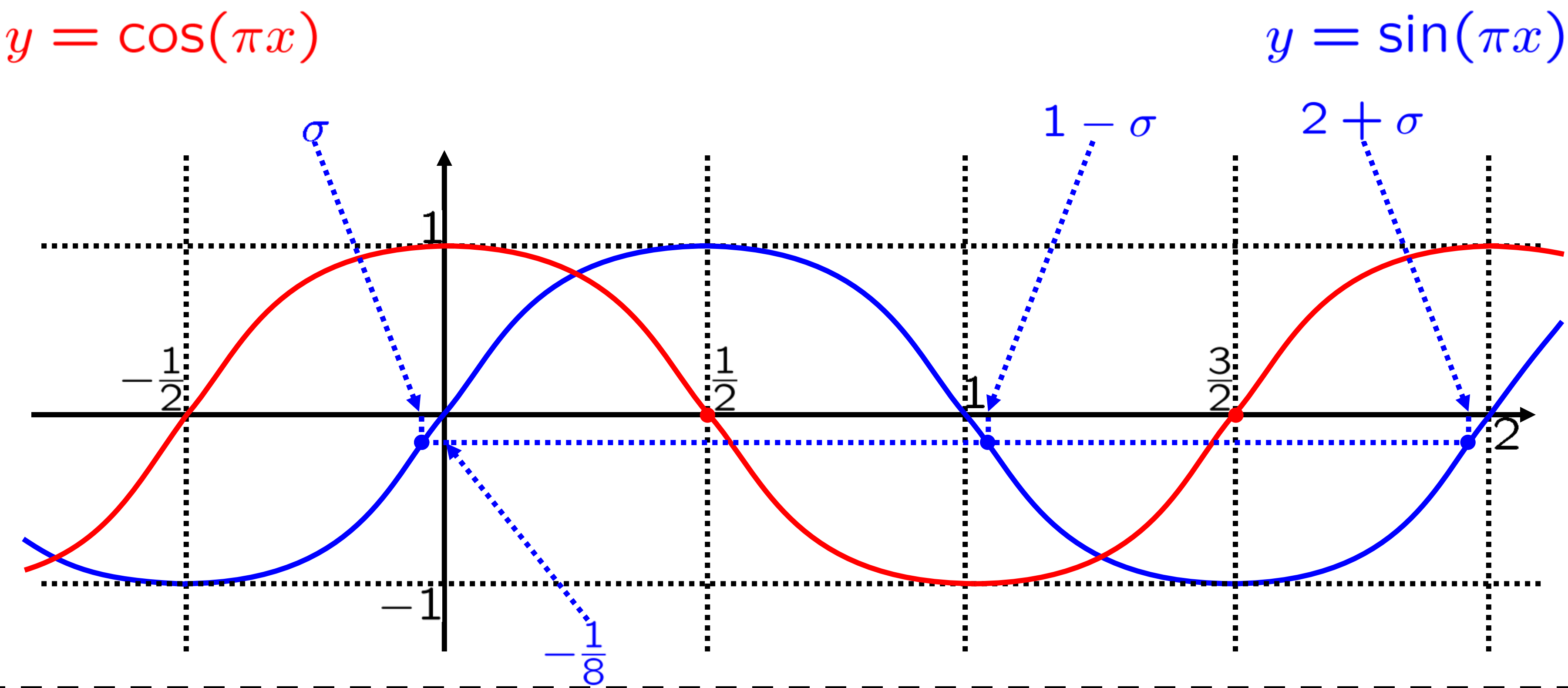
$$\text{Sin}(\pi\sigma) = -\frac{1}{8}$$

$$\pi\sigma = \text{Sin}^{-1}(-\tfrac{1}{8})$$

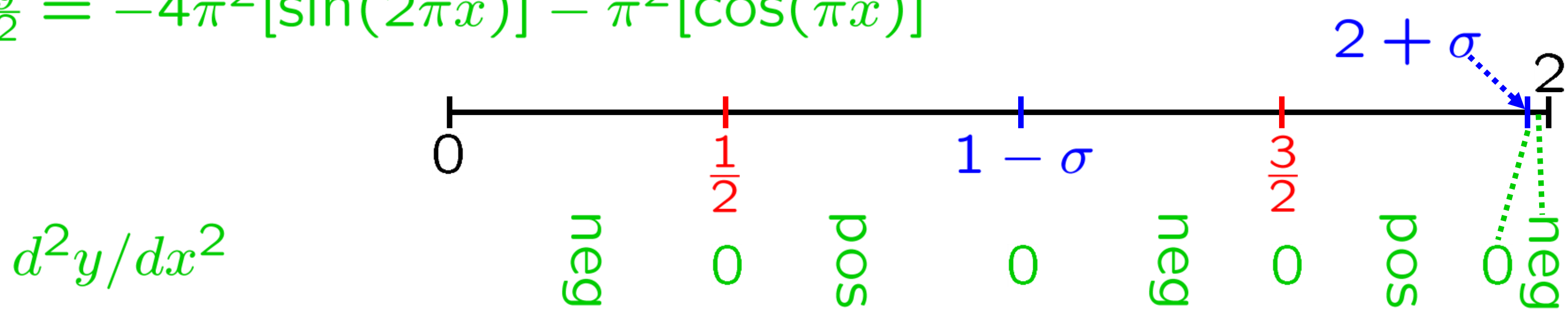
$$\sigma = \frac{1}{\pi}\left[\text{Sin}^{-1}(-\tfrac{1}{8})\right] \approx -0.0399$$

$x = 1 - \sigma$ or $x = 2 + \sigma$ or $x = \frac{1}{2}$ or $x = \frac{3}{2}$

$[-\frac{1}{8} = \sin(\pi x)$ or $0 = \cos(\pi x)]$ and $[0 \le x \le 2]$



$$\frac{d^2y}{dx^2} = -4\pi^2[\sin(2\pi x)] - \pi^2[\cos(\pi x)]$$



$x = 1 - \sigma$ or $x = 2 + \sigma$ or $x = \frac{1}{2}$ or $x = \frac{3}{2}$

$[-\frac{1}{8} = \sin(\pi x)$ or $0 = \cos(\pi x)]$ and $[0 \leq x \leq 2]$

EXAMPLE: Sketch the graph of $y = [\sin(2\pi x)] + [\cos(\pi x)]$.

A. Symmetry 2-periodic (over $[0,2]$; then repeat)

B. Intervals of Positivity or Negativity, and

pos$[0, \frac{1}{2})$, neg$(\frac{1}{2}, \frac{7}{6})$, pos$(\frac{7}{6}, \frac{3}{2})$, neg$(\frac{3}{2}, \frac{11}{6})$, pos$(\frac{11}{6}, 2]$

(i) domain$\supseteq[0,2]$ $\bullet(\frac{1}{2}, 0)$, $\bullet(\frac{7}{6}, 0)$, $\bullet(\frac{3}{2}, 0)$, $\bullet(\frac{11}{6}, 0)$

$\bullet(0,1)$ (ii) x,y-intercepts

no asymptotes

(iii) vertical, horizontal asymptotes

C. Intervals of Increase or Decrease

$\uparrow [0, \theta_+]$, $\downarrow [\theta_+, 1 - \theta_+]$, $\uparrow [1 - \theta_+, 1 - \theta_-]$, $\downarrow [1 - \theta_-, 2 + \theta_-]$, $\uparrow [2 + \theta_-, 2]$

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D. Concavity and Points of Inflection

$\cap[0, \frac{1}{2}]$, $\cup[\frac{1}{2}, 1 - \sigma]$, $\cap[1 - \sigma, \frac{3}{2}]$, $\cup[\frac{3}{2}, 2 + \sigma]$, $\cap[2 + \sigma, 2]$

$$\sigma = \frac{1}{\pi}\left[\text{Sin}^{-1}(-\tfrac{1}{8})\right]$$

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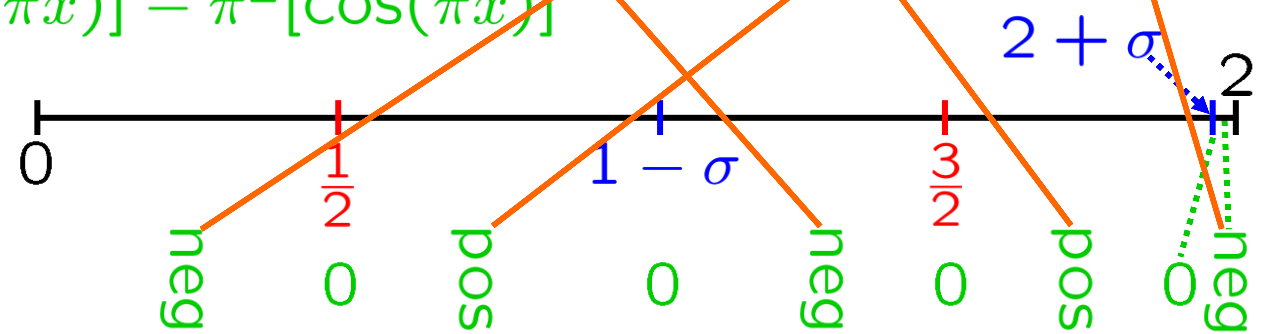

$x = 1 - \sigma$ or $x = 2 + \sigma$ or $x = \frac{1}{2}$ or $x = \frac{3}{2}$

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EXAMPLE: Sketch the graph of $y = [\sin(2\pi x)] + [\cos(\pi x)]$.

2-periodic (over $[0, 2]$) 2-periodic (over domain $\supseteq [0, 2]$

$\sigma = \frac{1}{\pi}\left[\mathrm{Sin}^{-1}(-\frac{1}{8})\right]$

$\bullet(0,1)$ $\bullet(\frac{1}{2}, 0)$, $\bullet(\frac{7}{6}, 0)$, $\bullet(\frac{3}{2}, 0)$, $\bullet(\frac{11}{6}, 0)$

pos$[0, \frac{1}{2})$, neg domain $\supseteq [0, 2]$ $\bullet(\frac{1}{2}, 0)$, $\bullet(\frac{7}{6}, 0)$, $\bullet(\frac{3}{2}, 0)$, $\bullet($

$\theta_\pm = \frac{\mathrm{Sin}^{-1}(\alpha_\pm)}{\pi}$

$\cap[0, \frac{1}{2}]$, $\cup[\frac{1}{2}, 1-\sigma]$, $\cap[1-\sigma, \frac{3}{2}]$, $\cup[\frac{3}{2}, 2+\sigma]$, $\cap[2+\sigma, 2]$

neg$(\frac{\pi}{2}, \frac{11}{6})$,

$\uparrow[0, \theta_+]$, $\downarrow[\theta_+, 1-\theta_+]$, $\uparrow[1-\theta_+, 1-\theta_-]$, $\downarrow[1-\theta_-, 2+\theta_-]$, $\uparrow[2+\theta_-, 2]$

$\downarrow[\theta_+, 1-\theta_+]$, pos$(\frac{11}{6}, 2]$

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$\uparrow[1-\theta_+, 1-\theta_-]$,

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$$\sigma = \frac{1}{\pi}\left[\mathrm{Sin}^{-1}(-\frac{1}{8})\right]$$

 $\sigma \approx -0.0399$ $\theta_+ \approx 0.202$ $\theta_- \approx -0.319$

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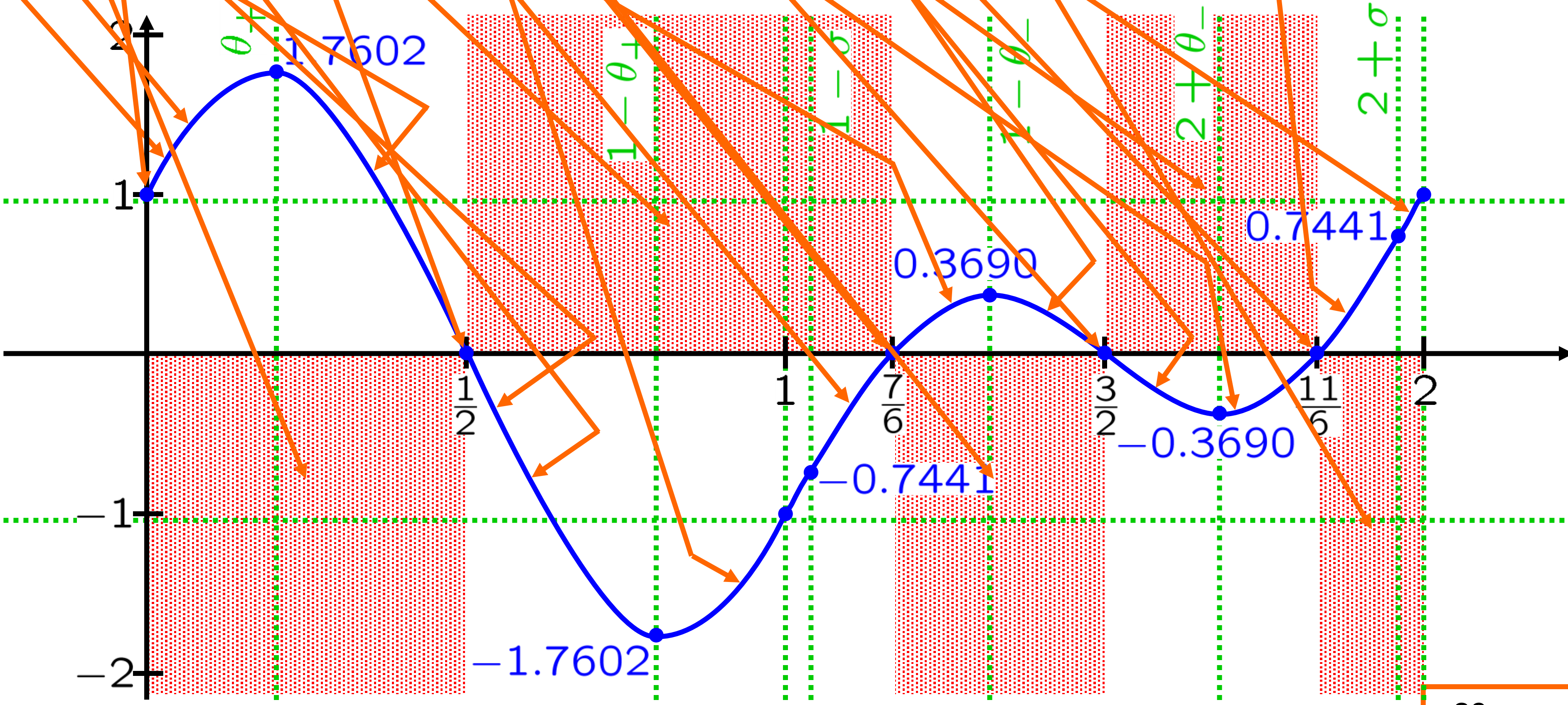

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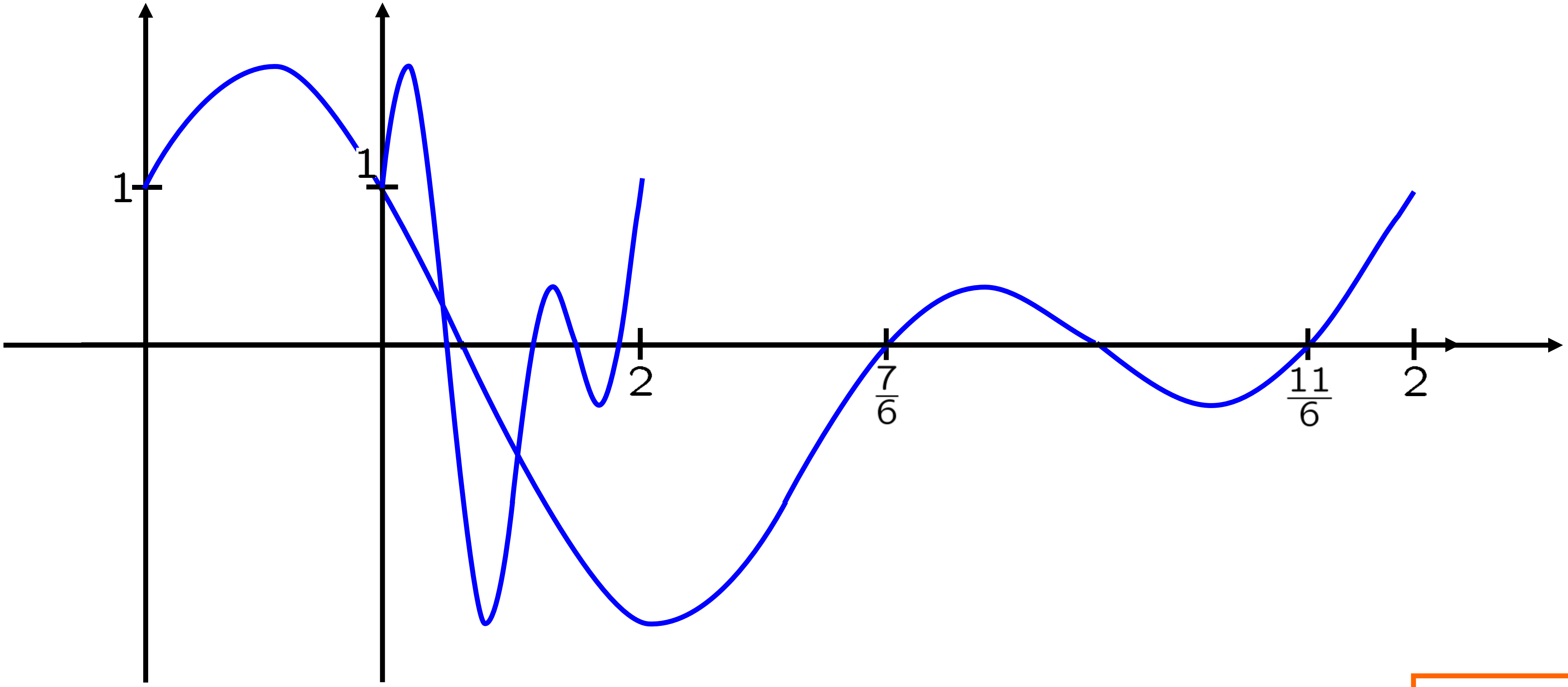

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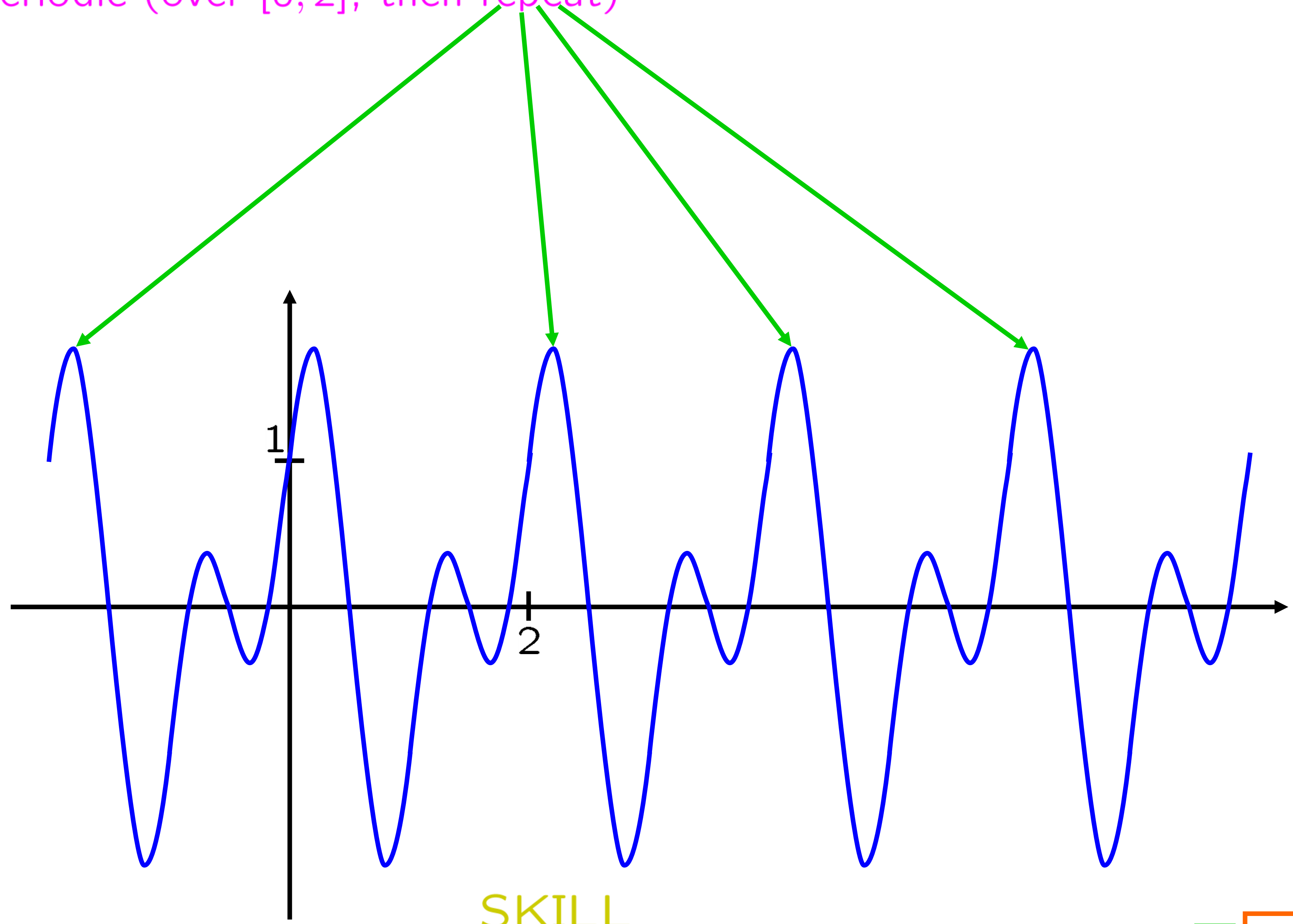


SKILL
curve sketching

EXAMPLE: Sketch the graph of $y = \dfrac{x+3}{x^2+4x+4}$.

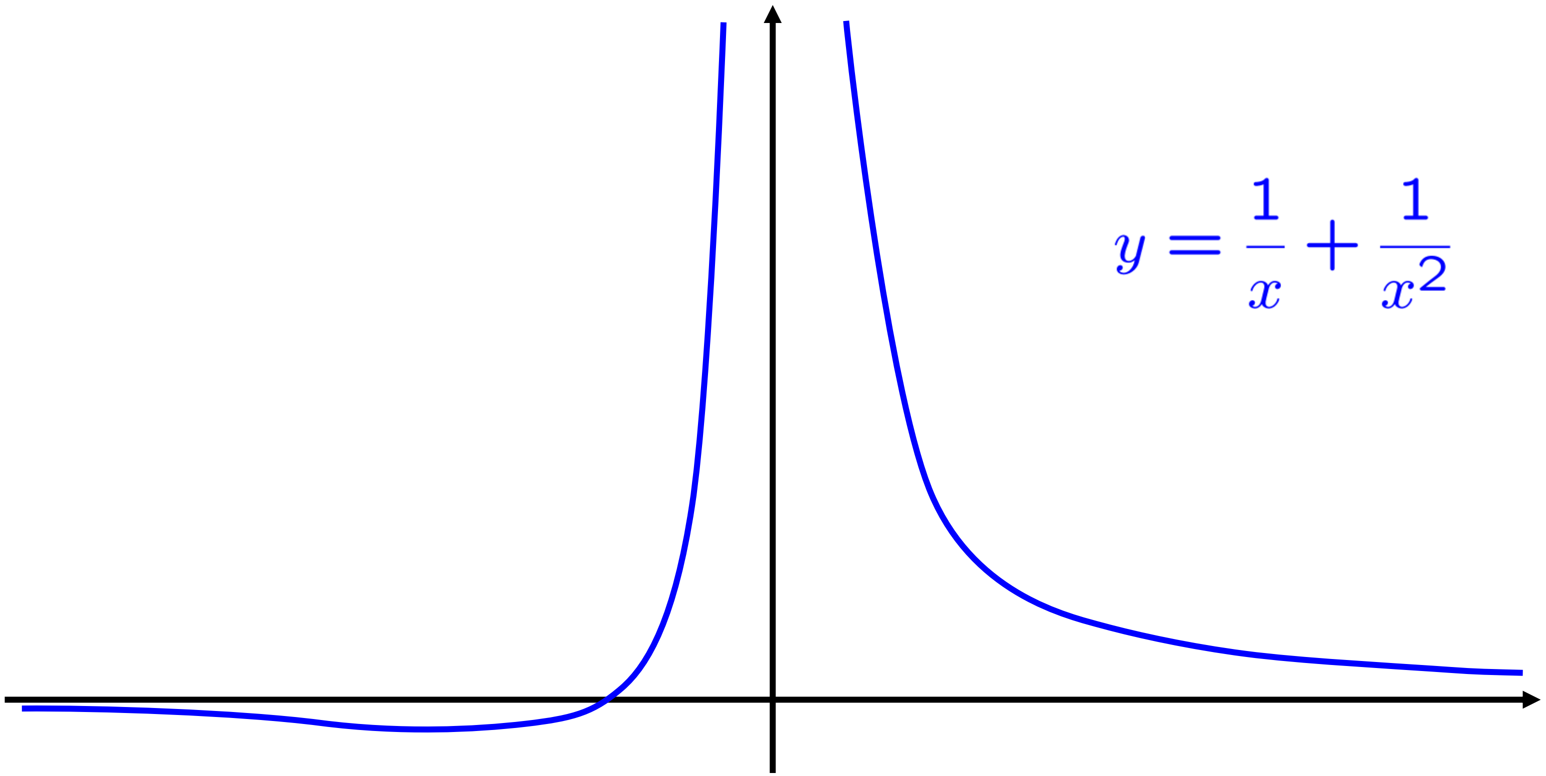
$x^2+4x+4 = (x+2)^2$

Suggestion: $x :\rightarrow x-2$ (translates graph 2 units to the right)

Graph $y = \dfrac{x+1}{x^2}$, *i.e.*, graph $y = \dfrac{1}{x} + \dfrac{1}{x^2}$,

then translate graph back 2 units to the left.

Or translate vertical axis 2 units to the right.



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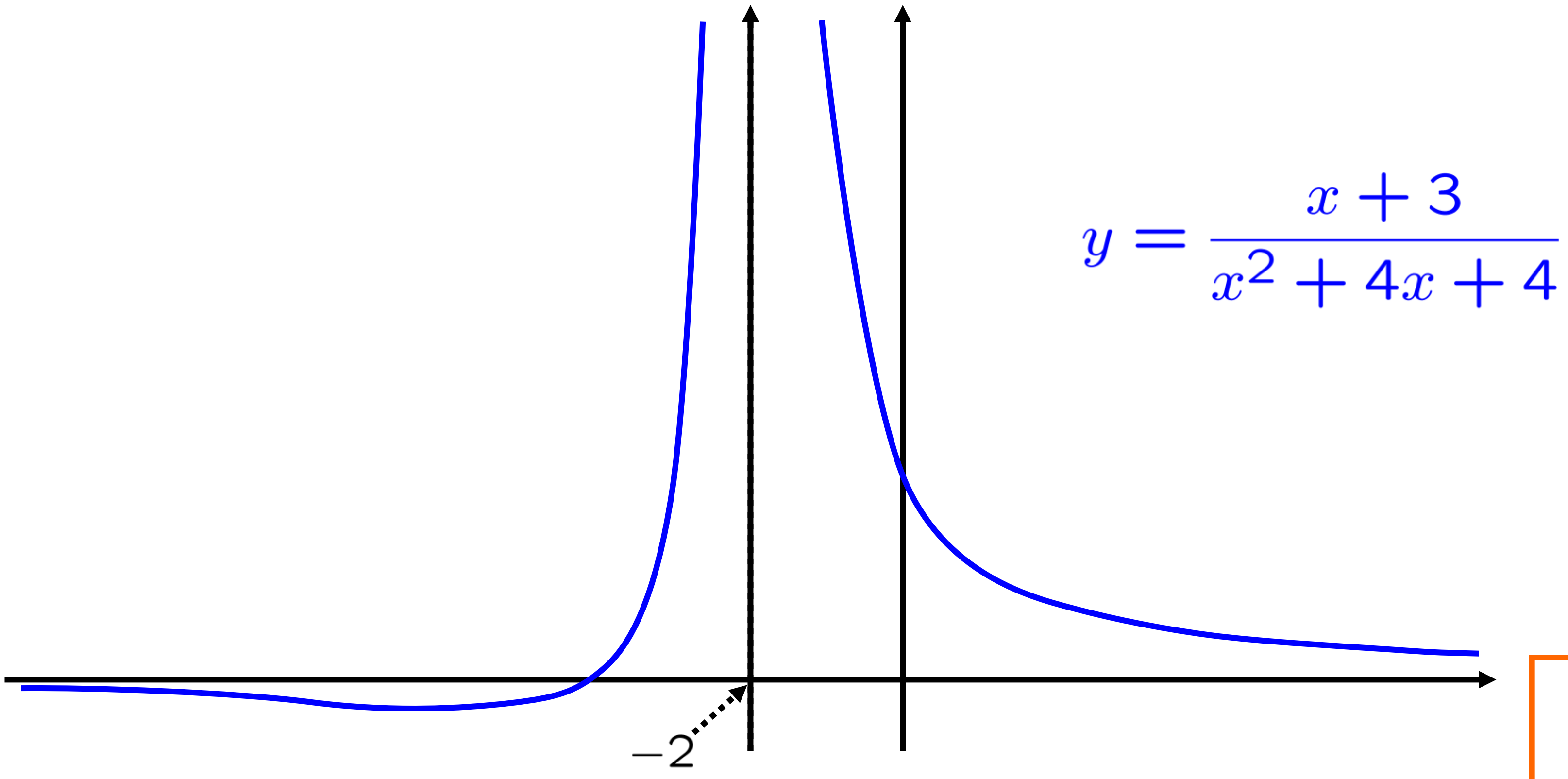

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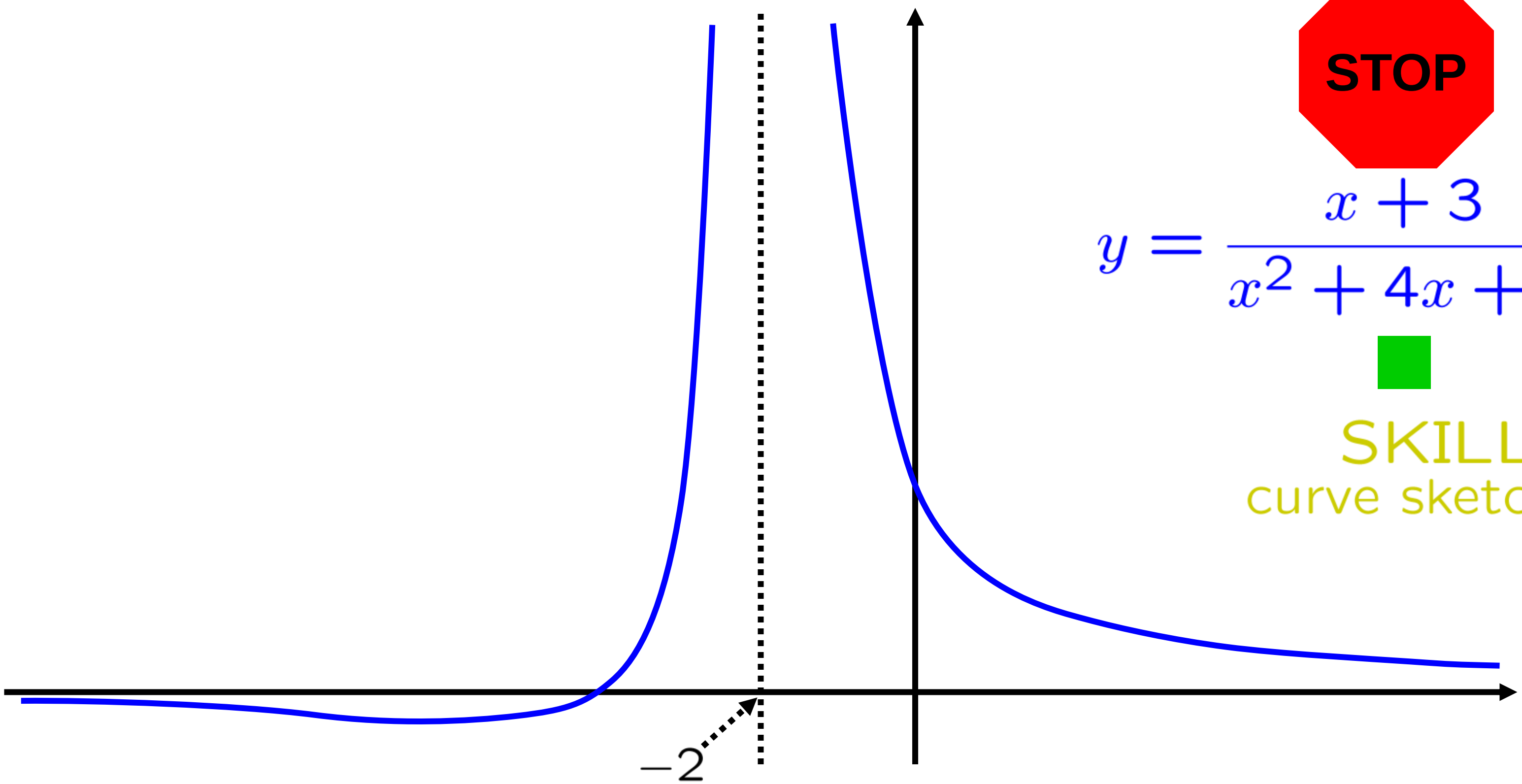

STOP

$y = \frac{x+3}{x^2+4x+4}$ ■

SKILL
curve sketching