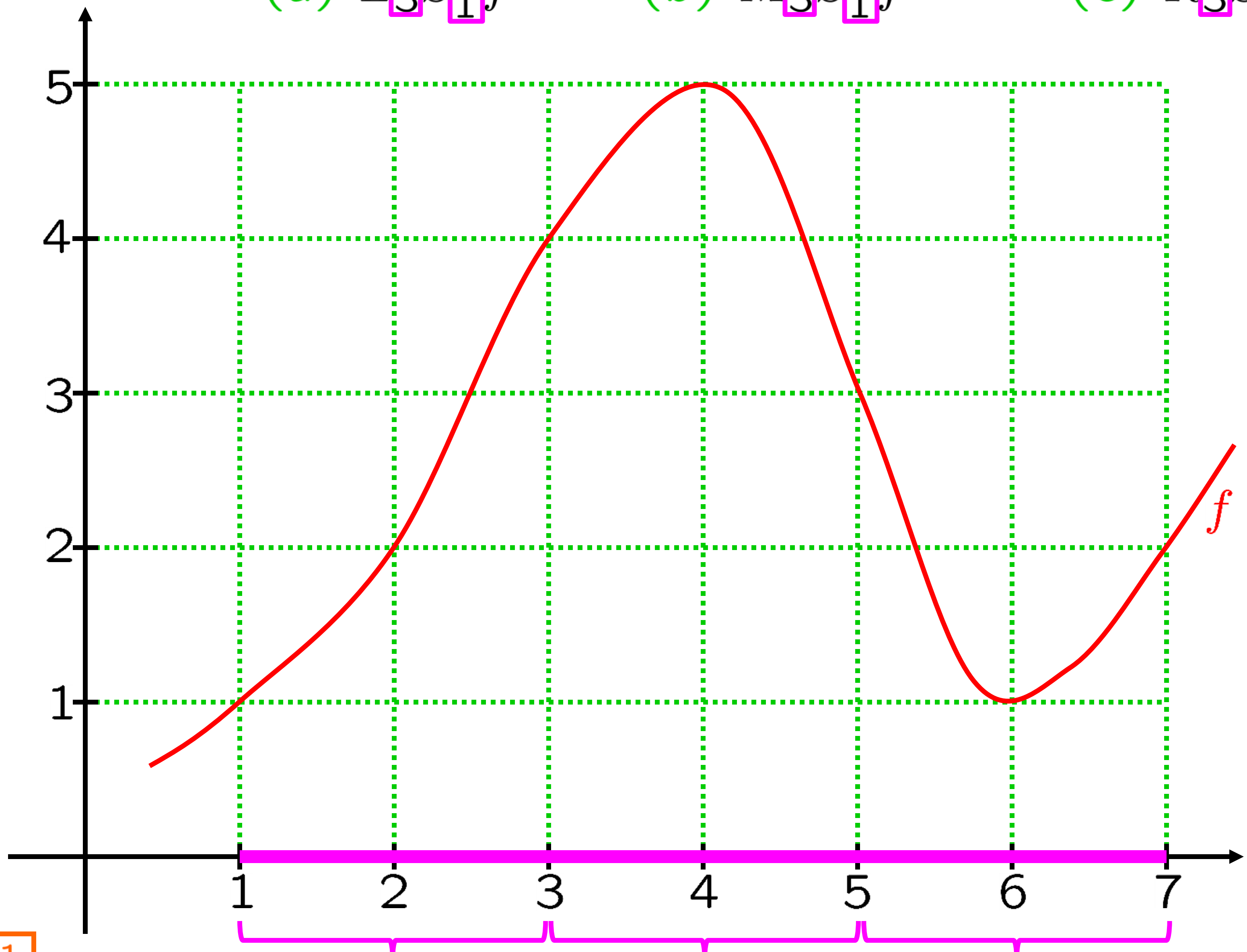
CALCULUS

Definite integration and Riemann sum problems

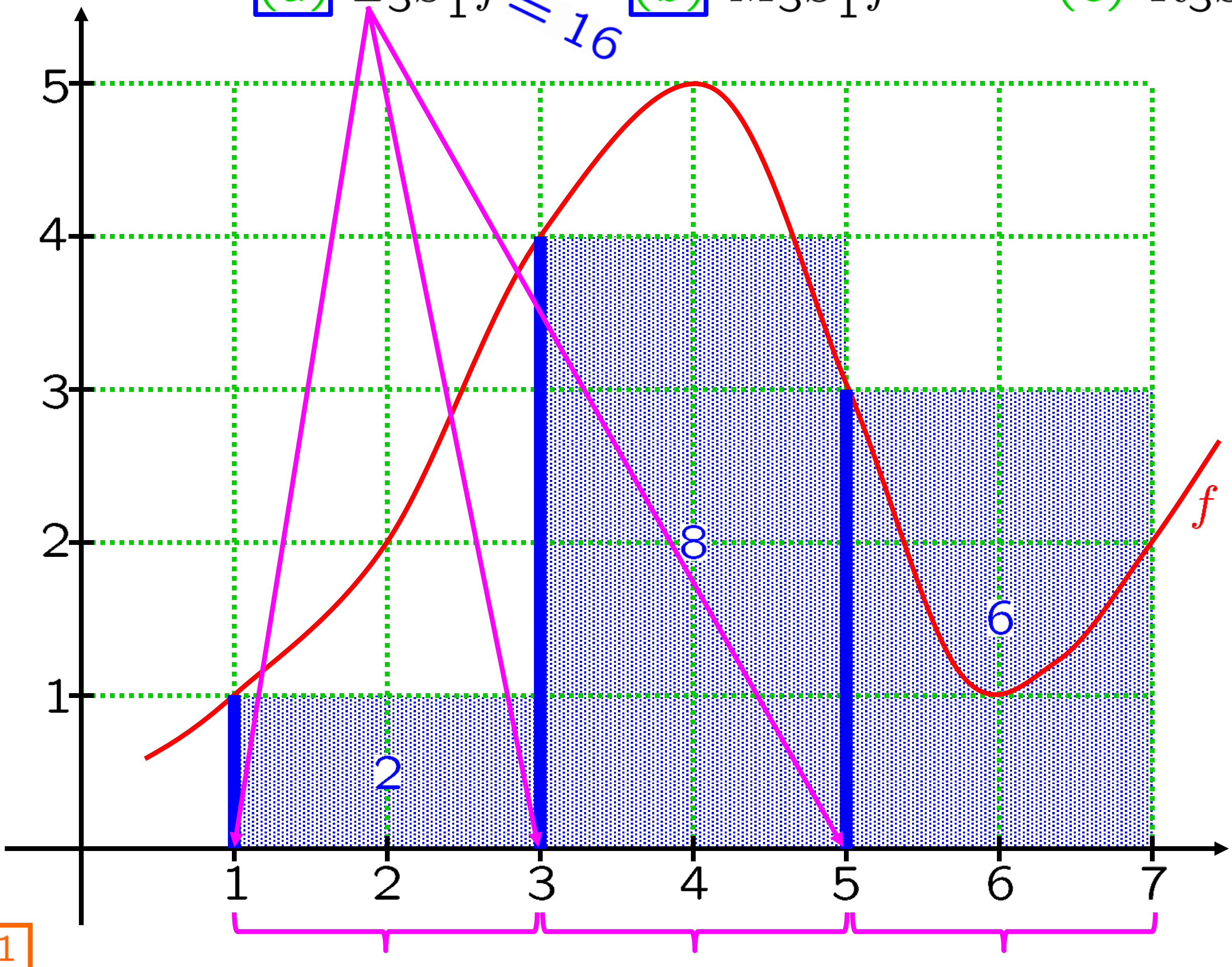
EXAMPLE: Estimate the area under $y = f(x)$ from $x = 1$ to $x = 7$ by calculating

(a) $L_{\boxed{3}}S\boxed{\substack{7\\1}}f$ (b) $M_{\boxed{3}}S\boxed{\substack{7\\1}}f$ (c) $R_{\boxed{3}}S\boxed{\substack{7\\1}}f$



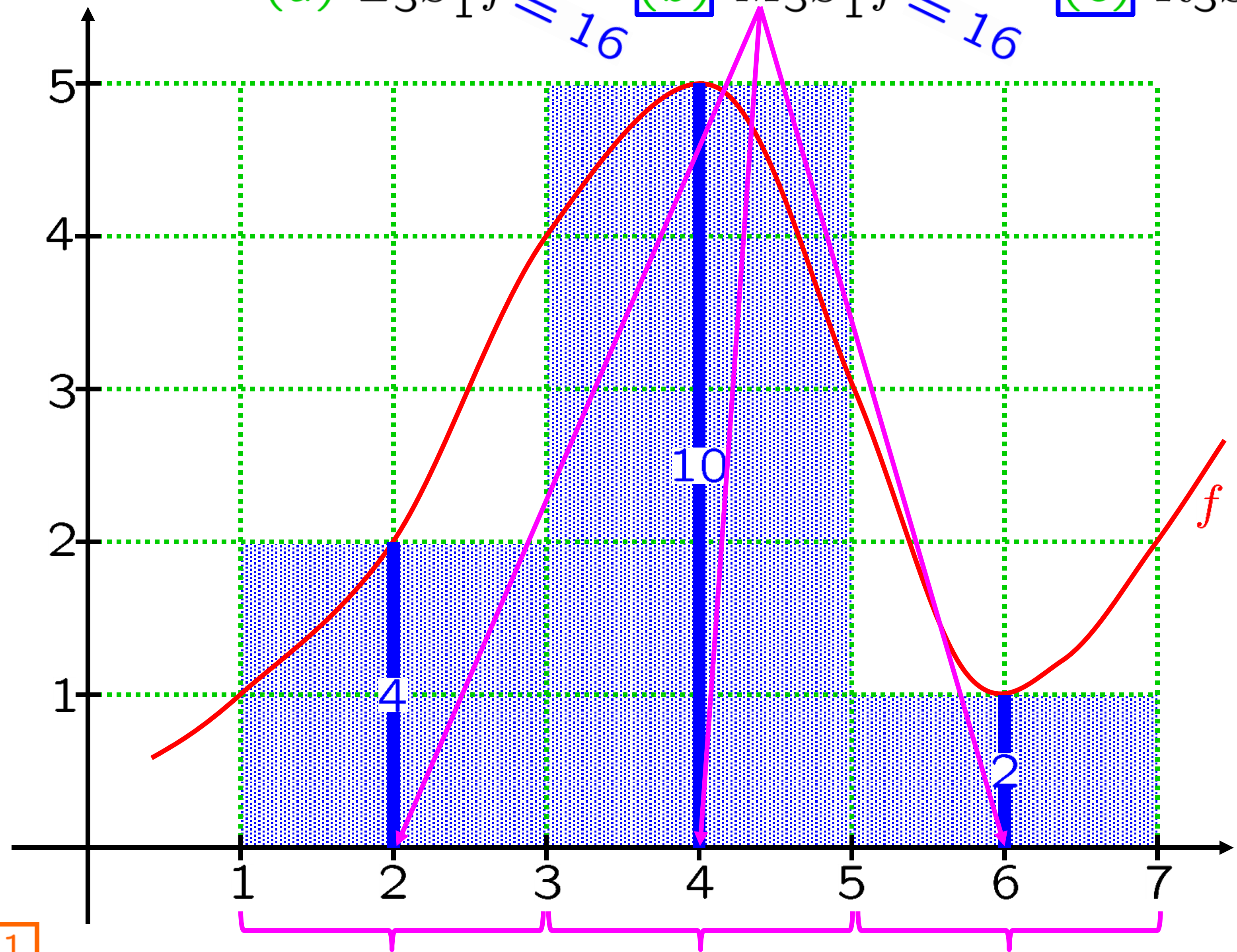
EXAMPLE: Estimate the area under $y = f(x)$ from $x = 1$ to $x = 7$ by calculating

$\boxed{\text{(a)}}$ $L_3 S_1^7 f = 16$ $\boxed{\text{(b)}}$ $M_3 S_1^7 f$ (c) $R_3 S_1^7 f$



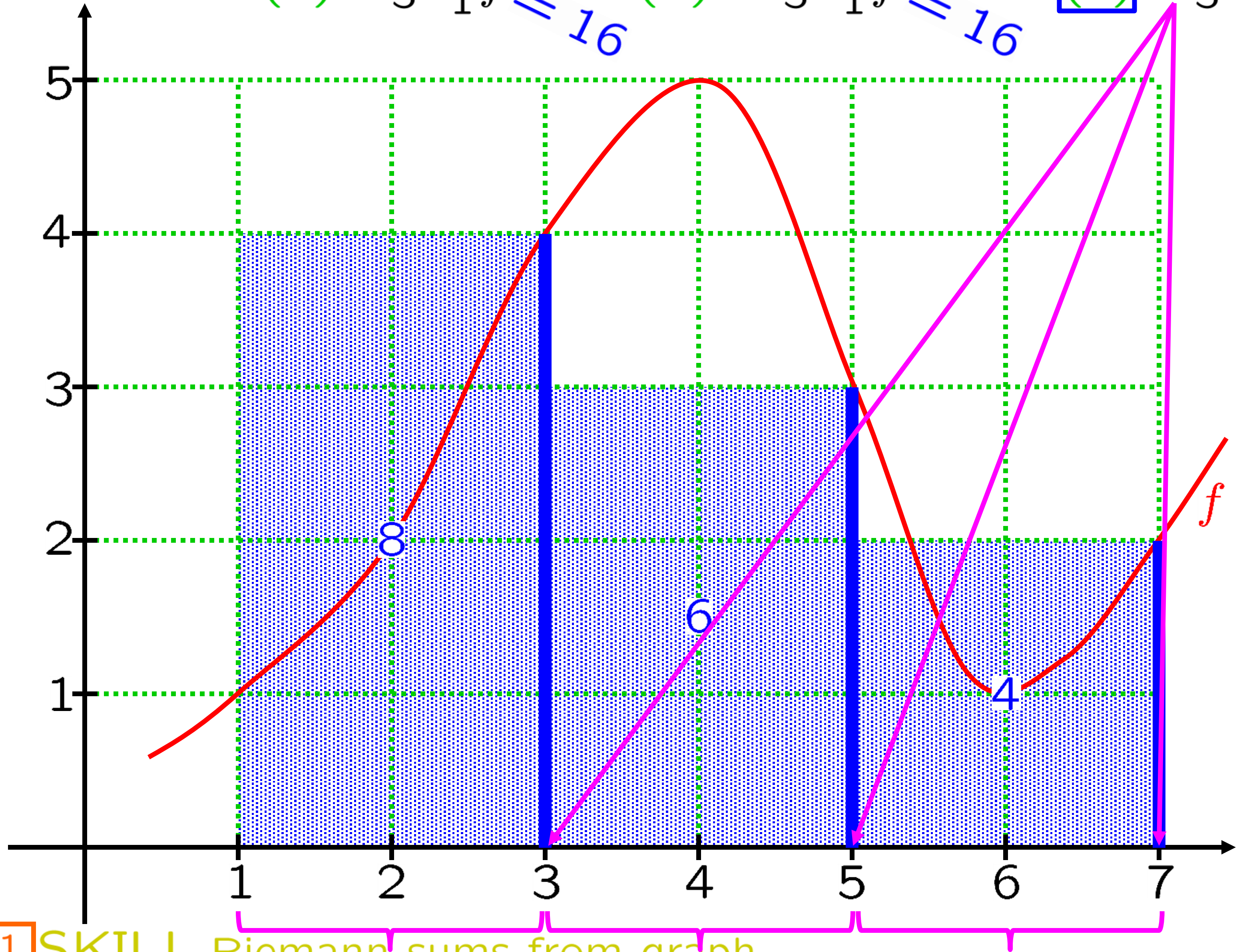
EXAMPLE: Estimate the area under $y = f(x)$ from $x = 1$ to $x = 7$ by calculating

(a) $L_3 S_1^7 f = 16$ $\boxed{\text{(b)}}$ $M_3 S_1^7 f = 16$ $\boxed{\text{(c)}}$ $R_3 S_1^7 f$

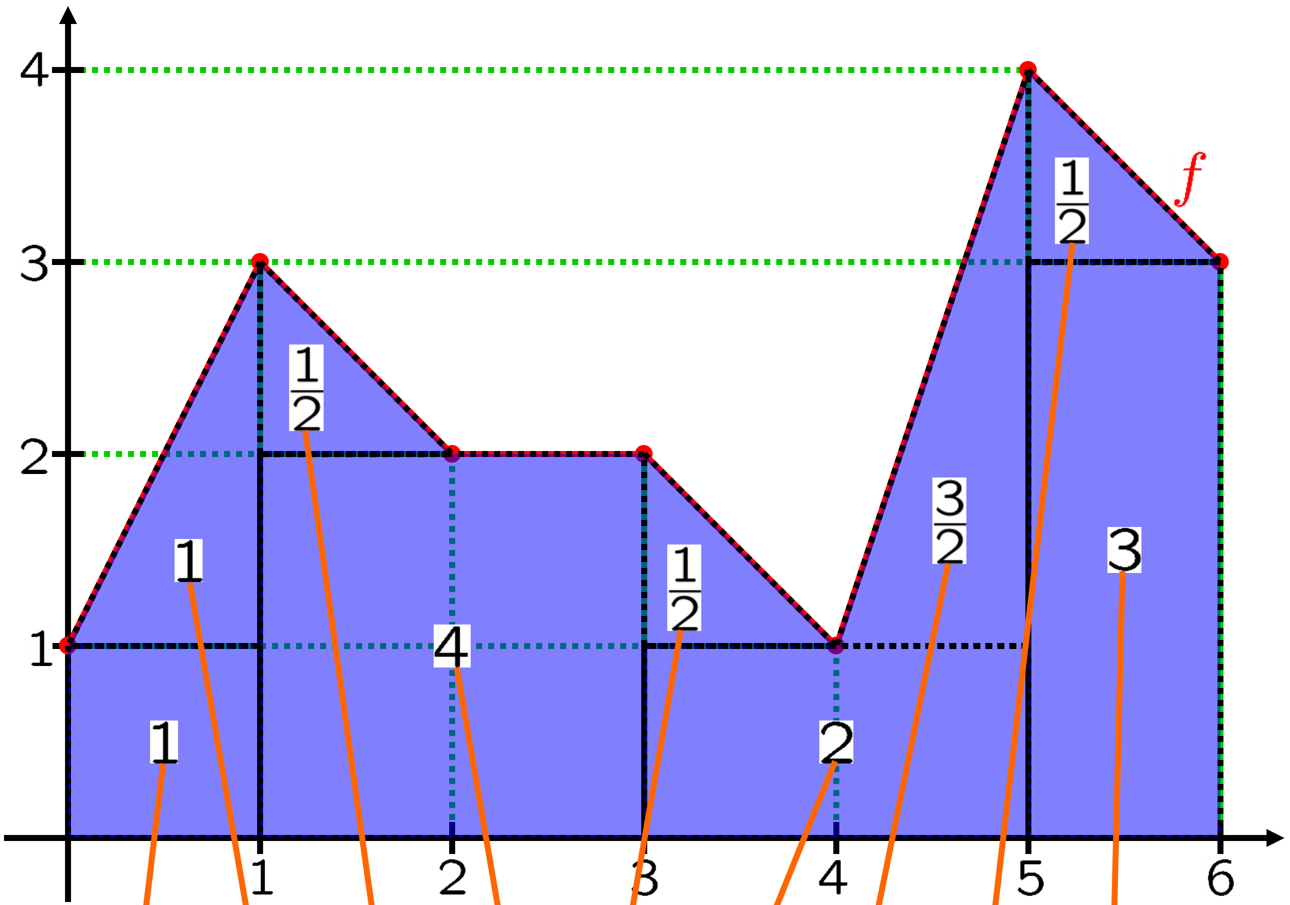


EXAMPLE: Estimate the area under $y = f(x)$ from $x = 1$ to $x = 7$ by calculating

(a) $L_3 S_1^7 f$ = 16 (b) $M_3 S_1^7 f$ = 16 $\boxed{\text{(c)}}$ $R_3 S_1^7 f$ = 18 ■



EXAMPLE: From the graph of f, given below, compute $\int_0^6 f(x)\,dx$ by interpreting it as an area.



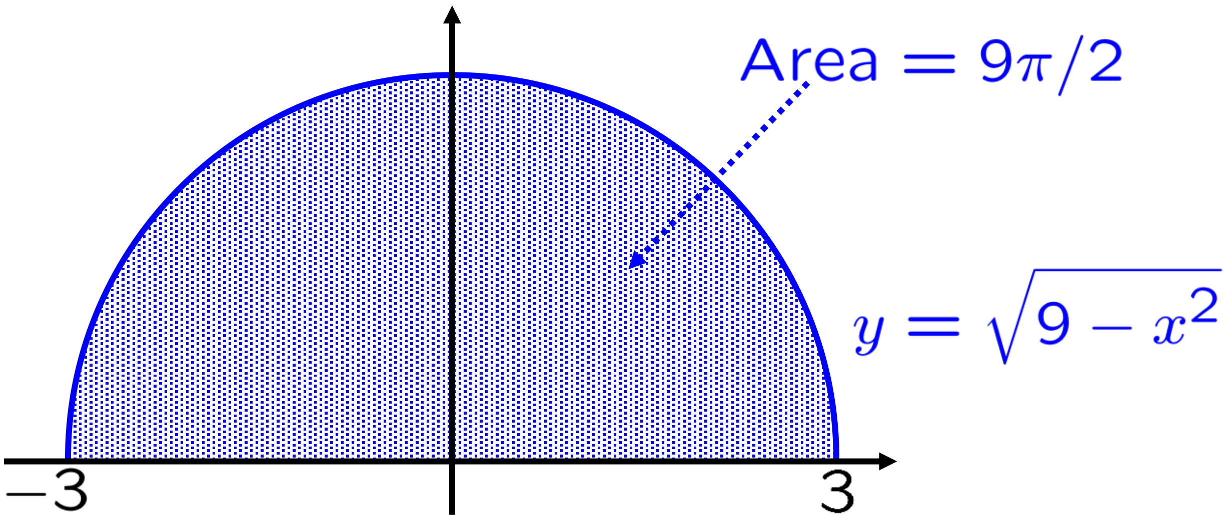
$$\int_0^6 f(x)\,dx = 1 + 1 + \tfrac{1}{2} + 4 + \tfrac{1}{2} + 2 + \tfrac{3}{2} + \tfrac{1}{2} + 3 = 14 \blacksquare$$

SKILL
Integral as area

EXAMPLE:

(a) Compute $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$ by interpreting it as an area.

(b) Compute $\int_{3}^{-3} \sqrt{9 - x^2}\, dx$.



Area inside circle of radius 3
$= \pi[3^2]$
$= 9\pi$

(a) $\int_{-3}^{3} \sqrt{9 - x^2}\, dx = 9\pi/2$

SKILL
Definite integral

(b) $\int_{3}^{-3} \sqrt{9 - x^2}\, dx = -\int_{-3}^{3} \sqrt{9 - x^2}\, dx = -9\pi/2$ ■

Def'n: $\boxed{\int_{b}^{a} f(x)\, dx} := -\int_{a}^{b} f(x)\, dx$, if $a < b$

EXAMPLE: Estimate $\int_1^7 f(x)\,dx$ by calculating

(a) $L_3 S_1^7 f$ (b) $M_3 S_1^7 f$ (c) $R_3 S_1^7 f$

1st SUBINT. 2nd SUBINT. 3rd SUBINT.

x	1	2	3	4	5	6	7
$f(x)$	2	-1	3	4	7	2	0

(a) $L_3 S_1^7 f = (2)((f(1)) + (2)(f(3)) + (2)(f(5))$

$= [2][((f(1)) + (f(3)) + (f(5))]$

$= [2][2 + 3 + 7] = 24$

EXAMPLE: Estimate $\int_1^7 f(x)\,dx$ by calculating

(a) $L_3 S_1^7 f$ (b) $M_3 S_1^7 f$ (c) $R_3 S_1^7 f$

1st SUBINT. 2nd SUBINT. 3rd SUBINT.

x	1	2	3	4	5	6	7
$f(x)$	2	-1	3	4	7	2	0

(a) $L_3 S_1^7 f = (2)((f(1)) + (2)(f(3)) + (2)(f(5))$
$= [2][((f(1)) + (f(3)) + (f(5))]$
$= [2][2 + 3 + 7] = 24$

(b) $M_3 S_1^7 f = (2)((f(2)) + (2)(f(4)) + (2)(f(6))$
$= [2][((f(2)) + (f(4)) + (f(6))]$
$= [2][-1 + 4 + 2] = 10$

EXAMPLE: Estimate $\int_1^7 f(x)\,dx$ by calculating

(a) $L_3S_1^7 f$ (b) $M_3S_1^7 f$ (c) $R_3S_1^7 f$

1st SUBINT. 2nd SUBINT. 3rd SUBINT.

x	1	2	3	4	5	6	7
$f(x)$	2	-1	3	4	7	2	0

(a) $L_3S_1^7 f = (2)((f(1)) + (2)(f(3)) + (2)(f(5))$
$= [2][((f(1)) + (f(3)) + (f(5))]$
$= [2][2 + 3 + 7] = 24$

(b) $M_3S_1^7 f = (2)((f(2)) + (2)(f(4)) + (2)(f(6))$
$= [2][((f(2)) + (f(4)) + (f(6))]$
$= [2][-1 + 4 + 2] = 10$

NOTE: Midpoint sum might not be between left and right sums.

(c) $R_3S_1^7 f = (2)((f(3)) + (2)(f(5)) + (2)(f(7))$
$= [2][((f(3)) + (f(5)) + (f(7))]$
$= [2][3 + 7 + 0] = 20$ ■

SKILL
Riemann sums from table

EXAMPLE: Let $f(x) = x^3 - 3x$, and compute $R_6 S_0^6 f$.

$$h_6 = \frac{6-0}{6} = 1$$ = width of every rectangle

$$R_6 S_0^6 f = \sum_{j=1}^{6} [\cancel{h_6}][f(\cancel{0} + j\cancel{h_6})]$$

right endpoint of jth subinterval . . .
height of jth rectangle . . .
area of jth rectangle . . .

$$= \sum_{j=1}^{6} f(j)$$

$$= [f(1)] + [f(2)] + [f(3)] + [f(4)] + [f(5)] + [f(6)]$$

$$= -2 + 2 + 18 + 52 + 110 + 198 = 378$$ ■

SKILL
Compute Riemann sum

EXAMPLE: Approximate $\int_2^6 x^3 e^x \, dx$

using midpoints and four subintervals.

I.e., let $f(x) = x^3 e^x$, and compute $M_4 S_2^6 f$.

$$h_4 = \frac{6-2}{4} = 1 \qquad R_4 S_2^6 f = \sum_{j=1}^{4} [h_4][f(2 + \boxed{j} h_4)]$$

$j - 1$ left-endpt

$j - \frac{1}{2}$ midpt

$$M_4 S_2^6 f = \sum_{j=1}^{4} [\cancel{h_4}][f(2 + \boxed{(j - \tfrac{1}{2})} \cancel{h_4})]$$

Next: Review $\sum j^k$

$$= \sum_{j=1}^{4} f(\boxed{2} + (j \boxed{- \tfrac{1}{2}})) = \sum_{j=1}^{4} f(j + \boxed{1.5})$$

$$= [f(2.5)] + [f(3.5)] + [f(4.5)] + [f(5.5)]$$

$$= [(2.5)^3 e^{2.5}] + [(3.5)^3 e^{3.5}] + [(4.5)^3 e^{4.5}] + [(5.5)^3 e^{5.5}]$$

$$= \cdots \blacksquare$$

SKILL
Compute Riemann sum

$$\sum_{j=1}^{n} 1 = \underbrace{1 + 1 + \cdots + 1}_{n \text{ terms}} = n$$

$$\sum_{j=1}^{n} j = 1 + 2 + \cdots + n = \frac{n^2 + n}{2}$$

$$\sum_{j=1}^{n} j^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{2n^3 + 3n^2 + n}{6}$$

$$\sum_{j=1}^{n} j^3 = 1^3 + 2^3 + \cdots + n^3 = \frac{n^4 + 2n^3 + n^2}{4}$$

EXAMPLE: Let $f(x) = x^2$. Recall that $\lim_{n\to\infty} R_n S_0^1 f = 1/3$.

Show, by computation, that $\lim_{n\to\infty} L_n S_0^1 f = 1/3$ ☺

and that $\lim_{n\to\infty} M_n S_0^1 f = 1/3$.

$$h_n = \frac{1-0}{n} = \frac{1}{n}$$

$$L_n S_0^1 f = \frac{1}{n}\sum_{j=1}^{n}\left[\cancel{0} + (\underline{j-1})\frac{1}{n}\right]^2 = \boxed{\frac{1}{n}}\sum_{j=1}^{n}\left(\underline{j^2 - 2j + 1}\right)\boxed{\frac{1}{n^2}}$$

$$\| $$

$$\frac{1}{\boxed{n^3}}\left[\left(\boxed{\frac{2n^3+3n^2+n}{6}}\right) - 2\left(\boxed{\frac{n^2+n}{2}}\right) + \boxed{n}\right] = \boxed{\frac{1}{n^3}}\boxed{\sum_{j=1}^{n}}\boxed{j^2} - 2\boxed{j} + \boxed{1}$$

$$\|$$

$$\left(\boxed{\frac{2}{6}} + \boxed{\frac{3}{6n} + \frac{1}{6n^2}}\right) - 2\left(\boxed{\frac{1}{2n} + \frac{1}{2n^2}}\right) + \boxed{\frac{1}{n^2}} \quad \xrightarrow{n\to\infty} \quad \boxed{\frac{2}{6}} = \frac{1}{3}$$

as $n \to \infty$

$0 \qquad 0 \qquad 0$

EXAMPLE: Let $f(x) = x^2$. Recall that $\lim_{n\to\infty} R_n S_0^1 f = 1/3$.

Show, by computation, that $\lim_{n\to\infty} L_n S_0^1 f = 1/3$

and that $\lim_{n\to\infty} M_n S_0^1 f = 1/3$.

$$h_n = \frac{1-0}{n} = \frac{1}{n}$$

$$M_n S_0^1 f = \frac{1}{n}\sum_{j=1}^{n}\left[\cancel{0} + \left(j - \frac{1}{2}\right)\frac{1}{n}\right]^2 = \boxed{\frac{1}{n}}\sum_{j=1}^{n}\left(j^2 - j + \frac{1}{4}\right)\boxed{\frac{1}{n^2}}$$

$$= \frac{1}{\boxed{n^3}}\left[\left(\boxed{\frac{2n^3+3n^2+n}{6}}\right) - \left(\boxed{\frac{n^2+n}{2}}\right) + \boxed{\frac{n}{4}}\right] = \boxed{\frac{1}{n^3}}\boxed{\sum_{j=1}^{n}}\boxed{j^2} - \boxed{j} + \boxed{\frac{1}{4}}$$

$$= \left(\boxed{\frac{2}{6}} + \boxed{\frac{3}{6n} + \frac{1}{6n^2}}\right) - \left(\boxed{\frac{1}{2n} + \frac{1}{2n^2}}\right) + \boxed{\frac{1}{4n^2}} \xrightarrow{n\to\infty} \boxed{\frac{2}{6}} = \frac{1}{3}$$

as $n \to \infty$

0 0 0

EXAMPLE: Let $f(x) = x^2$. Recall that $\lim_{n\to\infty} R_n S_0^1 f = 1/3$.

Show, by computation, that $\lim_{n\to\infty} L_n S_0^1 f = 1/3$

and that $\lim_{n\to\infty} M_n S_0^1 f = 1/3$.

SKILL
Limit of Riemann sums

$$M_n S_0^1 f = \frac{1}{n}\sum_{j=1}^{n}\left[\cancel{0} + \left(j - \frac{1}{2}\right)\frac{1}{n}\right]^2 = \frac{1}{n}\sum_{j=1}^{n}\left(j^2 - j + \frac{1}{4}\right)\frac{1}{n^2}$$

$$\|$$

$$\frac{1}{n^3}\left[\left(\frac{2n^3+3n^2+n}{6}\right) - \left(\frac{n^2+n}{2}\right) + \frac{n}{4}\right] = \frac{1}{n^3}\sum_{j=1}^{n} j^2 - j + \frac{1}{4}$$

$$\|$$

$$\left(\frac{2}{6} + \frac{3}{6n} + \frac{1}{6n^2}\right) - \left(\frac{1}{2n} + \frac{1}{2n^2}\right) + \frac{1}{4n^2} \xrightarrow{n\to\infty} \frac{2}{6} = \frac{1}{3}$$

IOU: An easier approach, via the Fundamental Theorem of Calculus (Later topic.)

Kinda hard. . .

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$$R_n S_2^7 f = \frac{5}{n}\sum_{j=1}^{n} f\left(\underline{2 + j\frac{5}{n}}\right) = \frac{5}{n}\sum_{j=1}^{n} 3\left(\underline{2 + j\frac{5}{n}}\right)^2 + 4\left(\underline{2 + j\frac{5}{n}}\right)^3$$

$$h_n = \frac{7-2}{n} = \frac{5}{n}$$

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$$R_n S_2^7 f = \frac{5}{n}\sum_{j=1}^{n} f\left(2 + j\frac{5}{n}\right) = \frac{5}{n}\sum_{j=1}^{n} 3\left(\boxed{2 + j\frac{5}{n}}\right)^2 + 4\left(\boxed{2 + j\frac{5}{n}}\right)^3$$

$$\mathbin{/\!/}$$

$$\frac{5}{n}\sum_{j=1}^{n} 3\left(\boxed{\frac{2n+5j}{n}}\right)^{2} + 4\left(\boxed{\frac{2n+5j}{n}}\right)^{3}$$

$$\|$$

$$\frac{5}{n}\sum_{j=1}^{n} \frac{3(2n+5j)^{2}}{\boxed{n^{2}}} + \frac{4(2n+5j)^{3}}{n^{3}}$$

$$\|$$

$$\frac{5}{\boxed{n}}\sum_{j=1}^{n} \frac{3\boxed{n}(2n+5j)^2}{\boxed{n^3}} + \frac{4(2n+5j)^3}{\boxed{n^3}}$$

COMMON DENOMINATOR

$$\|$$

$$\frac{5}{\boxed{n^4}}\sum_{j=1}^{n} 3n(2n+5j)^2 + 4(2n+5j)^3$$

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$$R_n S_2^7 f = \frac{5}{n^4} \sum_{j=1}^{n} 3n\boxed{(2n+5j)^2} + 4\boxed{(2n+5j)^3}$$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$$\frac{5}{n^4} \sum_{j=1}^{n} 3n(2n+5j)^2 + 4(2n+5j)^3$$

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$$R_n S_2^7 f = \frac{5}{n^4} \sum_{j=1}^{n} 3n\boxed{(2n+5j)^2} + 4\boxed{(2n+5j)^3}$$

$$\|$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} 3n\boxed{[(2n)^2 + 2(2n)(5j) + (5j)^2]} \\ + \\ 4\boxed{[(2n)^3 + 3(2n)^2(5j) + 3(2n)(5j)^2 + (5j)^3]} \end{array} \right)$$

1

1 1

1 2 1

1 3 3 1

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$$R_n S_2^7 f = \frac{5}{n^4} \sum_{j=1}^{n} 3n(2n+5j)^2 + 4(2n+5j)^3$$

$$\|$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} 3n[\boxed{(2n)^2} + 2(2n)(5j) + \boxed{(5j)^2}] \\ + \\ 4[\boxed{(2n)^3} + 3(2n)^2(5j) + 3(2n)(5j)^2 + \boxed{(5j)^3}] \end{array} \right)$$

$$\|$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} 3n[\boxed{4n^2} + \boxed{2(2n)(5j)} + \boxed{25j^2}] \\ + \\ 4[\boxed{8n^3} + \boxed{3(4n^2)(5j)} + \boxed{3(2n)(25j^2)} + \boxed{125j^3}] \end{array} \right)$$

$$\|$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} 3n[4n^2 + \boxed{20nj} + 25j^2] \\ + \\ 4[8n^3 + \boxed{60n^2j} + \boxed{150nj^2} + 125j^3] \end{array} \right)$$

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$R_n S_2^7 f =$

$$R_n S_2^7 f = \frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} \boxed{3n[4n^2 + 20nj + 25j^2]} \\ + \\ \boxed{4[8n^3 + 60n^2 j + 150nj^2 + 125j^3]} \end{array} \right)$$

$$\|$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} \boxed{12n^3 + 60n^2 j + 75nj^2} \\ + \\ \boxed{32n^3 + 240n^2 j + 600nj^2 + 500j^3} \end{array} \right)$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} 3n[4n^2 + 20nj + 25j^2] \\ + \\ 4[8n^3 + 60n^2 j + 150nj^2 + 125j^3] \end{array} \right)$$

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$$R_n S_2^7 f = \frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} 3n[4n^2 + 20nj + 25j^2] \\ + \\ 4[8n^3 + 60n^2 j + 150nj^2 + 125j^3] \end{array} \right)$$

$$\|$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} \boxed{12n^3} + \boxed{60n^2 j} + \boxed{75nj^2} \\ + \\ \boxed{32n^3} + \boxed{240n^2 j} + \boxed{600nj^2} + \boxed{500j^3} \end{array} \right)$$

$$\|$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \left(\begin{array}{c} \boxed{44n^3} \\ + \\ \boxed{300n^2 j} \\ + \\ \boxed{675nj^2} \\ + \\ \boxed{500j^3} \end{array} \right)$$

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$$R_n S_2^7 f = \frac{5}{n^4} \boxed{\sum_{j=1}^{n}} \begin{pmatrix} 44n^3 \cdot \boxed{1} \\ + \\ 300n^2 \boxed{j} \\ + \\ 675n \boxed{j^2} \\ + \\ 500 \boxed{j^3} \end{pmatrix} = \frac{5}{n^4} \begin{pmatrix} 44n^3 (\boxed{n}) \\ + \\ 300n^2 \boxed{(n^2+n)/2} \\ + \\ 675n \boxed{(2n^3+3n^2+n)/6} \\ + \\ 500 \boxed{(n^4+2n^3+n^2)/4} \end{pmatrix}$$

$$\frac{5}{n^4} \sum_{j=1}^{n} \begin{pmatrix} 44n^3 \\ + \\ 300n^2 j \\ + \\ 675nj^2 \\ + \\ 500j^3 \end{pmatrix}$$

EXAMPLE: Let $f(x) = 3x^2 + 4x^3$. Compute $\lim_{n\to\infty} R_n S_2^7 f$.

$$R_n S_2^7 f = \frac{5}{n^4} \sum_{j=1}^{n} \begin{pmatrix} 44n^3 \\ + \\ 300n^2 j \\ + \\ 675 n j^2 \\ + \\ 500 j^3 \end{pmatrix} = \frac{5}{\boxed{n^4}} \begin{pmatrix} 44n^3(n) \\ + \\ 300n^2(n^2+n)/2 \\ + \\ 675n(2n^3+3n^2+n)/6 \\ + \\ 500(n^4+2n^3+n^2)/4 \end{pmatrix}$$

$$\| $$

SKILL
Limit of
Riemann sums

$$■\, 2720 = 5 \begin{pmatrix} 44(1) \\ + \\ 300(1)/2 \\ + \\ 675(2)/6 \\ + \\ 500(1)/4 \end{pmatrix} \xleftarrow{n\to\infty} 5 \begin{pmatrix} 44(1) \\ + \\ 300(1+\boxed{n^{-1}})/2 \\ + \\ 675(2+3\boxed{n^{-1}}+\boxed{n^{-2}})/6 \\ + \\ 500(1+2\boxed{n^{-1}}+\boxed{n^{-2}})/4 \end{pmatrix}$$

0

Kinda hard...

IOU: An easier approach, via the Fundamental Theorem of Calculus (Later topic.)

EXAMPLE: Express $\int_2^6 \ln(x^2+7)\,dx$ as a limit of

$\boxed{\text{left}}$ $\boxed{\text{right}}$-endpoint Riemann sums.

For each $n = 1, 2, 3, \ldots,$

find width of rectangles, $(h_n) = \frac{6-2}{n} = \frac{4}{n}$

find $\boxed{\text{right}}$-endpoint of jth subinterval, $= 2 + \boxed{j}\left(\frac{4}{n}\right) = 2 + \frac{4\boxed{j}}{n}$ $\quad \boxed{j-1}$

find height of jth rectangle,

find area of jth rectangle,

find add over $j = 1, \ldots, n$.

Then take $\lim\limits_{n\to\infty}$.

$$\lim_{n\to\infty} \sum_{j=1}^{n} \left[\frac{4}{n}\right] \left[\ln\left(\left[2+\frac{4\boxed{j}}{n}\right]^2 + 7\right)\right] \quad \boxed{j-1}$$

SKILL
Write int as lim RS

EXAMPLE: Express $\int_2^6 \ln(x^2+7)\,dx$ as a limit of

$\boxed{\text{midpoint}} \rightarrow \boxed{\text{left-endpoint}}$ Riemann sums.

For each $n = 1, 2, 3, \ldots$, (h_n)

find width of rectangles, $= \dfrac{6-2}{n} = \dfrac{4}{n}$

find $\boxed{\text{left-endpoint}}$ of jth subinterval,

find height of jth rectangle,

find area of jth rectangle,

find add over $j = 1, \ldots, n$.

Then take $\lim\limits_{n\to\infty}$.

$$= 2 + (j - \boxed{1})\left(\frac{4}{n}\right) \qquad \boxed{\tfrac{1}{2}}$$

$$= 2 + \frac{4(j - \boxed{1})}{n} \qquad \boxed{\tfrac{1}{2}}$$

$\boxed{\tfrac{1}{2}}$

$$\lim_{n\to\infty} \sum_{j=1}^{n} \left[\frac{4}{n}\right] \left[\ln\left(\left[2 + \frac{4(j-\boxed{1})}{n}\right]^2 + 7\right)\right] \blacksquare$$

SKILL
Write int as lim RS

EXAMPLE: Express $\int_2^6 \ln(x^2+7)\,dx$ as a limit of midpoint Riemann sums.

For each $n = 1, 2, 3, \ldots$, (h_n) find width of rectangles, $= \frac{6-2}{n} = \frac{4}{n}$

find midpoint of jth subinterval, $= 2 + \left(j - \frac{1}{2}\right)\left(\frac{4}{n}\right) = 2 + \frac{4\left(j-\frac{1}{2}\right)}{n}$

find height of jth rectangle,

find area of jth rectangle,

find add over $j = 1, \ldots, n$.

Then take $\lim\limits_{n\to\infty}$.

$$\lim_{n\to\infty} \sum_{j=1}^{n} \left[\frac{4}{n}\right] \left[\ln\left(\left[2 + \frac{4\left(j-\frac{1}{2}\right)}{n}\right]^2 + 7\right)\right] \blacksquare$$

SKILL

Write int as lim RS

EXAMPLE: Express $\lim_{n\to\infty} \frac{2}{n} \sum_{j=1}^{n} (3+(2/n)j)e^{3+(2/n)j}$ as a definite integral.

Ignore $\lim_{n\to\infty}$. Identify width of subinterval $= \frac{b-a}{n}$

and right-endpoint $= a + j\left(\boxed{\frac{b-a}{n}}\right)$.

Might appear more than once.

Might be left-endpoint ($j :\to j-1$)

or midpoint ($j :\to j - \frac{1}{2}$).

Figure out a and b. Figure out $f(x)$, using endpoint $:\to x$

Answer is: $\int_a^b f(x)\,dx = \int_3^5 xe^x\,dx$

$f(x) = xe^x$

$$\lim_{n\to\infty} \boxed{\frac{2}{n}} \sum_{j=1}^{n} \left(\boxed{3+\left(\boxed{2/n}\right)j}\right)e^{\boxed{3+\left(\boxed{2/n}\right)j}} = \int_3^5 xe^x\,dx \quad \blacksquare$$

SKILL

Interpret limit of Riemann sum

$\frac{b-a}{n} = \frac{2}{n}$, $\quad b-a=2$, $\quad a=3$, $\quad b=5$

SKILL
Integration by summation

Whitman problems
§7.1, p. 143, #1-8