CALCULUS
The Fundamental Theorems of Calculus, problems

cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is contin. on $[a,b]$, then $\frac{d}{dx}\int_a^x f(t)\,dt = [f(t)]_{t:\to x} = f(x)$, for $x \in (a,b)$.

EXAMPLE: Find the derivative of

the function $g(x) = \int_0^x \sqrt{2+t^4}\,dt$.

Sol'n: $g'(x) = \frac{d}{dx}\int_0^x \sqrt{2+t^4}\,dt$

$\overset{\text{FTC}}{=} \left[\sqrt{2+t^4}\right]_{t:\to x}$

$= \sqrt{2+x^4}$ ■

SKILL
diff int const to variable

If f is contin. on $[a,b]$, then $\displaystyle\frac{d}{dx}\int_a^x f(t)\,dt = [f(t)]_{t:\to x} = f(x)$, for $x \in (a,b)$.

Example: Use the FTC to find the derivative of

$$g(s) = \int_0^s \sqrt[7]{x^3 - 5x + 2}\,dx.$$

Sol'n: $g'(s) \overset{\text{FTC}}{=} \left[\sqrt[7]{x^3 - 5x + 2}\right]_{x:\to s}$

$$= \sqrt[7]{s^3 - 5s + 2} \blacksquare$$

SKILL
diff int const to variable

cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is contin. on $[a, b]$, then $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = [f(t)]_{t:\to x} = f(x)$, for $x \in (a, b)$.

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cf. §7.2, p. 145 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.3

Let f be any function, contin. on $[a, b]$.

Let F be an antiderivative of f on $[a, b]$.

$$\text{Then } \int_a^b f(x)\,dx = [F(x)]_{x:\to a}^{x:\to b} = (F(b)) - (F(a)).$$

$$\text{e.g.: } \int_5^7 x^2\,dx \overset{\text{FTC}}{=} \left[\frac{x^3}{3}\right]_{x:\to 5}^{x:\to 7} = \frac{7^3}{3} - \frac{5^3}{3} = \frac{218}{3} \blacksquare$$

SKILL
eval def integral

cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is contin. on $[a, b]$, then $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = [f(t)]_{t:\to x} = f(x)$, for $x \in (a, b)$.

cf. §7.2, p. 145 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.3

Let f be any function, contin. on $[a, b]$.

Let F be an antiderivative of f on $[a, b]$.

Then $$\int_a^b f(x)\,dx = [F(x)]_{x:\to a}^{x:\to b} = (F(b)) - (F(a)).$$

EXAMPLE: Evaluate the integral $\displaystyle\int_1^5 e^{7x}\,dx$.

Sol'n: $\displaystyle\int_1^5 e^{7x}\,dx$

e^{7x}

cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is contin. on $[a,b]$, then $\frac{d}{dx}\int_a^x f(t)\,dt = [f(t)]_{t:\to x} = f(x)$, for $x \in (a,b)$.

cf. §7.2, p. 145 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.3

Let f be any function, contin. on $[a,b]$.

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Then $\int_a^b f(x)\,dx = [F(x)]_{x:\to a}^{x:\to b} = (F(b)) - (F(a))$.

EXAMPLE: Evaluate the integral $\int_1^5 e^{7x}\,dx$.

~~$7e^{7x}$~~

Sol'n: $\int_1^5 e^{7x}\,dx$

cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is contin. on $[a,b]$, then $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = [f(t)]_{t:\to x} = f(x)$, for $x \in (a,b)$.

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Let f be any function, contin. on $[a,b]$.

Let F be an antiderivative of f on $[a,b]$.

Then $\displaystyle\int_a^b f(x)\,dx = [F(x)]_{x:\to a}^{x:\to b} = (F(b)) - (F(a))$.

EXAMPLE: Evaluate the integral $\displaystyle\int_1^5 e^{7x}\,dx$.

Sol'n: $\displaystyle\int_1^5 e^{7x}\,dx \underset{\text{FTC}}{=} [e^{7x}/7]_{x:\to 1}^{x:\to 5} = (e^{35}/7) - (e^{7}/7)$ ■

($e^{7x}/7$ ☺)

SKILL
eval def integral

EXERCISE: Evaluate $\int_0^2 (1-\frac{1}{2}u^6-\frac{4}{5}u^9)\,du$.

$u - \frac{1}{2}(\frac{1}{7}u^7) - \frac{4}{5}(\frac{1}{10}u^{10}) = u - \frac{1}{14}u^7 - \frac{4}{50}u^{10}$

Sol'n: $\int_0^2 (1-\frac{1}{2}u^6-\frac{4}{5}u^9)\,du \overset{\text{FTC}}{=} \left[u - \frac{1}{14}u^7 - \frac{4}{50}u^{10}\right]_{u:\to 0}^{u:\to 2}$

$= \left[2 - \frac{1}{14}(128) - \frac{4}{50}(1024)\right] \cancel{- [0]}$

$= \frac{14}{7} - \frac{64}{7} - \frac{2048}{25} = -\frac{50}{7} - \frac{2048}{25}$

$= -\frac{(50)(25)+(2048)(7)}{(7)(25)}$

$= -\frac{1250+14336}{175}$

$= -\frac{15586}{175}$ ■

SKILL
eval def integral

EXAMPLE: Evaluate $\int_0^5 (x^4 - 8x)\,dx$.

$$\| \text{FTC}$$

$$\left[\frac{x^5}{5} - \frac{8x^2}{2}\right]_{x:\to 0}^{x:\to 5}$$

$$\|$$

$$\left[\frac{5^5}{5} - \frac{8 \cdot 5^2}{2}\right] \cancel{-0}$$

$$\|$$

$$5^4 - 4 \cdot 25$$

$$\|$$

$$625 - 100$$

$$\|$$

$$525$$

■

SKILL
eval def int

EXAMPLE: Compute $\int_1^8 (3u-5)(5u+2)\,du$.

$\|$ EXPAND

$$\int_1^8 \boxed{(3u)(5u)} + \boxed{(3u)(2)} - \boxed{(5)(5u)} - \boxed{(5)(2)}\,du$$

$\|$

$$\int_1^8 \boxed{15u^2} \boxed{+\boxed{6u} - \boxed{25u}} - \boxed{10}\,du$$

$\|$

$$\int_1^8 15u^2 \boxed{-19u} - 10\,du$$

$\|$ FTC

$$\left[5u^3 - \frac{19u^2}{2} - 10u\right]_{u:\to 1}^{u:\to 8}$$

LINEARITY OF $[\bullet]_{x:\to a}^{x:\to b}$

$\|$

$$5(8^3-1^3) - \frac{19(8^2-1^2)}{2} - 10(8-1)$$ ■

WARNING:

§8.1 Integration is not multiplicative.

SKILL
eval def integral

EXERCISE: Evaluate $\int_0^4 (\sqrt[5]{2} + x^3 \sqrt[7]{x})\, dx$.

Sol'n: $\int_0^4 (\sqrt[5]{2} + x^3 \sqrt[7]{x})\, dx = \int_0^4 (\sqrt[5]{2} + x^3 \cdot x^{1/7})\, dx$

$3 + (1/7) = (21/7) + (1/7) = 22/7$

$$= \int_0^4 (\sqrt[5]{2} + x^{22/7})\, dx$$

$1 + (22/7) = (7/7) + (22/7) = 29/7$

$$\overset{\text{FTC}}{=} \left[\sqrt[5]{2}x + \frac{x^{29/7}}{29/7} \right]_{x:\to 0}^{x:\to 4}$$

LINEARITY OF $[\bullet]_{x:\to a}^{x:\to b}$

$$= \sqrt[5]{2}\left([x]_{x:\to 0}^{x:\to 4}\right) + \frac{\left[x^{29/7}\right]_{x:\to 0}^{x:\to 4}}{29/7}$$

$$= \sqrt[5]{2}\,(4) + \frac{4^{29/7}}{29/7} \blacksquare$$

SKILL
eval def integral

EXERCISE: Evaluate $\int_1^2 \left(\frac{-7+w^3}{w^8}\right)\, dw.$

$$\| $$

$$\int_1^2 \left(-\frac{7}{w^8}+\frac{w^3}{w^8}\right)\, dw$$

$$\| $$

$$\int_1^2 \left(-\frac{7}{w^8}+\frac{1}{w^5}\right)\, dw$$

$$\| $$

$$\int_1^2 \left(-7w^{-8}+w^{-5}\right)\, dw$$

$$-8+1=-7 \quad \| \quad -5+1=-4$$

FTC $\left[-7\left(\frac{w^{-7}}{-7}\right)+\left(\frac{w^{-4}}{-4}\right)\right]_{w:\to 1}^{w:\to 2}$

$$\| $$

$$\left[w^{-7}-\frac{w^{-4}}{4}\right]_{w:\to 1}^{w:\to 2}$$

§7.2

EXERCISE: Evaluate $\int_1^2 \left(\frac{-7 + w^3}{w^8} \right) dw.$

$$\| $$

$$\left[w^{-7} - \frac{w^{-4}}{4} \right]_{w:\to 1}^{w:\to 2}$$

$$\| $$

$$\left[\frac{1}{w^7} - \frac{1}{4w^4} \right]_{w:\to 1}^{w:\to 2}$$

$$\left[w^{-7} - \frac{w^{-4}}{4} \right]_{w:\to 1}^{w:\to 2}$$

EXERCISE: Evaluate $\int_1^2 \left(\frac{-7+w^3}{w^8}\right)\,dw.$

$$\| $$

$$\left[w^{-7} - \frac{w^{-4}}{4}\right]_{w:\to 1}^{w:\to 2}$$

$$\| $$

$$\left[\frac{1}{w^7} - \frac{1}{4w^4}\right]_{w:\to 1}^{w:\to 2}$$

$$\| $$

$$\left[\frac{1}{2^7} - \frac{1}{4\cdot 2^4}\right] - \left[\frac{1}{1^7} - \frac{1}{4\cdot 1^4}\right]$$

$$\| $$

$$\left[\frac{1}{128} - \frac{1}{64}\right] - \left[1 - \frac{1}{4}\right]$$

$$\| $$

$$-\frac{1}{128} - \frac{3}{4} = -\frac{1}{128} - \frac{96}{128}$$

$$/\!/$$

$$-\frac{97}{128}$$

SKILL
eval def integral

EXAMPLE: Evaluate $\displaystyle\int_2^7 \frac{3t^2 + t^2\sqrt[5]{t} - 1}{\boxed{t^3}}\,dt.$

$$\|$$

$$\int_2^7 \frac{3t^2}{\boxed{t^3}} + \frac{t^2\sqrt[5]{t}}{\boxed{t^3}} - \frac{1}{\boxed{t^3}}\,dt$$

$$\|$$

$$\int_2^7 3t^{2-3} + t^{2+(1/5)-3} - t^{-3}\,dt$$

$$\|$$

$$\int_2^7 3t^{-1} + t^{-4/5} - t^{-3}\,dt$$

FTC $\|$

$$\left[3[\ln(|t|)] + \frac{t^{1/5}}{1/5} - \frac{t^{-2}}{-2}\right]_{t:\to 2}^{t:\to 7}$$

LINEARITY OF $[\bullet]_{x:\to a}^{x:\to b}$

$$\|$$

$$3[(\ln 7) - (\ln 2)] + \frac{7^{1/5} - 2^{1/5}}{1/5} - \frac{7^{-2} - 2^{-2}}{-2}$$ ■

SKILL
eval def int

EXERCISE: Evaluate $\displaystyle\int_0^1 \frac{3}{t^2+1}\,dt$.

$$\overset{\text{FTC}}{\|}$$

$$[3\arctan t]_{t:\to 0}^{t:\to 1}$$

LINEARITY OF $[\bullet]_{x:\to a}^{x:\to b}$

$$\|$$

$$3\left([\arctan t]_{t:\to 0}^{t:\to 1}\right)$$

$$\|$$

$$3\,([\arctan 1] - [\arctan 0])$$

$$\|$$

$$3\,([\pi/4] - [0])$$

$$\|$$

$$3\pi/4$$

■

SKILL
eval def integral

EXAMPLE: Find $\int_0^3 \left(\frac{7}{x^2+1}\right) dx$ and describe the result as an area under the graph.

FTC $\parallel$

$[7 \arctan x]_{x:\to 0}^{x:\to 3}$

$\parallel$

$[7 \arctan 3] - 0$

$\cdot\parallel$

8.743

■

SKILL
eval def int

Problems of the form

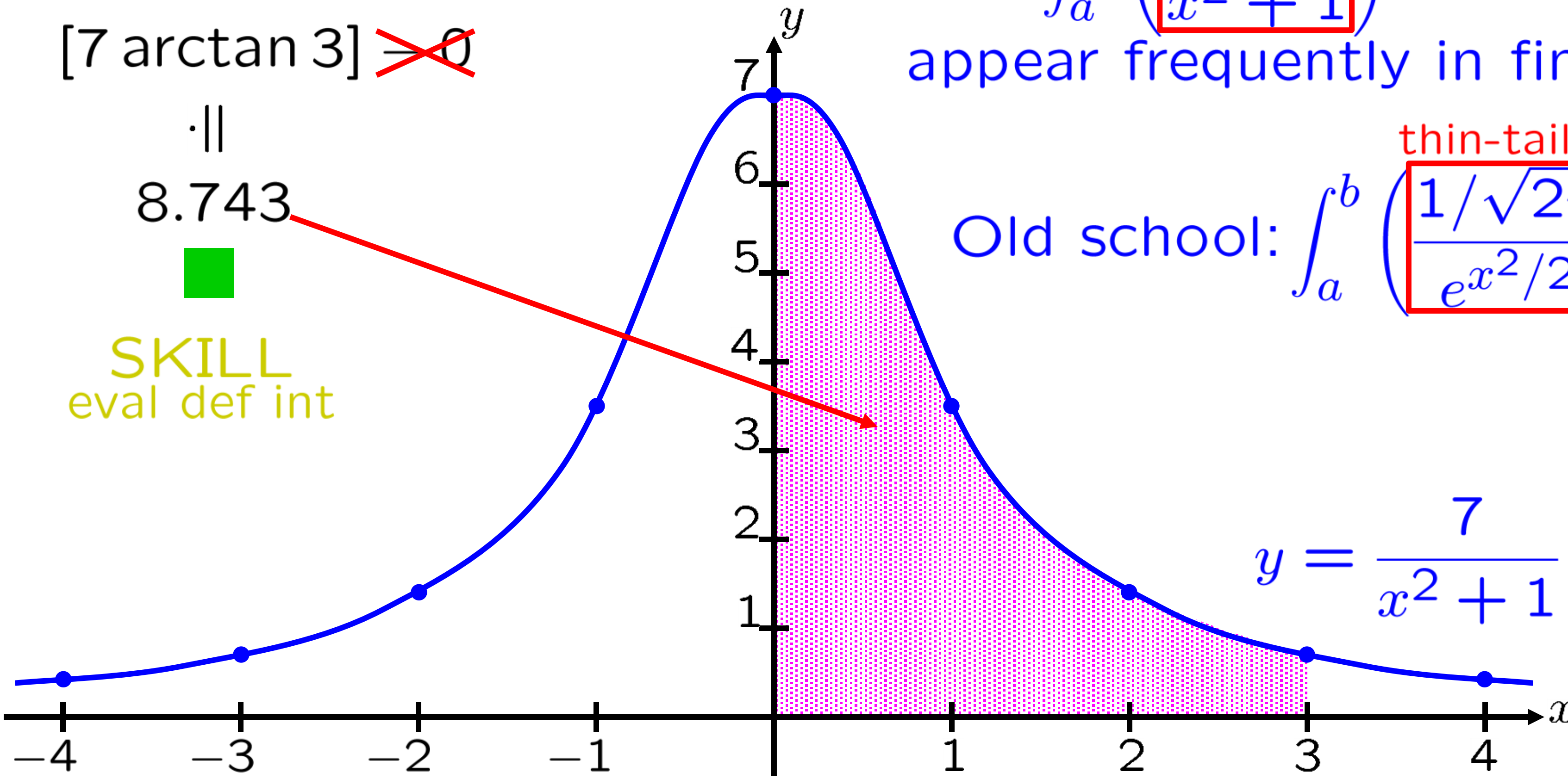
$$\int_a^b \left(\frac{1/\pi}{x^2+1}\right) dx$$

fat-tailed

appear frequently in finance.

thin-tailed

Old school: $\int_a^b \left(\frac{1/\sqrt{2\pi}}{e^{x^2/2}}\right) dx$



Fat-tailed distributions make the world of finance go 'round . . .

EXAMPLE: Compute $\int_{\pi/4}^{\pi/3} (\csc\theta)(\cot\theta)\,d\theta$.

FTC ||

$$[-\csc\theta]_{\theta:\to\pi/4}^{\theta:\to\pi/3}$$

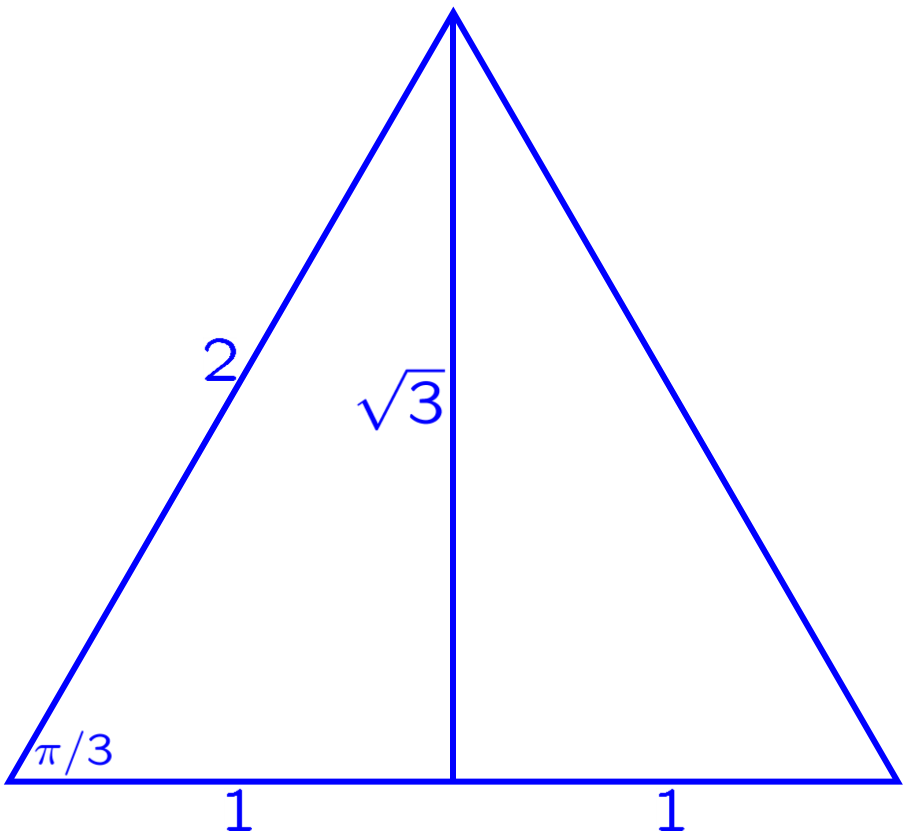
||

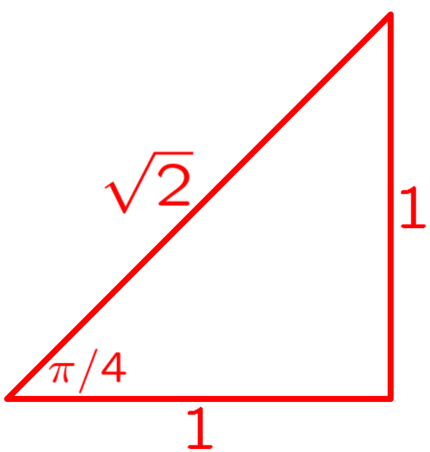
$$[-\csc(\pi/3)] - [-\csc(\pi/4)]$$

||

$$\left[-\frac{2}{\sqrt{3}}\right] - \left[-\sqrt{2}\right]$$ ■

SKILL
find def int





$$\csc(\pi/3) = \frac{2}{\sqrt{3}}$$

$$\csc(\pi/4) = \sqrt{2}$$

EXAMPLE: Compute $\int_3^5 \frac{dx}{x}$, $\int_{-8}^{-4} \frac{dx}{x}$, $\int_{-8}^{5} \frac{dx}{x}$.

$$\int \frac{dx}{x} = \int \frac{1}{x}\,dx = \begin{cases} (\ln(x)) + A, & \text{if } x > 0 \\ (\ln(-x)) + B, & \text{if } x < 0 \end{cases}$$

Setting $A = 0$ and $B = 0$, we see that $\ln(|x|)$ is an antiderivative of $1/x$ w.r.t. x

$$\int_3^5 \frac{dx}{x} \overset{\text{FTC}}{=} [\ln(|x|)]_{x:\to 3}^{x:\to 5} = (\ln 5) - (\ln 3)$$

Same answers for any choice of A and B.

$$\int_{-8}^{-4} \frac{dx}{x} \overset{\text{FTC}}{=} [\ln(|x|)]_{x:\to -8}^{x:\to -4} = (\ln 4) - (\ln 8)$$

SKILL
Definite integral

$\int_{-8}^{5} \frac{dx}{x}$ DNE ■

§8.1

EXAMPLE: Compute $\displaystyle\int_3^5 \frac{dx}{x}$, $\displaystyle\int_{-8}^{-4} \frac{dx}{x}$, $\displaystyle\int_{-8}^{5} \frac{dx}{x}$.

$$\int \frac{dx}{x} = [\ln(|x|)] + C$$

It's quite common to list one antiderivative "plus C", even in cases where it's technically wrong!
Gives the right answer here.

$$\int \frac{dx}{x} = \int \frac{1}{x}\,dx = \begin{Bmatrix} (\ln(x)) + A, & \text{if } x > 0 \\ (\ln(-x)) + B, & \text{if } x < 0 \end{Bmatrix}$$

Setting $A = 0$ and $B = 0$, we see that $\ln(|x|)$ is an antiderivative of $1/x$ w.r.t. x

SKILL
Definite integral

$$\int_3^5 \frac{dx}{x} = [\ln(|x|)]_{x:\to 3}^{x:\to 5} = (\ln 5) - (\ln 3)$$

$$\int_{-8}^{-4} \frac{dx}{x} = [\ln(|x|)]_{x:\to -8}^{x:\to -4} = (\ln 4) - (\ln 8)$$

$$\int_{-8}^{5} \frac{dx}{x} \quad \text{DNE}$$

cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is contin. on $[a,b]$, then $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dx = [f(t)]_{t:\to x} = f(x)$, for $x \in (a,b)$.

[[FTC, THEOREM 7.4, Corollary]]

$\ln(1+t^6)$ has an antideriv. (w.r.t. t)

$F(t)$ $\qquad F'(t) = \ln(1+t^6)$

EXAMPLE: Find $\dfrac{d}{dx}\displaystyle\int_{x^2}^{x^4} \ln(1+t^6)\,dt.$

$(F(x^4)) - (F(x^2))$

[[FTC, THEOREM 7.3]]

Sol'n:

CHAIN RULE

$$\frac{d}{dx}[(F(x^4)) - (F(x^2))] = [F'(x^4)][4x^3] - [F'(x^2)][2x]$$

$$= [\ln(1+(x^4)^6)][4x^3] - [\ln(1+(x^2)^6)][2x]$$

■

SKILL

diff int expr to expr

EXAMPLE: Use the FTC to find the derivative of

$$h(x) = \int_0^{x^3} \overbrace{\sqrt{1+2s^6}}^{F(s)}\, ds.$$

$$F'(s) = \sqrt{1+2s^6}$$

$$F' = \sqrt{1+2(\bullet)^6}$$

Solution: $h'(x) \overset{\text{FTC}}{=} \frac{d}{dx}[(F(x^3)) - (F(0))]$

$$\overset{\text{CR}}{=} [F'(x^3)][3x^2] - 0$$

$$= \left[\sqrt{1+2(x^3)^6}\right][3x^2] \blacksquare$$

SKILL
diff int expr to expr

EXAMPLE: Find the derivative of

$$g(x) = \int_{x^4}^{\cos x} \frac{8}{\sqrt{7+t^6}}\,dt.$$

$F(t)$

FTC $(F(\cos x)) - (F(x^4))$

Sol'n: $g'(x) = \frac{d}{dx}\left[(F(\cos x)) - (F(x^4))\right]$

CHAIN RULE

$$= [F'(\cos x)][-\sin x] - [F'(x^4)][4x^3]$$

$$= \left[\frac{8}{\sqrt{7+(\cos x)^6}}\right][-\sin x] - \left[\frac{8}{\sqrt{7+(x^4)^6}}\right][4x^3]$$

SKILL

diff int expr to expr

EXERCISE: Let $g(x) = \int_{-1-x^4}^{2+x^6} (1 + \sqrt[3]{t})\, dt$. $t + [t^{4/3}/(4/3)]$

(a) Compute $g(x)$ in "closed form",

then differentiate to get $g'(x)$.

(b) Compute $g'(x)$ using the FTC.

Sol'n:

(a) $$g(x) \overset{\text{FTC}}{=} \left[t + \frac{t^{4/3}}{4/3} \right]_{t:\to -1-x^4}^{t:\to 2+x^6}$$

$$= \left[(2 + x^6) + \frac{(2 + x^6)^{4/3}}{4/3} \right]$$

$$- \left[(-1 - x^4) + \frac{(-1 - x^4)^{4/3}}{4/3} \right]$$

$$g'(x) = \left[(6x^5) + \frac{(4/3)(2 + x^6)^{1/3}}{4/3} \overset{\text{CR}}{(6x^5)} \right]$$

$$- \left[(-4x^3) + \frac{(4/3)(-1 - x^4)^{1/3} \overset{\text{CR}}{(-4x^3)}}{4/3} \right]$$

EXERCISE: Let $g(x) = \int_{-1-x^4}^{2+x^6} (1 + \sqrt[3]{t})\, dt$.

(a) Compute $g(x)$ in "closed form",

then differentiate to get $g'(x)$.

(b) Compute $g'(x)$ using the FTC.

Sol'n:

(a) $$g'(x) = \left[(6x^5) + \frac{\cancel{(4/3)}(2 + x^6)^{1/3}}{\cancel{4/3}}(6x^5) \right] - \left[(-4x^3) + \frac{(4/3)(-1 - x^4)^{1/3}(-4x^3)}{4/3} \right]$$

$$g'(x) = \left[(6x^5) + \frac{(4/3)(2 + x^6)^{1/3}}{4/3}(6x^5) \right] - \left[(-4x^3) + \frac{(4/3)(-1 - x^4)^{1/3}(-4x^3)}{4/3} \right]$$

EXERCISE: Let $g(x) = \int_{-1-x^4}^{2+x^6} (1+\sqrt[3]{t})\,dt$.

(a) Compute $g(x)$ in "closed form",

then differentiate to get $g'(x)$.

(b) Compute $g'(x)$ using the FTC.

Sol'n:

(a) $$g'(x) = \left[(6x^5) + \frac{(4/3)(2+x^6)^{1/3}}{4/3}(6x^5)\right] - \left[(-4x^3) + \frac{(4/3)(-1-x^4)^{1/3}(-4x^3)}{4/3}\right]$$

$$= \left[1 + (2+x^6)^{1/3}\right](6x^5) - \left[1 + (-1-x^4)^{1/3}\right](-4x^3)$$

$$= \left[1 + \sqrt[3]{2+x^6}\right](6x^5) - \left[1 + \sqrt[3]{-1-x^4}\right](-4x^3)$$

EXERCISE: Let $g(x) = \int_{-1-x^4}^{2+x^6} (1 + \sqrt[3]{t})\, dt$.

(a) Compute $g(x)$ in "closed form",

then differentiate to get $g'(x)$.

(b) Compute $g'(x)$ using the FTC.

Sol'n:

(a) $g'(x) = \left[1 + \sqrt[3]{2 + x^6}\right](6x^5) - \left[1 + \sqrt[3]{-1 - x^4}\right](-4x^3)$

(b)

$$\left[1 + \sqrt[3]{2 + x^6}\right](6x^5)$$

$$- \left[1 + \sqrt[3]{-1 - x^4}\right](-4x^3)$$

EXERCISE: Let $g(x) = \int_{-1-x^4}^{2+x^6} (1 + \sqrt[3]{t})\, dt$.

$F(t)$

(a) Compute $g(x)$ in "closed form",
then differentiate to get $g'(x)$.

(b) Compute $g'(x)$ using the FTC.

Sol'n:

(a) $g'(x) = \left[1 + \sqrt[3]{2 + x^6}\right](6x^5) - \left[1 + \sqrt[3]{-1 - x^4}\right](-4x^3)$

(b) $g(x) \overset{\text{FTC}}{=} [F(2 + x^6))] - [F(-1 - x^4)]$

$g'(x) \overset{\text{CR}}{=} [F'(2 + x^6))](6x^5) - [F'(-1 - x^4)](-4x^3)$

$= \left[1 + \sqrt[3]{2 + x^6}\right](6x^5) - \left[1 + \sqrt[3]{-1 - x^4}\right](-4x^3)$

EXERCISE: Let $g(x) = \int_{-1-x^4}^{2+x^6} (1 + \sqrt[3]{t})\, dt.$

(a) Compute $g(x)$ in "closed form",

then differentiate to get $g'(x)$.

(b) Compute $g'(x)$ using the FTC.

Sol'n:

(a) $g'(x) = \left[1 + \sqrt[3]{2 + x^6}\right](6x^5) - \left[1 + \sqrt[3]{-1 - x^4}\right](-4x^3)$

(b) $g'(x) = \left[1 + \sqrt[3]{2 + x^6}\right](6x^5) - \left[1 + \sqrt[3]{-1 - x^4}\right](-4x^3)$ ■

SKILL

diff int expr to expr

(b)

$$g'(x) =$$

$$\left[1 + \sqrt[3]{2 + x^6}\right](6x^5) - \left[1 + \sqrt[3]{-1 - x^4}\right](-4x^3)$$

EXAMPLE: Let $f(s) := \underbrace{\int_3^{s^6} \overbrace{e^{-r^2}}^{G(r)}\, dr}_{\text{FTC } (G(s^6)) - (G(3))}$, $G'(r) = e^{-r^2}$, $G' = e^{-(\bullet)^2}$

and let $F(t) := \int_4^t f(s)\, ds$. Compute $F''(8)$.

Sol'n: $F'(t) \overset{\text{FTC}}{=} [f(s)]_{s:\to t} = f(t)$

$$F''(t) = f'(t) = \frac{d}{dt}[f(t)] = \frac{d}{dt}\left[(G(t^6)) - (G(3))\right]$$

$$\overset{\text{CR}}{=} [G'(t^6)][6t^5] \cancel{- 0}$$

$$= \left[e^{-(t^6)^2}\right][6t^5]$$

$$F''(8) = \left[e^{-(8^6)^2}\right][6(8^5)] \blacksquare$$

SKILL

diff^2 int^2 expr to expr

EXAMPLE: If water pours into a tank at a rate of $r(t)$ gallons per hour at time t,

what does $\int_0^{24} r(t)\,dt$ represent?

Sol'n: $A(t) :=$ amount of water in the tank at time t

$$A'(t) = r(t)$$

$$\int_0^{24} r(t)\,dt \stackrel{\text{FTC}}{=} [A(t)]_{t:\to 0}^{t:\to 24}$$

$$= [A(24)] - [A(0)]$$

= the change in the amount of water in the tank between time 0 and time 24 ■

SKILL
interpret def int

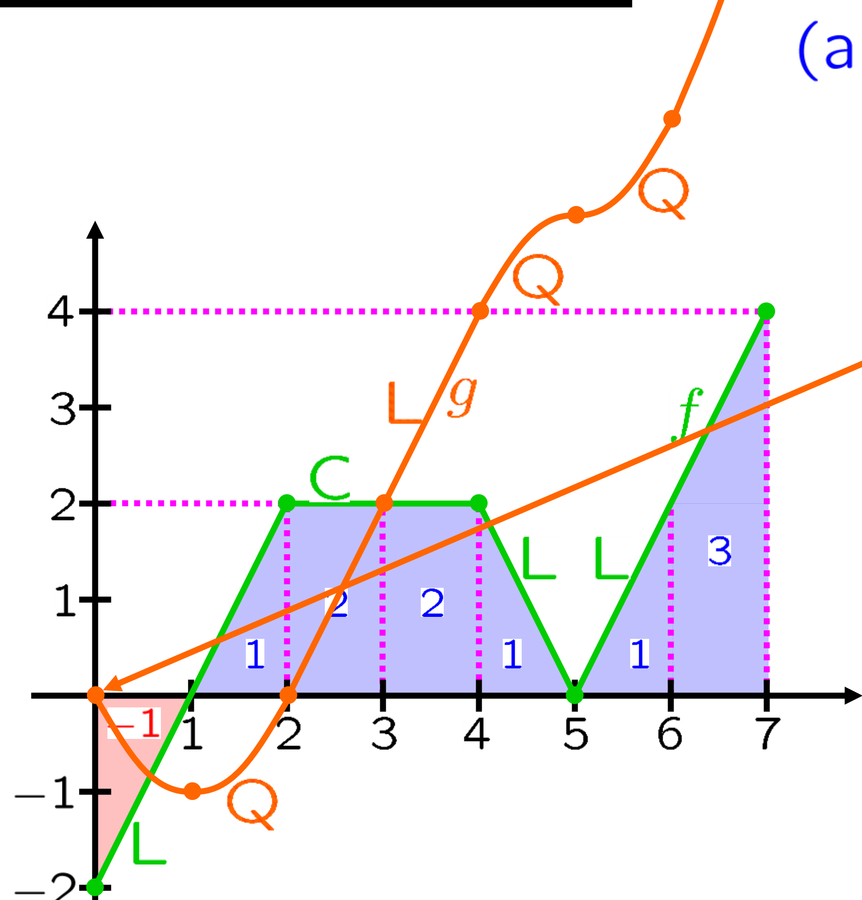
EXERCISE: Let f be the function whose graph is shown below and let $g(x) := \int_0^x f(s)\,ds$.

(a) Compute $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$, $g(5)$, $g(6)$, $g(7)$.

(b) Where does g attain a maximum value?

(c) Where does g attain a minimum value?

(d) Sketch the graph of g.



(a)
$g(0) = 0$ $(0,0)$
$g(1) = 0 - 1 = -1$ $(1,-1)$
$g(2) = -1 + 1 = 0$ $(2,0)$
$g(3) = 0 + 2 = 2$ $(3,2)$
$g(4) = 2 + 2 = 4$ $(4,4)$
$g(5) = 4 + 1 = 5$ $(5,5)$
$g(6) = 5 + 1 = 6$ $(6,6)$
$g(7) = 6 + 3 = 9$ $(7,9)$

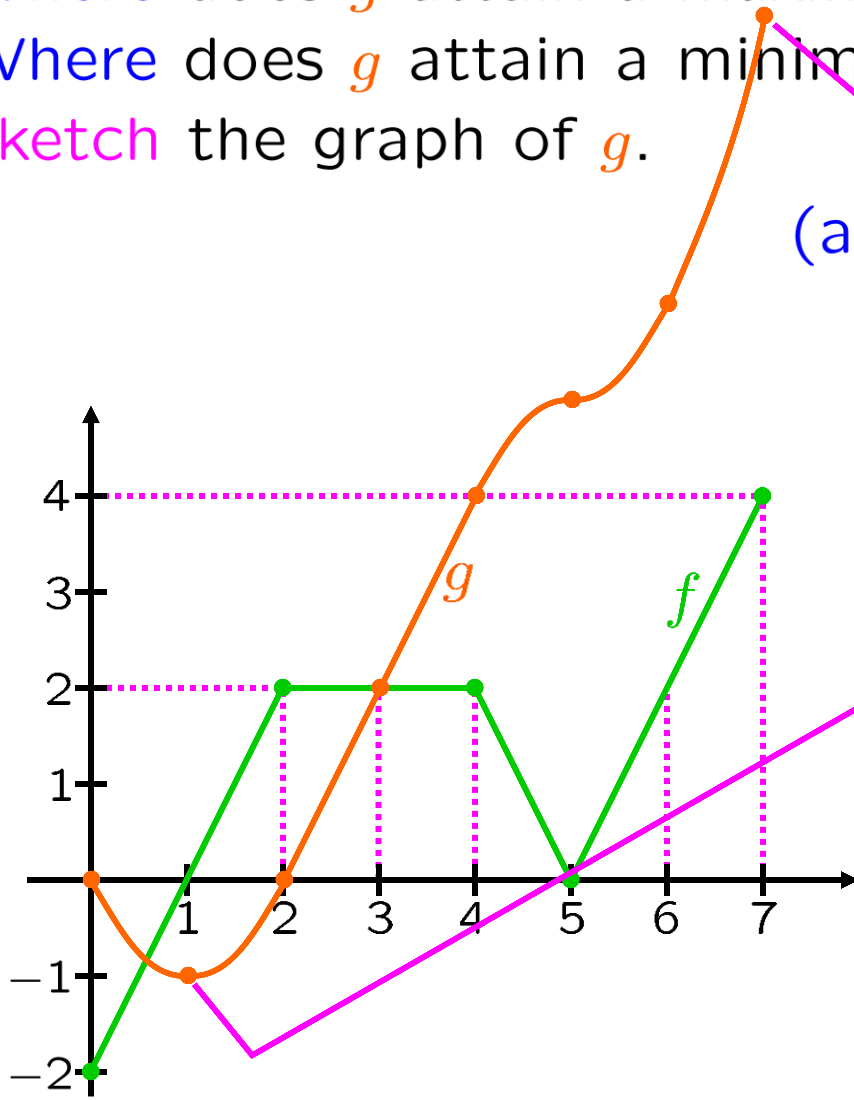
EXERCISE: Let f be the function whose graph is shown below and let $g(x) := \int_0^x f(s)\,ds$.

(a) Compute $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$, $g(5)$, $g(6)$, $g(7)$.

(b) Where does g attain a maximum value?

(c) Where does g attain a minimum value?

(d) Sketch the graph of g.



(a) $g(0) = 0$

$g(1) = 0 - 1 = -1$

$g(2) = -1 + 1 = 0$

$g(3) = 0 + 2 = 2$

$g(4) = 2 + 2 = 4$

$g(5) = 4 + 1 = 5$

$g(6) = 5 + 1 = 6$

$g(7) = 6 + 3 = 9$

(c) at 1

(b) at 7

SKILL
integrate gph

SKILL

fund th'm of calc

Whitman problems

§7.2, p. 150, #1-20

STOP